

# 具有边值条件的高维吉洪诺夫系统的阶梯状 空间对照结构\*

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**提要** 借助于首次积分构造高维的空间异宿轨道, 利用指数二分法的一些性质和 Fredholm 交换引理, 在求解高阶边界函数的同时确定了转移点  $t^*$ . 利用边界函数法构造形式渐近解, 用  $k + \sigma$  交换引理证明了高维吉洪诺夫系统阶梯状空间对照结构解的存在性和形式渐近解的一致有效性. 最后举例验证本文的结果.

**关键词** 空间对照结构, 奇异摄动, 渐近展开, 边界函数

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## 1 引言

考虑高维吉洪诺夫系统

$$\mu \frac{dz}{dt} = f(z, y, t), \quad \frac{dy}{dt} = g(z, y, t), \quad 0 \leq t \leq 1, \quad (1.1)$$

$$Az(0, \mu) = Az^0, \quad Bz(1, \mu) = Bz^1, \quad Cy(0, \mu) = Cy^0, \quad Dy(1, \mu) = Dy^1, \quad (1.2)$$

阶梯状空间对照结构解的存在性. 这里  $\mu > 0$  是小参数.  $z, f$  是  $M$  维向量,  $y, g$  是  $m$  维向量.  $A = \begin{pmatrix} E_{M_1} & 0 \\ 0 & E_{M_2} \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 & 0 \\ 0 & E_{M_2} \end{pmatrix}$  是  $M \times M$  阶矩阵.  $E_{M_1}$  是  $M_1 \times M_1$  阶矩阵,  $E_{M_2}$  是  $M_2 \times M_2$  阶矩阵.  $M_1 + M_2 = M$ .  $C = \begin{pmatrix} E_{m_1} & 0 \\ 0 & 0 \end{pmatrix}$ ,  $D = \begin{pmatrix} 0 & 0 \\ 0 & E_{m_2} \end{pmatrix}$  是  $m \times m$  阶矩阵.  $E_{m_1}$  是  $m_1 \times m_1$  阶矩阵,  $E_{m_2}$  是  $m_2 \times m_2$  阶矩阵.  $m_1 + m_2 = m$ .

奇异摄动系统的内部转移层解又称为空间对照结构, 它的基本特征是在所讨论的区间上存在一个内部转移点  $t^*$  (或者多个  $t^*$ ), 其位置事先是未知的, 需要在渐近解的构造过程中来确定. 在  $t^*$  的某个小邻域内, 问题的解  $y(t, \mu)$  会发生剧烈的结构变化. 近年来, 具有空间对照结构的奇摄动问题成为奇摄动理论研究的主流, 受到了国内外学者的广泛关注. 尤其是俄罗斯学派, 在这方面做了大量工作, 主要研究了半线性边值问题阶梯状空间对照结构<sup>[1]</sup>和脉冲状空间对照结构<sup>[2]</sup>, 拟线性方程<sup>[3]</sup>和拟线性方程组<sup>[4]</sup>的阶梯状空间对照结构, 弱非线性方程的阶梯状空间对照结构<sup>[5]</sup>和脉冲状空间对照结构<sup>[6]</sup>, 奇性相同的方程组的阶梯状空间对照结构<sup>[7-8]</sup>, 奇性不同的方程组的空间对照结构<sup>[9]</sup>, 这些工作主要涉及平面二阶方程和方程组的边值问题, 对高维系统研究较少. 只有个案涉及到三阶系统和特定的边值<sup>[10]</sup>. 欧美学派采用动力系统方法和几何方法对该类问题进行过探讨<sup>[11-12]</sup>, 目前我国对该领域的研究也有了一定的结果<sup>[13-14]</sup>.

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## 2 基本假设

(H<sub>1</sub>) 假设  $f(z, y, t)$  和  $g(z, y, t)$  在变量  $(z, y, t)$  空间的某个开区域  $G(z, y, t)$  内关于所有变量都具有  $n+2$  阶连续偏导数.

在 (1.1) 中令  $\mu = 0$ , 可得退化方程

$$f(\bar{z}, \bar{y}, t) = 0, \quad \frac{d\bar{y}}{dt} = g(\bar{z}, \bar{y}, t). \quad (2.1)$$

(H<sub>2</sub>) 假设退化方程  $f(\bar{z}, \bar{y}, t) = 0$  在  $\bar{D}$  上有两个退化根:

$$\bar{z} = \alpha(\bar{y}, t), \quad \bar{z} = \beta(\bar{y}, t),$$

这里

$$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_M)^T, \quad \beta = (\beta_1, \beta_2, \dots, \beta_M)^T.$$

(H<sub>3</sub>) 假设  $F_z(\alpha(\bar{y}(t), t), \bar{y}(t), t)$  和  $F_z(\beta(\bar{y}(t), t), \bar{y}(t), t)$  分别有特征根  $\bar{\lambda}_i^\alpha(t)$  和  $\bar{\lambda}_i^\beta(t)$  ( $i = 1, 2, \dots, M$ ), 这里  $0 \leq t \leq 1$ , 且满足

$$\begin{aligned} \operatorname{Re} \bar{\lambda}_i^j(t) < 0, & \quad i = 1, 2, \dots, k < M, \\ \operatorname{Re} \bar{\lambda}_i^j(t) > 0, & \quad i = k + 1, k + 2, \dots, M, \end{aligned}$$

其中  $j = \alpha, \beta$ .

假设 (H<sub>3</sub>) 说明, 对  $\bar{t} \in [0, 1]$  暂且固定, 问题 (1.1) 的辅助系统

$$\frac{d\tilde{z}}{d\tau} = f(\tilde{z}, \bar{y}(\bar{t}), \bar{t}) \quad (2.2)$$

的两个平衡点  $(\alpha(\bar{y}^{(-)}(\bar{t}), \bar{t}), \bar{y}^{(-)}(\bar{t}))$  和  $(\beta(\bar{y}^{(+)}(\bar{t}), \bar{t}), \bar{y}^{(+)}(\bar{t}))$  在  $(\tilde{y}, \tilde{z})$  上都是两个双曲鞍点.

## 3 形式渐近解的构造

设  $t^*(t^* \in (0, 1))$ , 它具有如下展开式

$$t^* = t_0 + \mu t_1 + \mu^2 t_2 + \dots + \mu^k t_k + \dots,$$

其中  $t_k (k = 0, 1, 2, \dots)$  是待定常数.

可以认为所设想的阶梯状解是由下面两个纯边界层的解复合而成.

左问题 ( $0 \leq t \leq t^*$ ):

$$\mu \frac{dz^{(-)}}{dt} = f(z^{(-)}, y^{(-)}, t), \quad \frac{dy^{(-)}}{dt} = g(z^{(-)}, y^{(-)}, t), \quad (3.1)$$

$$Az^{(-)}(0, \mu) = Az^0, \quad Bz^{(-)}(t^*, \mu) = Bz^*, \quad (3.2)$$

$$Cy^{(-)}(0, \mu) = y^0, \quad Dy^{(-)}(t^*, \mu) = Dy^*. \quad (3.3)$$

右问题 ( $t^* \leq t \leq 1$ ):

$$\mu \frac{dz^{(+)}}{dt} = f(z^{(+)}, y^{(+)}, t), \quad \frac{dy^{(+)}}{dt} = g(z^{(+)}, y^{(+)}, t), \quad (3.4)$$

$$Az^{(+)}(t^*, \mu) = Az^*, \quad Bz^{(+)}(1, \mu) = Bz^1, \quad (3.5)$$

$$Cy^{(+)}(t^*, \mu) = Cy^*, \quad Dy^{(+)}(1, \mu) = Dy^1, \quad (3.6)$$

这里  $z^*, y^*$  都是参数, 且与  $t^*$  有关. 为方便起见, 仍设

$$z^* = z_0^* + \mu z_1^* + \cdots, \quad y^* = y_0^* + \mu y_1^* + \cdots.$$

令  $x = (z, y)^T$ , 要想得到从  $\alpha(t)$  到  $\beta(t)$  的阶梯解, 必须满足下面的缝接条件

$$x^{(-)}(t^*, \mu) = x^{(+)}(t^*, \mu). \quad (3.7)$$

设左右问题的形式渐近解分别为

$$x^{(-)}(t, \mu) = \sum_{j=0}^{\infty} \mu^j (\bar{x}_j^{(-)}(t) + L_j x(\tau_0) + Q_j^{(-)} x(\tau)) \quad (3.8)$$

和

$$x^{(+)}(t, \mu) = \sum_{j=0}^{\infty} \mu^j (\bar{x}_j^{(+)}(t) + Q_j^{(+)} x(\tau) + R_j x(\tau_1)), \quad (3.9)$$

其中  $\tau_0 = \frac{t}{\mu} > 0$ ,  $\tau = \frac{t-t^*}{\mu}$ ,  $\tau_1 = \frac{t-1}{\mu} < 0$ .  $\bar{x}_j^{(\mp)}(t)$  是正则项系数;  $L_j x(\tau_0)$  是左边界函数项;  $Q_j^{(\mp)} x(\tau)$  是  $t = t^*$  处的边界函数项,  $R_j x(\tau_1)$  是右边界函数项. 且

$$\lim_{\tau_0 \rightarrow +\infty} L_j x(\tau_0) = 0, \quad \lim_{\tau \rightarrow \mp\infty} Q_j^{(\mp)} x(\tau) = 0, \quad \lim_{\tau_1 \rightarrow -\infty} R_j x(\tau_1) = 0 \quad (j = 0, 1, 2, \cdots).$$

将 (3.8)–(3.9) 分别代入 (3.1)–(3.6), 由边界函数法, 可知

$$f(\bar{z}_0^{(\mp)}(t), \bar{y}_0^{(\mp)}(t), t) = 0.$$

根据假设 (H<sub>2</sub>),

$$z_0^{(-)}(t) = \alpha(\bar{y}_0^{(-)}(t), t), \quad z_0^{(+)}(t) = \beta(\bar{y}_0^{(+)}(t), t).$$

而  $\bar{y}_0^{(\mp)}(t)$  满足的方程和边值分别为

$$\frac{d\bar{y}_0^{(-)}(t)}{dt} = g(\alpha(\bar{y}_0^{(-)}(t), t), \bar{y}_0^{(-)}(t), t), \quad (3.10)$$

$$C\bar{y}_0^{(-)}(0) = Cy^0, \quad D\bar{y}_0^{(-)}(t_0) = Dy_0^* \quad (3.11)$$

和

$$\frac{d\bar{y}_0^{(+)}(t)}{dt} = g(\beta(\bar{y}_0^{(+)}(t), t), \bar{y}_0^{(+)}(t), t), \quad (3.12)$$

$$C\bar{y}_0^{(+)}(t_0) = C\bar{y}_0^*, \quad D\bar{y}_0^{(+)}(1) = D\bar{y}_0^1. \quad (3.13)$$

(H<sub>4</sub>) 假设 (3.10)–(3.11) 的解和 (3.12)–(3.13) 的解在  $y_0^*$  处横截相交, 这里  $y_0^* = y_0^*(t_0)$ . 决定  $Q_0^{(\mp)} x(\tau)$  的方程和定解条件为

$$\frac{dQ_0^{(-)} z}{d\tau} = f(\alpha(t_0) + Q_0^{(-)} z(\tau), \bar{y}_0^{(-)}(t_0) + Q_0^{(-)} y(\tau), t_0), \quad \frac{dQ_0^{(-)} y}{d\tau} = 0; \quad (3.14)$$

$$B(\bar{z}_0^{(-)}(t_0) + Q_0^{(-)} z(0)) = Bz_0^*, \quad Q_0^{(-)} x(-\infty) = 0 \quad (3.15)$$

和

$$\frac{dQ_0^{(+)} z}{d\tau} = f(\beta(t_0) + Q_0^{(+)} z(\tau), \bar{y}_0^{(+)}(t_0) + Q_0^{(+)} y(\tau), t_0), \quad \frac{dQ_0^{(+)} y}{d\tau} = 0. \quad (3.16)$$

$$A(\bar{z}_0^{(+)}(t_0) + Q_0^{(+)} z(0)) = Az_0^*, \quad Q_0^{(+)} x(+\infty) = 0. \quad (3.17)$$

显然,  $Q_0^{(\mp)}y(\tau) \equiv 0$ . 记

$$\alpha(t_0) + Q_0^{(-)}z(\tau) = \tilde{z}^{(-)}(\tau), \quad \beta(t_0) + Q_0^{(+)}z(\tau) = \tilde{z}^{(+)}(\tau).$$

则

$$\frac{d\tilde{z}^{(-)}}{d\tau} = f(\tilde{z}^{(-)}, \bar{y}_0^{(-)}(t_0), t_0), \quad B\tilde{z}^{(-)}(0) = Bz_0^*, \quad \tilde{z}^{(-)}(-\infty) = \alpha(t_0), \quad (3.18)$$

$$\frac{d\tilde{z}^{(+)}}{d\tau} = f(\tilde{z}^{(+)}, \bar{y}_0^{(+)}(t_0), t_0). \quad A\tilde{z}^{(+)}(0) = Az_0^*, \quad \tilde{z}^{(+)}(+\infty) = \beta(t_0), \quad (3.19)$$

显然, (3.18)–(3.19) 与 (2.2) 一致, 所以平衡点  $M_-(\alpha(\bar{y}_0^{(-)}(t_0), t_0), \bar{y}_0^{(-)}(t_0))$ ,  $M_+(\beta(\bar{y}_0^{(+)}(t_0), t_0), \bar{y}_0^{(+)}(t_0))$  都是双曲鞍点.

(H<sub>5</sub>) 假设对固定的  $t_0 \in (0, 1)$ , 辅助系统 (2.2) 存在  $M-1$  个相互独立的首次积分

$$\Phi_j(\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_M, t_0) = C_j, \quad j = 1, 2, \dots, M-1, \quad (3.20)$$

其中  $C_j (j = 1, \dots, M-1)$  是独立的参数.

过平衡点  $M_-(t_0)$  的轨线是

$$\Phi_j(\tilde{z}_1^{(-)}, \tilde{z}_2^{(-)}, \tilde{z}_3^{(-)}, \dots, \tilde{z}_M^{(-)}, t_0) = \Phi_j(M_-(t_0), t_0), \quad j = 1, 2, \dots, M-1. \quad (3.21)$$

过平衡点  $M_+(t_0)$  的轨线是

$$\Phi_j(\tilde{z}_1^{(+)}, \tilde{z}_2^{(+)}, \tilde{z}_3^{(+)}, \dots, \tilde{z}_M^{(+)}, t_0) = \Phi_j(M_+(t_0), t_0), \quad j = 1, 2, \dots, M-1. \quad (3.22)$$

从 (3.21)–(3.22) 可得连接平衡点  $M_-$  和  $M_+$  的异宿轨道的必要条件

$$\Phi_j(M_-(t_0), t_0) = \Phi_j(M_+(t_0), t_0), \quad j = 1, 2, \dots, M-1. \quad (3.23)$$

(3.23) 是求  $t_0$  的方程.

(H<sub>6</sub>) 假设 (3.23) 是相容的, 且关于  $t_0$  有解,  $t_0 = \bar{t}_0$ .

如果假设 (H<sub>6</sub>) 的相容性条件成立, 则存在从  $M_-(\bar{t}_0)$  出发进入  $M_+(\bar{t}_0)$  的异宿轨道.

**引理 3.1** 如果假设 (H<sub>1</sub>)–(H<sub>6</sub>) 成立, 则问题 (3.18)–(3.19) 的解  $Q_0^{(\mp)}z(\tau)$  存在, 且满足指数衰减, 即

$$Q_0^{(-)}z(\tau) \leq Ce^{\gamma\tau}, \quad \tau < 0,$$

$$Q_0^{(+)}z(\tau) \geq Ce^{-\gamma\tau}, \quad \tau > 0,$$

其中  $C, \gamma$  是任意正常数.

记

$$\tilde{u}^{(\mp)} = (\tilde{z}_1^{(\mp)}, \tilde{z}_2^{(\mp)}, \dots, \tilde{z}_k^{(\mp)})^T, \quad \tilde{v}^{(\mp)} = (\tilde{z}_{k+1}^{(\mp)}, \tilde{z}_{k+2}^{(\mp)}, \dots, \tilde{z}_M^{(\mp)})^T.$$

过平衡点  $M_-$  的  $M-k$  维不稳定流形记为

$$\tilde{u}^{(-)}(\tau) = \phi_u(\tilde{v}^{(-)}(\tau)),$$

过平衡点  $M_+$  的  $k$  维不稳定流形记为

$$\tilde{v}^{(+)}(\tau) = \phi_v(\tilde{u}^{(+)}(\tau)).$$

决定  $L_0x(\tau_0)$  和  $R_0x(\tau_1)$  的方程和定解条件分别为

$$\frac{dL_0z}{d\tau_0} = f(\alpha(0) + L_0z(\tau_0), \bar{y}_0^{(-)}(0) + L_0y(\tau_0), 0), \quad \frac{dL_0y}{d\tau_0} = 0;$$

$$A(\bar{z}_0^{(-)}(0) + L_0 z(0)) = Az^0, \quad L_0 x(+\infty) = 0$$

和

$$\frac{dR_0 z}{d\tau_1} = f(\beta(1) + R_0 z(\tau_1), \bar{y}_0^{(+)}(1) + R_0 y(\tau_1), 1), \quad \frac{dL_0 y}{d\tau_0} = 0;$$

$$B(\bar{z}_0^{(+)}(1) + R_0 z(0)) = Bz^1, \quad R_0 x(-\infty) = 0.$$

易知

$$L_0 y(\tau_0) \equiv 0, \quad R_0 y(\tau_1) \equiv 0.$$

令

$$\alpha(0) + L_0 z(\tau_0) = \tilde{z}^l(\tau_0), \quad \beta(1) + R_0 z(\tau_1) = \tilde{z}^r(\tau_1),$$

则上述方程化为

$$\frac{d\tilde{z}^l}{d\tau_0} = f(\tilde{z}^l, \bar{y}_0^{(-)}(0), 0), \quad A\tilde{z}^l(0) = Az^0, \quad \tilde{z}^l(-\infty) = \alpha(0), \quad (3.24)$$

$$\frac{d\tilde{z}^r}{d\tau_1} = f(\tilde{z}^r, \bar{y}_0^{(+)}(1), 1), \quad B\tilde{z}^r(0) = Bz^1, \quad \tilde{z}^r(-\infty) = \beta(1). \quad (3.25)$$

显然, (3.24)–(3.25) 与辅助系统 (2.2) 一致. 由 (H<sub>3</sub>), (3.24) 的平衡点  $M^l(\alpha(0))$ , (3.25) 的平衡点  $M^r(\beta(1))$  均为双曲鞍点. 为保证  $L_0 z(\tau_0)$ ,  $R_0 z(\tau_1)$  的存在性, 我们给出下面条件.

(H<sub>7</sub>) 假设流形  $A\tilde{z}^l(0) = Az^0$  与过  $M_1$  的稳定流形  $W^s(M^l(0))$  横截相交, 流形  $B\tilde{z}^r(0) = Bz^1$  与过  $M_2$  的不稳定流形  $W^u(M^r(1))$  横截相交.

**引理 3.2** 在假设 (H<sub>1</sub>)–(H<sub>4</sub>) 和 (H<sub>7</sub>) 下, 方程 (3.24)–(3.25) 的解  $L_0 z(\tau_0)$ ,  $R_0 z(\tau_1)$  存在, 且

$$L_0 z(\tau_0) \leq Ce^{-\gamma\tau_0}, \quad \tau_0 > 0, \quad R_0 z(\tau_1) \leq Ce^{\gamma\tau_1}, \quad \tau_1 < 0,$$

这里  $C, \gamma$  是任意常数.

对于正则项系数  $\bar{x}_j^{(\mp)}(t)$  ( $j = 1, 2, \dots$ ), 它们满足的方程和边界条件为

$$\frac{d\bar{z}_{j-1}^{(-)}(t)}{dt} = \bar{f}_z^{(-)}(t)\bar{z}_j^{(-)}(t) + \bar{f}_y^{(-)}(t)\bar{y}_j^{(-)}(t) + f_j^{(-)}(t), \quad (3.26)$$

$$\frac{d\bar{y}_j^{(-)}(t)}{dt} = \bar{g}_z^{(-)}(t)\bar{z}_j^{(-)}(t) + \bar{g}_y^{(-)}(t)\bar{y}_j^{(-)}(t) + g_j^{(-)}(t), \quad (3.27)$$

$$C(\bar{y}_j^{(-)}(0) + L_j y(0)) = 0, \quad D(\bar{y}_0^{(-)}(t_0)t_j + \gamma^{(-)} + Q_j^{(-)}y(0)) = Dy_j^* \quad (3.28)$$

和

$$\frac{d\bar{z}_{j-1}^{(+)}(t)}{dt} = \bar{f}_z^{(+)}(t)\bar{z}_j^{(+)}(t) + \bar{f}_y^{(+)}(t)\bar{y}_j^{(+)}(t) + f_j^{(+)}(t), \quad (3.29)$$

$$\frac{d\bar{y}_j^{(+)}(t)}{dt} = \bar{g}_z^{(+)}(t)\bar{z}_j^{(+)}(t) + \bar{g}_y^{(+)}(t)\bar{y}_j^{(+)}(t) + g_j^{(+)}(t), \quad (3.30)$$

$$C(\bar{y}_0^{(+)}(t_0)t_j + \gamma^{(+)} + Q_j^{(+)}y(0)) = Cy_j^*, \quad D(\bar{y}_j^{(+)}(1) + R_j y(0)) = 0, \quad (3.31)$$

其中  $\bar{f}_z^{(-)}(t)$ ,  $\bar{f}_z^{(+)}(t)$  分别在  $(\alpha(t), \bar{y}_0^{(-)}(t), t)$  和  $(\beta(t), \bar{y}_0^{(+)}(t), t)$  取值.  $\bar{f}_y^{(\mp)}(t)$ ,  $\bar{g}_z^{(\mp)}(t)$  和  $\bar{g}_y^{(\mp)}(t)$  也有相同的意义.  $f_j^{(\mp)}(t)$ ,  $g_j^{(\mp)}(t)$  均由已知函数复合而成,  $\gamma_j^{(\mp)}$  是已知量.

决定高次项  $L_j x(\tau_0)$  的方程和边界条件为

$$\frac{dL_j z}{d\tau_0} = \tilde{f}_z^{(l)}(\tau_0)L_j z + \tilde{f}_y^{(l)}(\tau_0)L_j y + H_j^{(l)}(\tau_0), \quad \frac{dL_j y}{d\tau_0} = L_{j-1}g(\tau_0), \quad (3.32)$$

$$A(\bar{z}_j^{(-)}(0) + L_j z(0)) = 0, \quad L_j x(+\infty) = 0, \quad (3.33)$$

其中

$$\tilde{f}_x^{(l)}(\tau_0) = f_x(\alpha(0) + L_0 z, \bar{y}_0^{(-)}(0), 0),$$

而  $H_j^{(l)}(\tau_0)$ ,  $L_{j-1}g(\tau_0)$  由已知函数复合而成.

由 (3.32) 和  $L_j y(+\infty) = 0$  得

$$L_j y(\tau_0) = \int_{+\infty}^{\tau_0} L_{j-1}g(s)ds.$$

故

$$L_j y(0) = \int_{+\infty}^0 L_{j-1}g(s)ds.$$

由 (3.28) 的第 1 个等式

$$C\bar{y}_j^{(-)}(0) = -CL_j y(0) = \int_0^{+\infty} CL_{j-1}g(s)ds. \quad (3.34)$$

决定高次项系数  $R_j x(\tau_1)$  的方程和边界条件

$$\frac{dR_j z}{d\tau_1} = \tilde{f}_z^{(r)}(\tau_1)R_j z + \tilde{f}_y^{(r)}(\tau_1)R_j y + H_j^{(r)}(\tau_1), \quad \frac{dR_j y}{d\tau_1} = R_{j-1}g(\tau_1), \quad (3.35)$$

$$B(\bar{z}_j^{(+)}(1) + R_j z(0)) = 0, \quad R_j x(-\infty) = 0,$$

其中

$$\tilde{f}_x^{(r)}(\tau_1) = f_x(\beta(1) + R_0 z, \bar{y}_0^{(+)}(1), 1).$$

而  $H_j^{(r)}(\tau_1)$ ,  $R_{j-1}g(\tau_1)$  由已知函数复合而成.

由 (3.35) 和  $R_j y(-\infty) = 0$  可得

$$R_j y(\tau_1) = \int_{-\infty}^{\tau_1} R_{j-1}g(s)ds.$$

故

$$R_j y(0) = \int_{+\infty}^0 R_{j-1}g(s)ds.$$

由 (3.31) 的第 2 个方程, 可得

$$D\bar{y}_j^{(+)}(1) = -DR_j y(0) = \int_0^{-\infty} DR_{j-1}g(s)ds.$$

进而  $\bar{x}_j^{(\mp)}(t)$  存在.

(H<sub>8</sub>) 假设  $\bar{x}_j^{(-)}(t)$  和  $\bar{x}_j^{(+)}(t)$  在点  $y_j^*$  横截相交.

决定  $Q_j^{(-)}x(\tau)$  的方程和定解条件为

$$\frac{dQ_j^{(-)}z}{d\tau} = f_z^{(-)}(\tau)Q_j^{(-)}z + f_y^{(-)}(\tau)Q_j^{(-)}y + G_j^{(-)}(\tau), \quad \frac{dQ_j^{(-)}y}{d\tau} = Q_{j-1}^{(-)}g(\tau), \quad (3.36)$$

$$BQ_j^{(-)}z(0) = Bz_j^* - B\bar{z}_0^{(-)}(t_0)t_j + B\rho_j^{(-)}, \quad Q_j^{(-)}x(-\infty) = 0, \quad (3.37)$$

其中

$$G_j^{(-)}(\tau) = (\Delta f_z^{(-)}(\tau)\alpha'(t_0) + \Delta f_y^{(-)}(\tau)(\bar{y}_0^{(-)}(t_0))' + \Delta f_t^{(-)}(\tau)t_j + \bar{G}_j^{(-)}(\tau),$$

而

$$f_x^{(-)}(\tau) = f_x(\alpha(t_0) + Q_0^{(-)}z(\tau), \bar{y}_0^{(-)}(t_0), t_0),$$

$$\Delta f_x^{(-)}(\tau) = f_x(\alpha(t_0) + Q_0^{(-)}z, \bar{y}_0^{(-)}(t_0), t_0) - f_x(\alpha(t_0), \bar{y}_0^{(-)}(t_0), t_0),$$

$\Delta f_t^{(-)}(\tau)$  记法类似.  $\bar{G}_j^{(-)}(\tau)$ ,  $Q_{j-1}^{(-)}g(\tau)$  是不含  $t_j$  的已知函数的复合.  $\rho_j^{(-)}$  是已知量.

由  $Q_j^{(-)}y(-\infty) = 0$  和 (3.36), 可知

$$Q_j^{(-)}y(\tau) = \int_{-\infty}^{\tau} Q_{j-1}^{(-)}g(s)ds.$$

将其代入 (3.36) 的第 1 个方程, 并结合初始条件 (3.37), 可得  $Q_j^{(-)}z(\tau)$ , 且它与  $t_j$  有关.

决定  $Q_j^{(+)}x(\tau)$  的方程和定解条件为

$$\frac{dQ_j^{(+)}z}{d\tau} = f_z^{(+)}(\tau)Q_j^{(+)}z + f_y^{(+)}(\tau)Q_j^{(+)}y + G_j^{(+)}(\tau), \quad \frac{dQ_j^{(+)}y}{d\tau} = Q_{j-1}^{(+)}g(\tau), \quad (3.38)$$

$$AQ_j^{(+)}z(0) = Az_j^* - A\bar{z}_0^{(+)}(t_0)t_j + A\rho_j^{(+)}, \quad Q_j^{(+)}x(+\infty) = 0, \quad (3.39)$$

其中

$$G_j^{(+)}(\tau) = (\Delta f_z^{(+)}(\tau)\beta'(t_0) + \Delta f_y^{(+)}(\tau)(\bar{y}_0^{(+)}(t_0))' + \Delta f_t^{(+)}(\tau)t_j + \bar{G}_j^{(+)}(\tau).$$

而

$$f_x^{(+)}(\tau) = f_x(\beta(t_0) + Q_0^{(+)}z(\tau), \bar{y}_0^{(+)}(t_0), t_0),$$

$$\Delta f_x^{(+)}(\tau) = f_x(\beta(t_0) + Q_0^{(+)}z, \bar{y}_0^{(+)}(t_0), t_0) - f_x(\beta(t_0), \bar{y}_0^{(+)}(t_0), t_0),$$

$\Delta f_t^{(+)}(\tau)$  记法类似.  $\bar{G}_j^{(+)}(\tau)$ ,  $Q_{j-1}^{(+)}g(\tau)$  是不含  $t_j$  的已知函数的复合.  $\rho_j^{(+)}$  是已知量.

由  $Q_j^{(+)}y(+\infty) = 0$  和 (3.38), 可知

$$Q_j^{(+)}y(\tau) = \int_{+\infty}^{\tau} Q_{j-1}^{(+)}g(s)ds.$$

将其代入 (3.38) 的第 1 个方程, 并结合初始条件 (3.39) 可得  $Q_j^{(+)}z(\tau)$ , 且与  $t_j$  有关.

下面写出  $Q_j^{(\mp)}z(\tau)$  的具体的解析表达式. 因为  $Q_j^{(\mp)}y(\tau)$  已经解出. 现将 (3.36), (3.38) 的第 1 个方程重新记为

$$\frac{dQ_j^{(\mp)}z}{d\tau} = f_z^{(\mp)}(\tau)Q_j^{(\mp)}z + \tilde{G}_j^{(\mp)}(\tau), \quad (3.40)$$

其中

$$\begin{aligned} \tilde{G}_j^{(\mp)}(\tau) &= f_y^{(\mp)}(\tau)Q_j^{(\mp)}y + (\Delta f_z^{(\mp)}(\tau)\beta'(t_0) + \Delta f_y^{(\mp)}(\tau)(\bar{y}_0^{(\mp)}(t_0))' \\ &\quad + \Delta f_t^{(\mp)}(\tau)t_j + \bar{G}_j^{(\mp)}(\tau). \end{aligned}$$

将其写成分块的形式

$$\begin{pmatrix} \frac{dQ_j^{(\mp)}u}{d\tau} \\ \frac{dQ_j^{(\mp)}v}{d\tau} \end{pmatrix} = \begin{pmatrix} F_{11}^{(\mp)}(\tau) & F_{12}^{(\mp)}(\tau) \\ F_{21}^{(\mp)}(\tau) & F_{22}^{(\mp)}(\tau) \end{pmatrix} \begin{pmatrix} Q_j^{(\mp)}u(\tau) \\ Q_j^{(\mp)}v(\tau) \end{pmatrix} + \begin{pmatrix} \psi_1^{(\mp)}(\tau) \\ \psi_2^{(\mp)}(\tau) \end{pmatrix},$$

经计算, 可得

$$\begin{aligned} Q_j^{(-)}u &= H^{(-)}(\tau)(\Delta_j^0)^{(-)}\Phi^{(-)}(\tau)(\Phi^{(-)}(0))^{-1} + H^{(-)}\int_0^\tau \Phi^{(-)}(\tau)(\Phi^{(-)}(s))^{-1} \\ &\quad \cdot \left[ F_{21}^{(-)}(s)\int_{-\infty}^s \Psi^{(-)}(\tau)(\Psi^{(-)}(\xi))^{-1}(\psi_1^{(-)}(\xi) - H^{(-)}(\xi)\psi_2^{(-)}(\xi))d\xi \right. \\ &\quad \left. + \psi_2^{(-)}(s) \right] ds + \int_{-\infty}^\tau \Psi^{(-)}(\tau)(\Psi^{(-)}(s))^{-1}(\psi_1^{(-)}(s) - H^{(-)}(s)\psi_2^{(-)}(s))ds, \\ Q_j^{(-)}v &= (\Delta_j^0)^{(-)}\Phi^{(-)}(\tau)(\Phi^{(-)}(0))^{-1} + \int_0^\tau \Phi^{(-)}(\tau)(\Phi^{(-)}(s))^{-1} \\ &\quad \cdot \left[ F_{21}^{(-)}(s)\int_{-\infty}^s \Psi^{(-)}(\tau)(\Psi^{(-)}(\xi))^{-1}(\psi_1^{(-)}(\xi) - H^{(-)}(\xi)\psi_2^{(-)}(\xi))d\xi \right. \\ &\quad \left. + \psi_2^{(-)}(s) \right] ds, \end{aligned}$$

其中  $(\Delta_j^0)^{(-)} = Bz_j^* - B\bar{z}_0^{(-)'}(t_0)t_j + B\rho_j^{(-)}(\tau)$ ,  $H^{(-)}(\tau) = \frac{\partial\phi_u}{\partial Q_j^{(-)}v}$ . 而  $\Phi^{(-)}(\tau)$  和  $\Psi^{(-)}(\tau)$  分别是

$$\frac{dQ_0^{(-)}u}{d\tau} = (F_{21}(\tau)H^{(-)}(\tau) + F_{22}(\tau))Q_0^{(-)}u, \quad \Phi^{(-)}(0) = E_k$$

和

$$\frac{dQ_0^{(-)}v}{d\tau} = (F_{11}(\tau) - H^{(-)}(\tau)F_{21}(\tau))Q_0^{(-)}v, \quad \Psi^{(-)}(0) = E_{M-k}$$

的解.

$$\begin{aligned} Q_j^{(+)}u &= (\Delta_j^0)^{(+)}\Phi^{(+)}(\tau)(\Phi^{(+)}(0))^{-1} + \int_0^\tau \Phi^{(+)}(\tau)(\Phi^{(+)}(s))^{-1} \\ &\quad \cdot \left[ F_{12}^{(+)}(s)\int_{+\infty}^s \Psi^{(+)}(\tau)(\Psi^{(+)}(\xi))^{-1}(\psi_2^{(+)}(\xi) - H^{(+)}(\xi)\psi_1^{(+)}(\xi))d\xi \right. \\ &\quad \left. + \psi_1^{(+)}(s) \right] ds, \\ Q_j^{(+)}v &= H^{(+)}(\tau)(\Delta_j^0)^{(+)}\Phi^{(+)}(\tau)(\Phi^{(+)}(0))^{-1} + H^{(+)}\int_0^\tau \Phi^{(+)}(\tau)(\Phi^{(+)}(s))^{-1} \\ &\quad \cdot \left[ F_{12}^{(+)}(s)\int_{+\infty}^s \Psi^{(+)}(\tau)(\Psi^{(+)}(\xi))^{-1}(\psi_2^{(+)}(\xi) - H^{(+)}(\xi)\psi_1^{(+)}(\xi))d\xi \right. \\ &\quad \left. + \psi_1^{(+)}(s) \right] ds + \int_{+\infty}^\tau \Psi^{(+)}(\tau)(\Psi^{(+)}(s))^{-1}(\psi_2^{(+)}(s) - H^{(+)}(s)\psi_1^{(+)}(s))ds, \end{aligned}$$

其中  $(\Delta_j^0)^{(+)} = Az_j^* - A\bar{z}_0^{(+)'}(t_0)t_j + A\rho_j^{(+)}(\tau)$ ,  $H^{(+)}(\tau) = \frac{\partial\phi_v}{\partial Q_j^{(+)}u}$ . 而  $\Phi^{(+)}(\tau)$  分别  $\Psi^{(+)}(\tau)$  是

$$\frac{dQ_0^{(+)}u}{d\tau} = (F_{11}(\tau) + F_{12}(\tau)H^{(+)}(\tau))Q_0^{(+)}u, \quad \Phi^{(+)}(0) = E_k$$

和

$$\frac{dQ_0^{(+)}v}{d\tau} = (F_{22}(\tau) - H^{(+)}(\tau)F_{12}(\tau))Q_0^{(+)}v, \quad \Psi^{(+)}(0) = E_{M-k}$$

的解. 决定高阶项边界函数  $L_jx(\tau_0)$ ,  $R_jx(\tau_1)$  的方程与  $Q_j^{(\mp)}x(\tau)$  类似, 所以可类似求出. 因其与讨论问题无实质影响, 在此不赘述.



**引理 3.3** 如果满足假设 (H<sub>1</sub>)–(H<sub>7</sub>), 则边界函数  $L_j x(\tau_0)$ ,  $Q_j^{(\mp)} x(\tau)$ ,  $R_j x(\tau_1)$ ,  $j = 1, 2, \dots$  存在, 且满足下面的不等式

$$\begin{aligned} L_j x(\tau_0) &\leq C e^{-\gamma \tau_0}, \quad \tau_0 > 0, \quad Q_j^{(-)} x(\tau) \leq C e^{\gamma \tau}, \quad \tau < 0, \\ Q_j^{(+)} x(\tau) &\leq C e^{-\gamma \tau}, \quad \tau > 0, \quad R_j x(\tau_1) \leq C e^{\gamma \tau_1}, \quad \tau_0 < 0, \end{aligned}$$

其中  $C, \gamma$  是任意正常数.

由 (H<sub>6</sub>), 左右问题的解满足缝接条件 (3.7). 所以 (3.40) 的联合系统

$$\frac{d}{d\tau} Q_j z = f_z(\tau) Q_j z + \tilde{G}_j(\tau) \quad (3.41)$$

满足  $Q_j z(-\infty) = 0$ ,  $Q_j z(+\infty) = 0$  的解一定存在, 其中

$$\tilde{G}_j(\tau) = f_y(\tau) Q_j y + (\Delta f_z(\tau) \beta'(t_0) + \Delta f_y(\tau) (\bar{y}_0(t_0))' + \Delta f_t(\tau)) t_j + \bar{G}_j(\tau).$$

易知 (3.41) 在  $R^-$  和  $R^+$  上均存在指数二分法 (见 [15]). 而  $F(Q_j z) = \frac{d}{d\tau} Q_j z - f_z(\tau) Q_j z$  是 Fredholm 算子, 且 Fredholm 指标为  $\text{Ker } F - \text{Ker } F^* = 0$ . 由于  $\frac{d}{d\tau} Q_0 z(\tau) \in \text{Ker } F$ , 不妨设  $\frac{d}{d\tau} Q_0 z(\tau)$  是算子  $F$  的唯一的核. 必存在唯一的  $\psi(\tau)$ , 使得  $\psi(\tau) \in \text{Ker } F^*$ . 由 Fredholm 交引理得 (3.41) 的解存在的充要条件为

$$\int_{-\infty}^{+\infty} \psi^*(t) \tilde{G}_j(\tau) d\tau = \int_{-\infty}^{+\infty} \psi^*(t) \{f_y(\tau) Q_j y + P(\tau)\} d\tau = 0, \quad (3.42)$$

其中

$$P(\tau) = (\Delta f_z(\tau) \beta'(t_0) + \Delta f_y(\tau) (\bar{y}_0(t_0))' + \Delta f_t(\tau)) t_j + \bar{G}_j(\tau). \quad (3.43)$$

将 (3.42) 重新记为

$$\begin{aligned} &t_j \int_{-\infty}^{+\infty} \psi^*(t) (\Delta f_z(\tau) \beta'(t_0) + \Delta f_y(\tau) (\bar{y}_0(t_0))' + \Delta f_t(\tau)) d\tau \\ &= - \int_{-\infty}^{+\infty} \psi^*(t) \{f_y(\tau) Q_j y + \bar{G}_j(\tau)\} d\tau. \end{aligned}$$

(H<sub>9</sub>) 假设

$$\int_{-\infty}^{+\infty} \psi^*(t) (\Delta f_z(\tau) \beta'(t_0) + \Delta f_y(\tau) (\bar{y}_0(t_0))' + \Delta f_t(\tau)) d\tau \neq 0.$$

在此假设下,  $t_j$  ( $j = 1, 2, \dots$ ) 完全确定. 这样渐近展开适当各项系数完全确定.

## 4 阶梯解的存在性

**定理 4.1** 如果满足假设 (H<sub>1</sub>)–(H<sub>9</sub>), 则存在  $\mu_0 > 0$ , 当  $0 < \mu < \mu_0$  时问题 (1.1)–(1.2) 存在具有结题状空间对照结构的解  $y(t, \mu)$ , 并有下面的渐近表达式成立

$$x(t, \mu) = \begin{cases} \sum_{i=0}^n \mu^i (\bar{x}_i^{(-)}(t) + L_i x(\tau_0) + Q_i^{(-)} x(\tau)) + O(\mu^{n+1}), & 0 \leq t \leq t^*, \\ \sum_{i=0}^n \mu^i (\bar{x}_i^{(+)}(t) + Q_i^{(+)} x(\tau) + R_i x(\tau_1)) + O(\mu^{n+1}), & t^* \leq t \leq 1. \end{cases}$$

**证** 为了借助  $k + \sigma$  交引理 (见 [16]) 证明本定理的结论, 考虑系统 (1.1) 的连接问题:

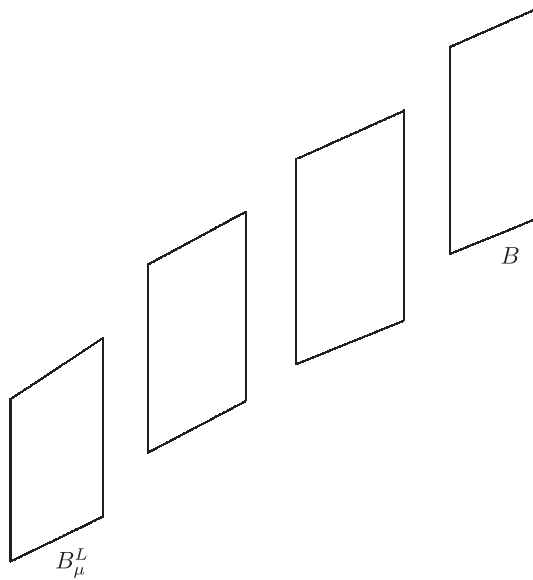
$$\mu z' = f(z, y, t), \quad y' = g(z, y, t), \quad t' = 1.$$

将边界条件 (1.2) 重新记为

$$B_\mu^L = \{(z, y, t) \mid Az(0, \mu) = Az^0, Cy(0, \mu) = Cy^0, t = 0\},$$

$$B_\mu^R = \{(z, y, t) \mid Bz(1, \mu) = Bz^1, Dy(1, \mu) = Dy^1, t = 1\}.$$

退化方程的两组解  $\phi_1(t, \alpha(t))$  和  $\phi_2(t, \beta(t))$  所在的曲面分别记为  $S_1$  和  $S_2$  (见图 1).



记  $\sigma = \dim(W^s(\chi^2) \cap W^u(U^1))$ . 由横截相交的定义  $\sigma = (m_2 + 1 + M_2) + (m_1 + 1 + M_1) - (M + m + 1) = 1$ , 即存在  $S_1$  到  $S_2$  的异宿轨道. 综上所述, 定理成立.

## 5 例子

考虑系统

$$\mu u'_1 = u_2, \quad \mu u'_2 = (u_1^2 - 1)(u_1 - t), \quad v'_1 = v_2, \quad v'_2 = u_2 v_2, \quad (5.1)$$

$$u_1(-1) = 0, \quad v_1(-1) = 1, \quad u_1(1) = 2, \quad v_2(1) = 0. \quad (5.2)$$

按照前面的算法, 可解得退化方程的 3 组解:

$$S_1 = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = 1, u_2 = 0, v_1 = v_1^*, v_2 = v_2^*, t = t^*\},$$

$$S_2 = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = -1, u_2 = 0, v_1 = v_1^*, v_2 = v_2^*, t = t^*\}$$

和

$$S_3 = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = t, u_2 = 0, v_1 = v_1^*, v_2 = v_2^*, t = t^*\},$$

这里  $v_1^*, v_2^*, t^*$  是参数, 它们可以不同. 易证  $\bar{\lambda}_{1,2}|_{S_1} = \pm\sqrt{2(1-t)}$ ,  $\bar{\lambda}_{1,2}|_{S_2} = \pm\sqrt{2(1+t)}$ ,  $\bar{\lambda}_{1,2}|_{S_3} = \pm i\sqrt{1-t^2}$ ,  $t \in [-1, 1]$ . 所以  $S_1, S_2$  是两个双曲鞍点,  $S_3$  是中心.

考虑 (5.1) 的连接问题:

$$\mu u'_1 = u_2, \quad \mu u'_2 = (u_1^2 - 1)(u_1 - t), \quad v'_1 = v_2, \quad v'_2 = u_2 v_2, \quad t' = 1. \quad (5.3)$$

边界条件重新记为

$$B^L = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = 0, v_1 = 1, t = -1\},$$

$$B^R = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = 2, v_1 = 1, t = 1\}.$$

(5.3) 的快系统为

$$\begin{aligned} \dot{u}_1 &= u_2, & \dot{u}_2 &= (u_1^2 - 1)(u_1 - t), \\ \dot{v}_1 &= 0, & \dot{v}_2 &= 0, \\ \dot{t} &= 0. \end{aligned} \quad (5.4)$$

(5.4) 存在首次积分

$$H_1 = \frac{u_1^4}{4} - \frac{1}{3}tu_1^3 - \frac{u_1^2}{2} + tu_1 - \frac{u_2^2}{2}, \quad H_2 = v_1, \quad H_3 = v_2, \quad H_4 = t.$$

记过  $S_1$  的稳定 (不稳定) 流形为  $W^{s,u}(S_1)$ ,

$$\begin{aligned} \frac{u_1^4}{4} - \frac{1}{3}tu_1^3 - \frac{u_1^2}{2} + tu_1 - \frac{u_2^2}{2} &= \frac{2}{3}t^* - \frac{1}{4}, \\ v_1 &= v_1^*, \quad v_2 = v_2^*, \quad t = t^*. \end{aligned}$$

易证

$$B^L \cap W^s(S_1) = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = 0, u_2 = \pm\sqrt{\frac{11}{6}}, v_1 = 1, v_2 = v_2^*, t = -1\},$$

$$\omega(B^L \cap W^s(S_1)) = \{(u_1, u_2, v_1, v_2, t) \mid u_1 = 1, u_2 = 0, v_1 = 1, v_2 = v_2^*, t = -1\}.$$

过  $S_2$  的稳定 (不稳定) 流形为  $W^{s,u}(S_2)$

$$\frac{u_1^4}{4} - \frac{1}{3}tu_1^3 - \frac{u_1^2}{2} + tu_1 - \frac{u_2^2}{2} = -\frac{2}{3}t^* - \frac{1}{4},$$

$$v_1 = v_1^*, \quad v_2 = v_2^*, \quad t = t^*.$$

易证

$$B^R \cap W^u(S_2) = \left\{ (u_1, u_2, v_1, v_2, t) \mid u_1 = 2, u_2 = \pm \frac{3}{\sqrt{2}}, v_1 = v_1^*, v_2 = 0, t = 1 \right\},$$

$$\alpha(B^R \cap W^u(S_2)) = \left\{ (u_1, u_2, v_1, v_2, t) \mid u_1 = -1, u_2 = 0, v_1 = v_1^*, v_2 = 0, t = 1 \right\}.$$

在  $S_1, S_2$  上的慢流形为

$$v_1' = v_2, \quad v_2' = 0, \quad t' = 1.$$

过  $\omega(B^L \cap W^s(S_1))$  的解为

$$\{(u_1, u_2, v_1, v_2, t) \mid u_1 = 1, u_2 = 0, v_1 = v_2^*(s+1) + 1, v_2 = v_2^*, t = s\}.$$

过  $\alpha(B^R \cap W^u(S_2))$  的解为

$$\{(u_1, u_2, v_1, v_2, t) \mid u_1 = -1, u_2 = 0, v_1 = v_1^*, v_2 = 0, t = s\},$$

其中  $s \in [-1, 1]$  是一个参数. 要使得存在连接  $\omega(B^L \cap W^s(S_1))$  和  $\alpha(B^R \cap W^u(S_2))$  的异宿轨道, 需满足  $v_1^* = 1, v_2^* = 0$ .

由

$$W^u(S_1) |_{(1,0,1,0,s)} \cap W^s(S_2) |_{(-1,0,1,0,s)},$$

可得  $\frac{2}{3}s - \frac{1}{4} = -\frac{2}{3}s - \frac{1}{4}$ , 所以  $s = 0$ . 故异宿轨道存在, 其方程为

$$\frac{u_1^4}{4} - \frac{u_1^2}{2} - \frac{u_2^2}{2} = -\frac{1}{4}, \quad v_1 = 1, \quad v_2 = 0.$$

当  $s = 0$  时,  $t = 0$ , 即在时刻  $t = 0$  发生跳跃, 存在空间对照结构解 (见图 2).

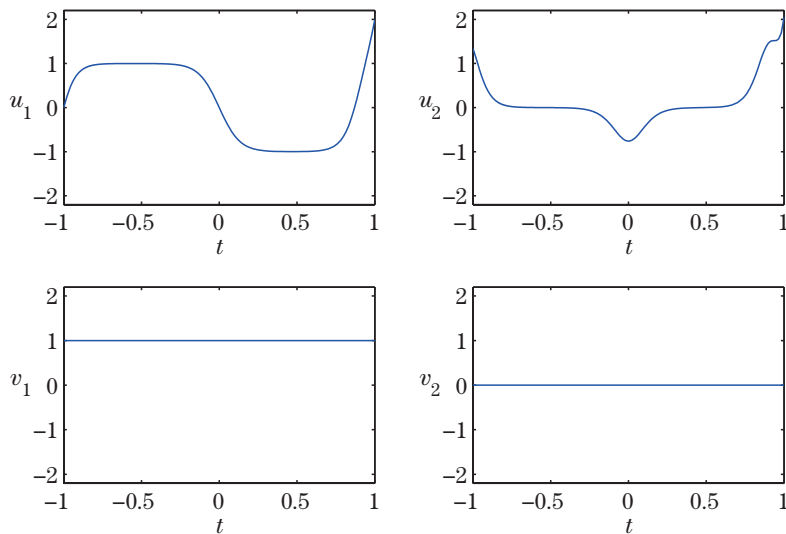


图 2 (5.1)–(5.2) 的解

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# The Step-Type Contrast Structure for High-Dimensional Tikhonov System with Boundary Conditions

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**Abstract** By means of the first integral method, the author finds a high-dimensional heteroclinic orbit in a fast phase space. He uses the properties of exponential dichotomies and the Fredholm alternatives to determine the internal transition time  $t^*$ . Using the method of boundary function, he constructs the formal asymptotic solution. Using the method of  $k + \sigma$  changing lemma, the existence of a step-type contrast structure for high-dimensional Tikhonov system with boundary conditions is shown and the asymptotic solution is proved to be uniformly effective in the whole interval. Finally, an example is given to demonstrate the effectiveness of the result.

**Keywords** Contrast structure, Singular perturbation, Asymptotic expansion,  
Boundary function

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