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THE GLOBAL SOLUTION AND "BLOW UP" PHENOMENON FOR A CLASS OF SYSTEM OF NONLINEAR SCHRÖDINGER EQUATIONS WITH THE MAGNETIC FIELD EFFECT

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Abstract

In this paper, the auther considers following initial value problem for the system of nonlinear Schrödinger equation with the magnetic field effect

$$i\varepsilon_t-\Delta\varepsilon+\beta q(|\varepsilon|^2)\varepsilon+\eta\varepsilon\times(\varepsilon\times\varepsilon)=0 \tag{1.1}$$

$$\varepsilon|_{t=0}=\varepsilon_0(x),\ x\in R^2, \tag{1.2}$$

where β, η are real constants, $\varepsilon=(\varepsilon^1, \varepsilon^2, \varepsilon^3)$. First, the existence of the global solution for problem (1.1), (1.2) is established by means of the method of integral estimates, and then the "blow up" theorem is obtained nuder some conditions.

§ 1. Introduction

One class of system of nonlinear Schrödinger equations was proposed in [1, 2], and its scattering inverse method was studied in [2]. The existence of the global solution for some systems of nonlinear Schrödinger equations has been proved in [3]. In [4, 5, 6] the system of Zakharov equations (including the system of nonlinear Schrödinger equations) with the longitudial and transverse oscillating and magnetic effect has been examined, the soliton properties and collapse in multi-dimensions have been revealed. In this paper, we shall consider the following initial value problem for the system of nonlinear Schrödinger equations with the magnetic field effect:

$$\begin{cases} i\varepsilon_t-\Delta\varepsilon+\beta q(|\varepsilon|^2)\varepsilon+\eta\varepsilon\times(\varepsilon\times\bar{\varepsilon})=0, & (1.1)\\ \varepsilon|_{t=0}=\varepsilon_0(x),\ x\in R^2, & (1.2)\end{cases}$$

where β, η are real constaants, $\varepsilon=(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is a 3-dimensional unknown functional vector, "$\times$" denotes the cross product operator of two 3-dimensional vectors, $\bar{\varepsilon}$ denotes the complex conjugate vector of ε, and $q(s)$ is a real function, $s\in[0, \infty)$. We shall first establish the existence of the global solution for problem(1.1), (1.2) by means of the method of integral eatimates, and then obtain the "blow up" theorem under

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some conditions. Here we shall apply the usual notations indicated in [3]. We shall always assume that as $|x|\to\infty$ the solution and its derivatives tend to zero.

§ 2. The Integral Estimates and Existence

Lemma 1. *If $\varepsilon_0(x)\in L_2$, then for the solution of problem* (1.1), (1.2), *we have*

$$\|\varepsilon(t)\|^2_{L_2}=\|\varepsilon_0(x)\|^2_{L_2}. \tag{2.1}$$

Proof The lemma is proved by multiplying (1.1) by $\bar{\varepsilon}$ and integrating the resulting equality with respect to x, taking the imaginary and noticing

$$\varepsilon\times(\varepsilon\times\bar{\varepsilon})=(\varepsilon\cdot\bar{\varepsilon})\varepsilon-\bar{\varepsilon}(\varepsilon\cdot\varepsilon)=|\varepsilon|^2\varepsilon-(\varepsilon\cdot\varepsilon)\bar{\varepsilon},$$

where $\varepsilon\cdot\varepsilon$ denotes the point product, i. e.,

$$\varepsilon\cdot\bar{\varepsilon}=|\varepsilon|^2=\sum_{j=1}^{3}|\varepsilon^j|^2,\ \varepsilon\cdot\varepsilon=\sum_{j=1}^{3}(\varepsilon^j)^2.$$

Lemma 2. *If $\beta\geqslant0$, $\eta\geqslant0$ and $\varepsilon_0(x)\in H^1$, $Q(|\varepsilon_0|^2)\in L$, where*

$$Q(s)=\int_0^s q(z)dz\geqslant0,$$

there is the estimation

$$\|\nabla\varepsilon\|^2_{L_2}\leqslant E_1,\ \beta\int Q(|\varepsilon|^2)dx\leqslant E_1, \tag{2.2}$$

where the constant E_1 only depends on L_2 norms of the initial function and its derivatives.

Proof Taking the inner product for (1.1) with ε_t, we have

$$(i\varepsilon^l_t,\ \varepsilon^l_t)-(\Delta\varepsilon^l,\ \varepsilon^l_t)+\beta(q(|\varepsilon|^2)\varepsilon^l,\ \varepsilon^l_t)+\eta(|\varepsilon|^2\varepsilon^l-\bar{\varepsilon}^l(\varepsilon\cdot\varepsilon),\ \varepsilon^l_t)=0. \tag{2.3}$$

Since

$$\mathrm{Re}[-(\Delta\varepsilon^l,\ \varepsilon^l_t)]=\frac{1}{2}\frac{d}{dt}\|\nabla\varepsilon^l\|^2_{L_2},$$

$$\mathrm{Re}\sum_{l=1}^{N}(\beta q(|\varepsilon|^2)\varepsilon^l,\ \varepsilon^l_t)=\frac{1}{2}\beta\frac{d}{dt}\int Q(|\varepsilon|^2)dx,$$

$$\mathrm{Re}\,\eta\sum_{l=1}^{N}(|\varepsilon|^2\varepsilon^l,\ \varepsilon^l_t)=\frac{\eta}{4}\frac{d}{dt}\int|\varepsilon|^4dx,$$

$$\mathrm{Re}\,\eta\sum_{l=1}^{N}(-\bar{\varepsilon}^l(\varepsilon\cdot\varepsilon),\ \varepsilon^l_t)=-\frac{\eta}{4}\frac{d}{dt}\int(\varepsilon\cdot\varepsilon)(\bar{\varepsilon}\cdot\bar{\varepsilon})dx.$$

Taking the real part of (2.3), summing up over l, it follows that

$$\frac{d}{dt}\|\nabla\varepsilon\|^2_{L_2}+\frac{d}{dt}\int\beta Q(|\varepsilon|^2)dx+\frac{\eta}{2}\frac{d}{dt}\int|\varepsilon|^4dx$$
$$-\frac{\eta}{2}\frac{d}{dt}\int|(\varepsilon\cdot\varepsilon)|^2dx=0.$$

Therefore

$$E(t)=\|\nabla\varepsilon(t)\|^2_{L_2}+\beta\int Q(|\varepsilon(t)|^2)\,dx+\frac{\eta}{2}\int|\varepsilon|^4dx-\frac{\eta}{2}\int|\varepsilon\cdot\varepsilon|^2dx=E(0). \tag{2.4}$$

As $|(\varepsilon\cdot\varepsilon)|\leqslant|\varepsilon|^2$, $|(\bar{\varepsilon}\cdot\bar{\varepsilon})|\leqslant|\varepsilon|^2$, so if $\eta\geqslant0$, $\beta\geqslant0$, (2.2) follows.

Lemma 3. (Sobolev estimate)[7] *Assume that $u\in L_q(R^n)$, $D^m u\in L_r(R^n)$, $1\leqslant q$, $r\leqslant\infty$, $0\leqslant j\leqslant m$. We have the estimation*

$$\|D^j u\|_{L_p} \leqslant C \|D^m u\|_{L_r}^a \|u\|_{L_q}^{1-a}, \tag{2.5}$$

where C is a positive constant,

$$\frac{1}{p} = \frac{j}{n} + a\left(\frac{1}{r} - \frac{m}{n}\right) + (1-a)\frac{1}{q}, \quad j/m \leqslant a < 1.$$

In particular, as $n=2$, $j=0$, $p=4$, $r=2$, $q=2$, we have

$$\|u\|_{L_4}^4 \leqslant \|\nabla u\|_{L_2}^2 \|u\|_{L_2}^2 \tag{2.6}$$

Lemma 4. *Suppose that the following conditions are satisfied:*

(1) $\int_0^s q(z)dz \geqslant -c_1 s^2$, $s>0$, $c_1 = \text{const.} > 0$.

(2) $\beta \leqslant 0$, $\eta \leqslant 0$.

(3) $\|\varepsilon_0(x)\|_{L_2}^2 < (|\beta| c_1 + |\eta|)^{-1}$, $\varepsilon_0(x) \in H^1$.

Then we have

$$\|\nabla \varepsilon\|_{L_2} \leqslant E_1', \tag{2.7}$$

where the constent E_1' only depends on the norms of the initial function $\varepsilon_0(x)$ and its derivative.

Proof From (2.4), we have

$$\beta \int Q(|\varepsilon|^2)dx + \frac{\eta}{2}\int |\varepsilon|^4 dx - \frac{\eta}{2}\int |(\varepsilon\cdot\varepsilon)|^2 dx$$
$$\leqslant -\beta C_1 \|\varepsilon\|_{L_4}^4 + |\eta| \, \|\varepsilon\|_{L_4}^4 \leqslant (|\beta| C_1 + |\eta|) \|\varepsilon_0\|_{L_2}^2 \|\nabla \varepsilon\|_{L_2}^2.$$

Hence, as $(|\beta| C_1 + |\eta|) \|\varepsilon_0(x)\|_{L_2}^2 < 1$, (2.7) follows.

Lemma 5.[8] *If $S(t)$ is a semigroup generated by the operator $i\Delta$, for*

$$1 \leqslant q \leqslant 2 \leqslant p \leqslant \infty, \ \frac{1}{p} + \frac{1}{q} = 1,$$

then we have

$$\|S(t)\varphi\|_{L_p} \leqslant t^{-n\left(\frac{1}{q}-\frac{1}{2}\right)} \|\varphi\|_{L_q}, \ \forall \varphi \in L_q(R^n) \tag{2.8}$$

and

$$\|S(t)\varphi\|_{L_2} = \|\varphi\|_{L_2}.$$

Lemma 6. *Suppose that the conditions in Lemma 2 or Lemma 4 are satisfied, and assume that*

(1) $|q^{(\nu)}(s)| \leqslant A s^{l+1-\nu}$, $A = \text{const.} > 0$, $s>0$, $\nu = 0, 1$, $l \geqslant 0$,

(2) $\varepsilon_0(x) \in H^2$.

Then we have

$$\|D\varepsilon(t)\|_{L_p} \leqslant C, \ p>2, \tag{2.9}$$

where the constant C only depends on L_2 norms of the initial function and its second order derivative, and D denotes the derivative with respect to x.

Proof Suppose that $S(t)$ is a semigroup generated by the operator $i\Delta$, the expression of the solution for the problem (1.1), (1.2) is

$$\varepsilon(t) = S(t)\varepsilon_0(t) + \int_0^t S(t-\xi)[-\beta q(|\varepsilon|^2)\varepsilon - \eta\varepsilon \times (\varepsilon \times \bar{\varepsilon})]d\xi.$$

From Lemma 2, Lemma 4, Lemma 5 and $H^1(R^2) \subsetneq L_m(R^2)$, $(2 \leqslant m < \infty)$ we have

$$\|D\varepsilon\|_{L_p}\leqslant\|S(t)D\varepsilon_0(x)\|_{L_p}+C_5\int_0^t (t-\xi)^{-\frac{(2-q)}{q}}\|D[-\beta q(|\varepsilon|^2)\varepsilon-\eta\varepsilon\times(\varepsilon\times\bar{\varepsilon})]\|_{L_q}d\xi$$

$$\leqslant C_3\|\varepsilon_0(x)\|_{H^2}+C_4\int_0^t (t-\xi)^{-\frac{(2-q)}{q}}\||\varepsilon|^{2l+2}|D\varepsilon|\|_{L_q}d\xi$$

$$\leqslant C_3\|\varepsilon_0(x)\|_{H^2}+C_5\int_0^t (t-\xi)^{-\frac{(2-q)}{q}}\|\varepsilon\|_{L_r}\|D\varepsilon\|_{L_2}d\xi$$

$$\leqslant C_3\|\varepsilon_0(x)\|_{H^2}+C_0\int_0^t (t-\xi)^{-\frac{(2-q)}{q}}d\xi\leqslant C(t),$$

where $\dfrac{2l+2}{r}=\dfrac{1}{2}-\dfrac{1}{p},\ \dfrac{1}{p}+\dfrac{1}{q}=1,\ p>2.$

Corollary.

$$\|\varepsilon(t)\|_{L_\infty}\leqslant C, \tag{2.10}$$

where C is a definite constant.

Thus we have the following theorem.

Theorem 1. *Suppose that the following conditions are satisfied:*

(1) $\beta\geqslant 0,\ \eta\geqslant 0$;

(2) $q(s)\geqslant 0,\ q(s)\in C^m,\ |q^{(\nu)}(s)|\leqslant AS^{l+1-\nu},\ A=\text{const}>0,\ s>0,\ \nu=0,1,\ l\geqslant 0.$

(3) $\varepsilon_0(x)\in H^m(m\geqslant 2)$.

Then there exists the global solution $\varepsilon(x,t)$ of problem (1.1), (1.2), *and* $\varepsilon(x,t)\in C^\circ(0,T;\ H^m(R^2))\cap C^1(0,T;\ H^{m-2}(R^2))$.

Proof (1.1), (1.2) can be written as

$$i\varepsilon_t-\Delta\varepsilon=J\varepsilon, \tag{2.11}$$

$$\varepsilon(0)=\varepsilon_0(x), \tag{2.12}$$

where $J\varepsilon=-\beta q(|\varepsilon|^2)\varepsilon-\eta\varepsilon\times(\varepsilon\times\varepsilon)$.

If we define the set Σ and the distance d as follows

$$\Sigma=\{\varepsilon\in C^\circ(0,T:H^m(R^2)):\|\varepsilon\|_{L^\infty(0,T;H^m(R^2))}\leqslant M,\ M>\|\varepsilon_0(x)\|_{H^m}\},$$

$$d(\varepsilon_1,\varepsilon_2)=\|\varepsilon_1-\varepsilon_2\|_{L^\infty(0,T;H^m(R^2))},$$

Σ is complete. For $\varepsilon\in\Sigma$, define

$$\varphi\varepsilon=S(t)\varepsilon_0+\int_0^t S(t-\xi)J\varepsilon(\xi)d\xi \tag{2.13}$$

the same as in [9]. If t is suitably small, it is easily shown that there is a fixed point $\varphi\varepsilon=\varepsilon$, which is the local solution of problem (2.11), (2.12),

$$\varepsilon(x,t)\in C^\circ(0,T_1;\ H^m)\cap C^1(0,T_1;\ H^{m-2}),$$

where T_1 depends on $\|\varepsilon_0(x)\|_{H^m}$. Then from the priori estimates in Lemma 2 and Lemma 6, we can know

$$\|\nabla\varepsilon\|_{L_2}\leqslant\text{const.},\|\varepsilon\|_{L_\infty}\leqslant\text{const.}$$

In addtion, using the inequality

$$\|f(u(t))\|_{H^k}\leqslant CM_k(f_0,b)(1+\|u(t)\|_{H^{k-1}})^{k-1}\|u(t)\|_{H^k},$$

where $f_0=\max\limits_{s\leqslant k}\sup\limits_{v}|D^sf(v)|,\ |v|\leqslant b=\sup\limits_{\tau}\|u(\tau)\|_{L_\infty}$, M_k is a constant which depends on k, f_0 and b, it follows that

$$\|\varepsilon\|_{H^m}\leqslant C_1\|\varepsilon_0\|_{H^m}+C_2\int_0^t\|\varepsilon(\xi)\|_{H^m}d\xi$$

By Gronwall's inequality, we have the boundness of $\|\varepsilon(t)\|_{H^m}$. Thus the theorem is true.

Theorem 2. *Suppose that the following conditions are satisfied:*

(1) $\beta\leqslant 0,\ \eta\leqslant 0$;

(2) $|q(s)|\leqslant As$, $A=\text{const.}>0$, $s>0$;

(3) $\varepsilon_0(x)\in H^m(R^2)$, $m\geqslant 2$;

(4) $\|\varepsilon_0(x)\|^2_{L_2}\leqslant\left(\frac{A}{2}|\beta|+|\eta|\right)^{-1}$.

Then there exists the global solution $\varepsilon(x, t)$ *of problem* (1.1), (1.2)

$$\varepsilon(x, t)\in C^0(0, T;\ H^m(R^2))\cap C^1(0, T;\ H^{m-2}(R^2)).$$

Proof By Lemma 4 and in a way similar to the proof of Theorem 1, the theorem can be proved.

§ 3. "Blow up" Phenomenon of the Solution.

In the following, we consider "blow up" problem for the solution of problem (1.1), (1.2) in R^3 as $\beta\leqslant 0$, $\eta\leqslant 0$. For simplicity, we consider a class of system of nonlinear Schrödinger equations with magnetic field effect inspherical symmetry case

$$i\varepsilon_t-\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\varepsilon}{\partial r}-|\varepsilon|^2\varepsilon-\eta\varepsilon\times(\varepsilon\times\bar{\varepsilon})=0 \tag{3.1}$$

with initial and boundary conditions

$$\begin{aligned}&\varepsilon|_{t=0}=\varepsilon_0(r),\quad 0\leqslant r<\infty,\\ &\left.\frac{\partial\varepsilon}{\partial r}\right|_{r=0}=0,\qquad t\geqslant 0,\end{aligned} \tag{3.2}$$

where $\varepsilon=(\varepsilon_1, \varepsilon_2, \varepsilon_3)$. For the solution of problem (3.1), (3.2), there are following estimations

Lemma 7. *If* $\varepsilon_0(x)\in L_2$, *we have*

$$E_0(t)=\int_0^\infty r^2|\varepsilon(r, t)|^2dr=\int_0^\infty r^2|\varepsilon_0(r)|^2dr=E_0(0). \tag{3.3}$$

Lemma 8. *If*

$$\int_0^\infty r^2\left|\frac{d\varepsilon_0}{dr}\right|^2dr<\infty,\quad \int_0^\infty r^2|\varepsilon_0|^4dr<\infty,$$

we have

$$\begin{aligned}E(t)=\frac{1}{2}\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr-\frac{1}{4}\int_0^\infty(1+\eta)r^2|\varepsilon|^4dr\\ +\frac{\eta}{4}\int_0^\infty r^2|(\varepsilon\cdot\varepsilon)|^2dr=E(0).\end{aligned} \tag{3.4}$$

Proof Multiplying (3.1) by $r^2\bar{\varepsilon}_t$, taking the inner product, it follows that

$$(i\varepsilon_t,\ \varepsilon_t)-\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\varepsilon}{\partial r},\ \varepsilon_t\right)-(|\varepsilon|^2\varepsilon,\ \varepsilon_t)-\eta(\varepsilon\times(\varepsilon\times\bar{\varepsilon}),\ \varepsilon_t)=0. \tag{3.5}$$

Since

$$-\mathrm{Re}\int_0^\infty\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial\varepsilon}{\partial r}\cdot r^2\bar{\varepsilon}_t dr=\frac{1}{2}\frac{d}{dt}\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr,$$

$$-\mathrm{Re}\int_0^\infty|\varepsilon|^2\varepsilon\cdot r^2\bar{\varepsilon}_t dr=-\frac{1}{4}\frac{d}{dt}\int_0^\infty r^2|\varepsilon|^4dr,$$

$$-\mathrm{Re}\,\eta(\varepsilon\times(\varepsilon\times\bar{\varepsilon}),\ \varepsilon_t)=-\mathrm{Re}\,\eta(|\varepsilon|^2\varepsilon,\ \varepsilon_t)+\mathrm{Re}\,\eta((\varepsilon\cdot\varepsilon)\bar{\varepsilon},\ \varepsilon_t)$$

$$=-\frac{\eta}{4}\frac{d}{dt}\int_0^\infty r^2|\varepsilon|^4dr+\frac{\eta}{4}\int_0^\infty r^2(\varepsilon\cdot\varepsilon)(\bar{\varepsilon}\cdot\bar{\varepsilon})dr,$$

from this (3.4) follows.

From Lemma 7 and Lemma 8, we can get the following "blow up" theorem.

Theorem 3. *Suppose that the following conditions are satisfied:*

(1) $\eta>0$.

(2) *The initial function satisfies the following conditions:*

$$\int_0^\infty r^2\left|\frac{d\varepsilon_0}{dr}\right|^2dr<\infty,\quad \int_0^\infty r^4|\varepsilon_0|^2dr<\infty,\int_0^\infty r^2|\varepsilon_0|^4dr<\infty$$

in (3.4), $E_0(0)<0$ *and* $\mathrm{Im}\int_0^\infty r^3\frac{d\varepsilon_0}{dr}\cdot\bar{\varepsilon}_0dr>0$.

Then the solution of the problem blow up, i. e., there is $t_0>0$ *such that*

$$\lim_{t\to t_0^-}\|\nabla\varepsilon\|_{L_2}^2=\lim_{t\to t_0^-}\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr=\infty. \tag{3.6}$$

Proof We can express equation (3.1) as an integral operator form. Using the contraction principle, it is easy to show that the local smooth solution for problem (3.1), (3.2) exists, and by means of Lemma 7 and Lemma 8, letting $D=\int_0^\infty|\varepsilon|^2r^4dr$, we have

$$\frac{dD}{dt}=\int_0^\infty\frac{\partial|\varepsilon|^2}{\partial t}r^4dr=-2\,\mathrm{Im}\int_0^\infty r^2\left(\bar{\varepsilon}\cdot\frac{\partial}{\partial r}r^2\frac{\partial\varepsilon}{\partial r}\right)dr$$

$$=-4\,\mathrm{Im}\int_0^\infty r^3\left(\frac{\partial\varepsilon}{\partial r}\cdot\bar{\varepsilon}\right)dr.$$

By calculation and using equation (3.1), and integrating by parts, it follows that

$$\frac{d^2D}{dt^2}=-4\,\mathrm{Im}\int_0^\infty r^3\cdot\sum_{l=1}^N\frac{\partial\varepsilon^l}{\partial r}\cdot\frac{\partial\bar{\varepsilon}^l}{\partial t}dr-4\,\mathrm{Im}\int_0^\infty r^3\sum_{l=1}^N\bar{\varepsilon}_l\frac{\partial^2\varepsilon^l}{\partial r\partial t}dr$$

$$=8\int_0^\infty\left|\frac{\partial\varepsilon}{\partial r}\right|^2r^2dr-6(1+\eta)\int_0^\infty r^2|\varepsilon|^4dr+6\eta\int_0^\infty r^2(\varepsilon\cdot\varepsilon)(\bar{\varepsilon}\cdot\bar{\varepsilon})dr.$$

Let $y(t)=-\frac{dD}{dt}$. Then

$$\frac{dy}{dt}=-8\int_0^\infty\left|\frac{\partial\varepsilon}{\partial r}\right|^2r^2dr+6(1+\eta)\int_0^\infty r^2|\varepsilon|^4dr-6\eta\int_0^\infty r^2|(\varepsilon\cdot\varepsilon)|^2dr.$$

Using equality (3.1), it follows that

$$\frac{dy}{dt}=4\int_0^\infty\left|\frac{\partial\varepsilon}{\partial r}\right|^2r^2dr-24E_0(0)\geqslant4\int_0^\infty\left|\frac{\partial\varepsilon}{\partial r}\right|^2r^2dr, \tag{3.7}$$

Due to the assumption $y(0)>0$, from (3.7) we have $y(t)>0$. Thus $\frac{dD}{dt}<0$. Since

$$\begin{aligned}|y(t)|=y(t)&=4\,\mathrm{Im}\int_0^\infty r^3\left(\frac{\partial\varepsilon}{\partial r}\cdot\bar{\varepsilon}\right)dr\\ &\leqslant 4\left(\int_0^\infty r^4|\varepsilon|^2dr\right)^{\frac{1}{2}}\left(\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr\right)^{\frac{1}{2}}\\ &\leqslant 4\left(\int_0^\infty r^4|\varepsilon_0|^2dr\right)^{\frac{1}{2}}\left(\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr\right)^{\frac{1}{2}}\\ &=k\left(\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr\right)^{\frac{1}{2}},\ k=4\left(\int_0^\infty r^4|\varepsilon_0|^2dr\right)^{\frac{1}{2}},\end{aligned}$$

we have

$$\int_0^\infty r^2\left|\frac{\partial\varepsilon}{\partial r}\right|^2dr\geqslant y^2/k^2.$$

Thus from (3.7), it follows that

$$\frac{dy}{dt}\geqslant\frac{4}{k^2}y^2,\quad y(0)>0.$$

Hence there is the estsmation on the interval $0\leqslant t\leqslant\frac{k^2}{4y(0)}$:

$$y(t)\geqslant\frac{y(0)k^2}{k^2-4y(0)t},$$

$$\|\nabla\varepsilon\|_{L_2}^2\geqslant\frac{y^2}{k^2}\geqslant\frac{1}{k^2}\left(\frac{y(0)k^2}{k^2-4y(0)t}\right)^2,$$

$$\lim_{t\to t_0^-}\|\nabla\varepsilon\|_{L_2}^2=\infty,\quad t_0^-=\frac{k^2}{4y(0)}.$$

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Due to the assumption $y(0)>0$, from (3.7) we have $y(t)>0$. Thus $\frac{dD}{dt}<0$. Since

$$
\begin{aligned}
|y(t)| = y(t) &= 4\,\mathrm{Im}\int_0^\infty r^3\left(\frac{\partial \varepsilon}{\partial r}\cdot\bar{\varepsilon}\right)dr \\
&\leqslant 4\left(\int_0^\infty r^4|\varepsilon|^2 dr\right)^{\frac{1}{2}}\left(\int_0^\infty r^2\left|\frac{\partial \varepsilon}{\partial r}\right|^2 dr\right)^{\frac{1}{2}} \\
&\leqslant 4\left(\int_0^\infty r^4|\varepsilon_0|^2 dr\right)^{\frac{1}{2}}\left(\int_0^\infty r^2\left|\frac{\partial \varepsilon}{\partial r}\right|^2 dr\right)^{\frac{1}{2}} \\
&= k\left(\int_0^\infty r^2\left|\frac{\partial \varepsilon}{\partial r}\right|^2 dr\right)^{\frac{1}{2}}, \quad k = 4\left(\int_0^\infty r^4|\varepsilon_0|^2 dr\right)^{\frac{1}{2}},
\end{aligned}
$$

we have

$$\int_0^\infty r^2\left|\frac{\partial \varepsilon}{\partial r}\right|^2 dr \geqslant y^2/k^2.$$

Thus from (3.7), it follows that

$$\frac{dy}{dt} \geqslant \frac{4}{k^2}y^2, \quad y(0)>0.$$

Hence there is the estsmation on the interval $0\leqslant t\leqslant\frac{k^2}{4y(0)}$:

$$y(t) \geqslant \frac{y(0)k^2}{k^2-4y(0)t},$$

$$\|\nabla\varepsilon\|_{L_2}^2 \geqslant \frac{y^2}{k^2} \geqslant \frac{1}{k^2}\left(\frac{y(0)k^2}{k^2-4y(0)t}\right)^2,$$

$$\lim_{t\to t_0^-}\|\nabla\varepsilon\|_{L_2}^2 = \infty, \quad t_0^- = \frac{k^2}{4y(0)}.$$

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