Chin. Ann. of Math.
10B(1) 1989

GLOBAL DISCONTINUOUS SOLUTIONS TO A CLASS OF DISCONTINUOUS INITIAL VALUE PROBLEMS FOR THE SYSTEM OF ISENTROPIC FLOW AND APPLICATIONS***

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Abstract

In this paper, the authors discuss a kind of discontinuous initial value problems for tl system of one-dimensional isentropic flow and prove the existence or nonexistence of glob discontinuous solutions only containing one shock in a class of piecewise continuous ar piecewise smooth functions. As applications, various interaction problems of a typical shoc with a rarefaction wave are considered.

§ 1. Introduction

In this paper, by means of the results in [1] on the global existence of cla solutions to some free boundary problems with characteristic boundary for quasil hyperbolic systems, we discuss a class of discontinuous initial value problems fc system of one-dimensional isentropic flow and prove the existence or nonexis of global discontinuous solutions only containing one shock in a class of piec continuous and piecewise smooth functions. As applications, various intera problems of a typical shock with a rarefaction wave are considered.

§ 2. Preliminaries

The system of one-dimensional isentropic flow can be written in Lagran representation as follows:

Manuscript received February 27, 1986.

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*** This research is supported by the National Natural Science Foundation of China and by a TWAS Research Grant under Contract No. 87-93.

$$\begin{cases}\dfrac{\partial\tau}{\partial t}-\dfrac{\partial u}{\partial x}=0,\\ \dfrac{\partial u}{\partial t}+\dfrac{\partial p(\tau)}{\partial x}=0,\end{cases}\tag{2.1}$$

ere u is the velocity, $\tau=1/\rho>0$ is the specific volume and $p=p(\tau)$ is the pressure. r polytropic gases

$$p=p(\tau)=A\tau^{-\gamma},\tag{2.2}$$

ere A is a positive constant and $\gamma>1$ is the adiabatic exponent.

Introducing the Riemann invariants

$$\begin{cases}r=\dfrac{1}{2}\left(u-\int_\tau^\infty\sqrt{-p'(\eta)}\,d\eta\right),\\ s=\dfrac{1}{2}\left(u+\int_\tau^\infty\sqrt{-p'(\eta)}\right)d\eta\end{cases}\tag{2.3}$$

new unknown functions, system (2.1) can be equivalently rewritten as

$$\begin{cases}\dfrac{\partial r}{\partial t}+\lambda(r,s)\dfrac{\partial r}{\partial x}=0,\\ \dfrac{\partial s}{\partial t}+\mu(r,s)\dfrac{\partial s}{\partial x}=0,\end{cases}\tag{2.4}$$

which

$$-\lambda(r,s)=\mu(r,s)=\sqrt{-p'(\tau(s-r))}=a(s-r)^{(\gamma+1)/(\gamma-1)}\tag{2.5}$$

d a is a positive constant.

Noting (2.2), the local speed of sound

$$c=\sqrt{A\gamma}\,\tau^{(1-\gamma)/2},\tag{2.6}$$

en, by (2.3) we have

$$\begin{cases}u=r+s,\\ \tilde{c}\triangleq\dfrac{2}{\gamma-1}\,c=s-r.\end{cases}\tag{2.7}$$

herefore

$$\tau'(s-r)<0.\tag{2.8}$$

It is easy to see that only in a domain where there never exists the vacuum ate, i. e.

$$s-r>0,\tag{2.9}$$

rstem (2.4) is strictly hyperbolic

$$\lambda(r,s)<\mu(r,s)\tag{2.10}$$

nd genuinely nonlinear in the sense of P. D. Lax

$$\frac{\partial\lambda}{\partial r}(r,s)>0,\quad \frac{\partial\mu}{\partial s}(r,s)>0,\tag{2.11}$$

ıoreover, we have

$$\frac{\partial\lambda}{\partial s}(r,s)<0,\ \frac{\partial\mu}{\partial r}(r,s)<0.\tag{2.12}$$

We now give some basic facts about shock. The Rankine–Hugoniot condition on a forward shock $x=x_2(t)$ can be written as

$$(u-u_+)^2=-(p-p_+)(\tau-\tau_+), \qquad (2.13)$$

$$\frac{dx}{dt}=\sqrt{-\frac{p-p_+}{\tau-\tau_+}}, \qquad (2.14)$$

where $(u_+,\ \tau_+)$ denotes the state just on the right side of the shock. Moreover, according to the entropy condition, we have

$$u>u_+ \qquad (2$$

and

$$\tau<\tau_+. \qquad (2$$

In terms of the Riemann invariants $(r,\ s)$, the preceding conditions on a forward shock $x=x_2(t)$ can be rewritten as

$$(r+s)-(r_++s_+)=\sqrt{-(p(\tau(s-r))-p(\tau(s_+-r_+)))(\tau(s-r)-\tau(s_+-r_+))}, \qquad (2$$

$$\frac{dx}{dt}=\sqrt{-\frac{p(\tau(s-r))-p(\tau(s_+-r_+))}{\tau(s-r)-\tau(s_+-r_+)}} \qquad (2$$

and

$$s-r>s_+-r_+>0, \qquad (2$$

where $(r_+,\ s_+)$ stands for the state just on the right side of the shock, $(r,\ s)$, state on the left side, can be connected with $(r_+,\ s_+)$ by a forward shock, and (2 shows that there never exists the vacuum state on both sides of the shock.

It is easy to see from (2.19) that the characteristic directions on both sides forward shock $x=x_2(t)$ satisfy the following condition

$$\begin{cases}\dfrac{dx_2}{dt}>\mu(r_+,\ s_+)>\lambda(r_+,\ s_+),\\[2mm] \mu(r,s)>\dfrac{dx_2}{dt}>\lambda(r,\ s),\end{cases} \qquad (2$$

which is an equivalent from of the entropy condition.

It easily follows from (2.17) that

$$\begin{cases}(1+\bar{A})\dfrac{dr}{dr_+}=1+\bar{B},\\[2mm] (1+\bar{A})\dfrac{dr}{ds_+}=1-\bar{B},\end{cases} \qquad (2$$

where

$$\bar{A}=\frac{p'(\tau(s-r))\tau'(s-r)(\tau(s_+-r_+)-\tau(s-r))-\tau'(s-r)(p(\tau(s-r))-p(\tau(s_+-r_+))}{2\sqrt{(p(\tau(s-r))-p(\tau(s_+-r_+)))(\tau(s_+-r_+)-\tau(s-r))}} \qquad (2$$

$$\bar{B}=\frac{p'(\tau(s_+-r_+))\tau'(s_+-r_+)(\tau(s_+-r_+)-\tau(s-r))-\tau'(s_+-r_+)(p(\tau(s-r))-p(\tau(s_+-r_+))}{2\sqrt{(p(\tau(s-r))-p(\tau(s_+-r_+)))(\tau(s_+-r_+)-\tau(s-r))}}. \qquad (2.23)$$

Noting

$$\begin{cases} p'(\tau) = -A\gamma\tau^{-\gamma-1}, \\ \tau'(s-r) = -\dfrac{1}{\sqrt{A\gamma}}\tau^{(\gamma+1)/2}, \end{cases} \tag{2.24}$$

e have

$$-p'(\tau(s-r))\tau'^2(s-r) = 1. \tag{2.25}$$

oticing (2.19), (2.8) and

$$p'(\tau) < 0, \tag{2.26}$$

nd using the familiar formula

$$\frac{a+b}{2} \geqslant \sqrt{ab} \quad (a,\ b \geqslant 0) \tag{2.27}$$

he sign of equality holds if and only if $a=b$, it follows from (2.22)–(2.23) that

$$\bar{A} \geqslant 1,\ \bar{B} \geqslant 1, \tag{2.28}$$

n which the sign of equality holds if and only if $(r,\ s) = (r_+, s_+)$, namely, no discontinuity. Therefore, it holds on a forward shock that

$$\frac{dr}{dr_+} > 0 \tag{2.29}$$

nd

$$\frac{dr}{ds_+} \leqslant 0 \tag{2.30}$$

the sign of equality of (2.30) holds if and only if $(r,\ s) = (r_+,\ s_+)$, namely, no discontinuity).

Furthermore, it follows from (2.17) in a similar way that

$$\frac{d(r+s)}{d(s-r)} = \frac{-\tau'(s-r)\left(p(\tau(s-r)) - p(\tau(s_+-r_+))\right) - p'(\tau(s-r))\tau'(s-r)\left(\tau(s-r) - \tau(s_+-r_+)\right)}{2\sqrt{\left(p(\tau(s-r)) - p(\tau(s_+-r_+))\right)\left(\tau(s_+-r_+) - \tau(s-r)\right)}} \geqslant 1, \tag{2.31}$$

ence, it holds on a forward shock that

$$0 \leqslant \frac{dr}{ds} < 1 \tag{2.32}$$

nd the sign of equality holds if and only if $(r,\ s) = (r_+,\ s_+)$, i. e., no discontinuity.

Rewriting conditions (2.17), (2.18) on a forward shock $x = x_2(t)$ as

$$r = g(r_+,\ s_+,\ s), \tag{2.33}$$

$$\frac{dx}{dt} = G(r_+,\ s_+,\ r,\ s), \tag{2.34}$$

we have, by (2.32) and (2.29), (2.30), that

$$0 \leqslant \frac{\partial g}{\partial s} < 1, \tag{2.35}$$

$$\frac{\partial g}{\partial r_+} > 0, \tag{2.36}$$

$$\frac{\partial g}{\partial s_+} \leqslant 0 \tag{2.37}$$

and the signs of equality in both (2.35) and (2.37) hold if and only if $(r, s) = (r_+, s_+)$, i. e., no discontinuity.

§ 3. A Class of Discontinuous Initial Value Problems for the System of Isentropic Flow

Now we consider the following discontinuous initial value problem for the sy of one-dimensional isentropic flow (2.4) with the initial data

$$t=0:\ r=\begin{cases} r_0^-(x) \\ r_0^+(x) \end{cases},\quad s=\begin{cases} s_-, & x\leqslant 0 \\ s_+, & x\geqslant 0 \end{cases} \qquad (\ $$

where s_- and s_+ are constants, while $r_0^-(x)$ and $r_0^+(x)$ are smooth functions de on $x\leqslant 0$ and $x\geqslant 0$, respectively. Setting

$$r_\pm = r_0^\pm(\pm 0), \qquad (\ $$

we suppose that

$$(r_+, s_+) \neq (r_-, s_-) \qquad (\ $$

and moreover

$$\begin{cases} s_+ - r_0^+(x) > 0, & \forall x \geqslant 0, \\ s_- - r_0^-(x) > 0, & \forall x \leqslant 0, \end{cases} \qquad (\ $$

i. e., there is no vacuum state at the initial time.

For the corresponding Riemann problem with the following piecewise ons initial data

$$t=0:\ r=\begin{cases} r_- \\ r_+ \end{cases},\quad s=\begin{cases} s_-, & x\leqslant 0, \\ s_+, & x\geqslant 0, \end{cases} \qquad (\ $$

suppose that its similarity solution is composed of constant states, a backw centered rarefaction wave and a forward typical shock, that is to say, there exis state (r_0, s_0) such that (r_-, s_-) and (r_+, s_+) can be connected with (r_0, s_0) by a b ward centered rarefaction wave and a forward typical shock $x = Vt$, respectiv Hence, we have

$$s_0 = s_-,\quad r_0 > r_- \qquad (3$$

and

$$r_0 = g(r_+, s_+, s_0), \qquad (3$$

$$V = G(r_+, s_+, r_0, s_0), \qquad (3$$

moreover, we have

$$\begin{cases} V > \mu(r_+, s_+) > \lambda(r_+, s_+) \\ \mu(r_0, s_0) > V > \lambda(r_0, s_0) \end{cases}, \qquad (3$$

and

$$s_0 - r_0 > s_+ - r_+ > 0. \qquad (3.10)$$

Theorem 1. *Suppose that* $r_0^+(x)$ *and* $r_0^-(x)$ *are bounded,* C^1 *functions on* $x\geqslant 0$ *and* $x\leqslant 0$ *respectively. Suppose that* (3.4) *holds and*

$$\begin{cases} r_0^{+\prime}(x) \geqslant 0, \ \forall x \geqslant 0, \\ r_0^{-\prime}(x) \geqslant 0, \ \forall x \leqslant 0. \end{cases} \tag{3.11}$$

ppose furthermore that the similarity solution to the corresponding Riemann problem .4), (3.5) is composed of constant states, a backward centered rarefaction wave and forward typical shock. Then, the discontinuous initial value problem (2.4), (3.1) mits a unique global discontinuous solution on $t \geqslant 0$ in a class of piecewise continuous d piecewise smooth functions. This solution has a global structure similar to that of e corresponding Riemann problem (2.4), (3.5), namely, the solution only contains a ckward centered rarefaction wave with the origin as its center and a forward shock $=x_2(t)$ passing through the origin. Moreover, on both sides of the shock, the solution a backward rarefaction wave $s \equiv s_-$ and $s \equiv s_+$ respectively, and there never exists any cuum state on $t \geqslant 0$.

Proof By solving the corresponding Cau hy problem for system (2.4) by means the initial data on $x \leqslant 0$, we can get a backward rarefaction wave solution

$$r = r_-(t, x), \ s = s_- \tag{3.12}$$

ı the maximum determinate domain

$$\hat{R}_- = \{(t, x) \mid t \geqslant 0, \ x \leqslant \hat{x}_1(t) \triangleq \lambda(r_-, s_-)t\}. \tag{3.13}$$

oreover, we have

$$\frac{\partial r_-(t, x)}{\partial x} \geqslant 0, \ \forall (t, x) \in \hat{R}_-, \tag{3.14}$$

$$s_- - r_-(t, x) > 0, \ \forall (t, x) \in \hat{R}_-, \tag{3.15}$$

nd $r_-(t, x)$ is bounded.

Since the right boundary $x = \hat{x}_1(t)$ of $\hat{R}_-$ is a backward straight characteristic assing through the origin, on which (r, s) takes the constant value (r_-, s_-), we can niquely determine a backward centered rarefaction wave on the angular domain

$$\hat{\hat{R}}_- = \{(t, x) \mid t \geqslant 0, \ \hat{x}_1(t) \leqslant x \leqslant x_1(t) \triangleq \lambda(r_0, s_-)t\} \tag{3.16}$$

o that (r_-, s_-) can be connected with $(r_0, s_0) = (r_0, s_-)$ by virtue of this centered arefaction wave. In (3.16), $x = x_1(t)$ denotes the backward characteristic passing hrough the origin, on which (r, s) takes theconstant value (r_0, s_-).

Thus, on the domain

$$R_- = \hat{R}_- \cup \hat{\hat{R}}_- = \{(t, x) \mid t \geqslant 0, \ x \leqslant x_1(t)\} \tag{3.17}$$

we obtain a backward rarefaction, wave solution, still denoted by (3.12), and we ıave

$$\frac{\partial r_-(t, x)}{\partial x} \geqslant 0, \ \forall (t, x) \in R_- \backslash \{(0, 0)\}. \tag{3.18}$$

Moreover, by (3.6), (3.10) and (3.15), it still holds that

$$s_- - r_-(t, x) > 0, \ \forall (t, x) \in R_-. \tag{3.19}$$

In a similar manner, by solving the corresponding Cauchy problem for fsystem

(2.4) by means of the initial date on $x \geqslant 0$, we can get a backward rarefaction wave solution

$$r = r_+(t, x), \quad s = s_+ \tag{3.20}$$

on the maximum determinate domain

$$\hat{R}_+ = \{(t, x) \mid t \geqslant 0, \ x \geqslant \hat{x}_2(t)\}, \tag{3.21}$$

where $x = \hat{x}_2(t)$ denotes the forward characteristic passing through the origin, i. e., we have

$$\frac{d\hat{x}_2(t)}{dt} = \mu(r_+(t, \hat{x}_2(t)), s_+), \ \hat{x}_2(0) = 0. \quad ($$

Here, $r_+(t, x)$ is bounded and it holds that

$$\frac{\partial r_+(t, x)}{\partial x} \geqslant 0, \ \forall (t, x) \in \hat{R}_+ \quad ($$

and

$$s_+ - r_+(t, x) > 0, \ \forall (t, x) \in \hat{R}_+. \quad ($$

According to the local existence theorem (cf. [2] or Chapter 6 in [3]), the assumptions of Theorem 1, this discontinuous initial value problem (2.4), admits a unique discontinuous solution in a class of piecewise continuou piecewise smooth functions at least on a localdomain $\{(t, x) \mid 0 \leqslant t \leqslant \delta_0, \ |x|$ with $\delta_0 > 0$ suitably small. Moreover, this solution only contains a backward ce rarefaction wave with the origin as its center and a forward shock $x = x_2(t)$ p through the origin. Observing the entropy condition (2.20) on a forward (which becomes (3.9) at $t = 0$), $x = x_2(t)$ must lie in the interior of the doma and on the right side of $x = x_1(t)$. Therefore, the solution on the right side of $x =$ should be furnished by $(r_+(t, x), s_+)$, and then in order to construct a discontinuous solution containing only a forward shock on the do main $\{t \geqslant 0$ it is only necessary to solve the following typical free boundary problem characteristic boundary for the system of isentropic flow (2.4) on the angular d

$$R = \{(t, x) \mid t \geqslant 0, \ x_1(t) \leqslant x \leqslant x_2(t)\}: \quad ($$

on the given straight characteristic $x = x_1(t)$,

$$s = s_- \quad ($$

with

$$r = r_0, \quad ($$

$$x_1'(t) = \lambda(r_0, s_-), \ x_1(0) = 0; \quad ($$

on the free boundary $x = x_2(t)$,

$$r = g(r_+(t, x), s_+, s), \quad ($$

$$\frac{dx}{dt} = \hat{G}(r_+(t, x), s_+, s), \quad ($$

where

$$\hat{G}(r_+, s_+, s) = G(r_+, s_+, g(r_+, s_+, s), s). \tag{3.31}$$

Moreover, by (2.19), the solution should sat sfy the follow ng property

$$s-r>s_+-r_+(t, x)>0. \tag{3.32}$$

Now we use the results of Theorem 2 and Remark 3 in [1] to prove that on the gular domain R, this typical free boundary problem with characteristic boundary mits a unique globally defined classical solution: $(r(t, x), s(t, x))\in C^1$ and $x_2(t)$ C^2. To do this, it is only necessary to justify all the hypotheses mentioned in §3 [1].

The hypothesis of smoothness (H1) is clearly satisfied. Next we verify (H2). It only necessary to prove that if the typical free boundary problem with characteristic undary (2.4), (3.26)-(3.31) admits a classical solution on the domain

$$R(T)=\{(t, x)\,|\,0\leqslant t\leqslant T, x_1(t)\leqslant x\leqslant x_2(t)\}, \tag{3.33}$$

en the solution must satisfy the following condition:

$$\text{on} \quad x=x_2(t)\ (0\leqslant t\leqslant T),\ \lambda(r, s)<x_2'(t)<\mu(r, s). \tag{3.34}$$

fact, noting (3.31), (2.18) and (2.34), (3.30) can be rewritten as

$$\frac{dx}{dt}=\sqrt{-\int_0^1 p'(h\tau(s-r)+(1-h)\tau(s_+-r_+(t, x)))\,dh}, \tag{3.35}$$

nce, noticing

$$p''(\tau)>0, \tag{3.36}$$

soon as (3.32) holds, we have

$$\sqrt{-p'(\tau(s_+-r_+(t, x)))}<x_2'(t)<\sqrt{-p'(\tau(s-r))}, \tag{3.37}$$

., (3.34) holds.

It remains then to prove (3.32) for $0\leqslant t\leqslant T$ under the assumption that the free undary problem (2.4), (3.26)-(3.31) fadmits a classical solution on $R(T)$. By 10) and noting that $s_0=s_-$ and $r_+(0, 0)=r_+$, (3.32) evidently holds at $t=0$. nce, by continuity, (3.32) holds on an interval $0\leqslant t\leqslant\delta$. Suppose that there exists >0 $(T\geqslant\delta_0>\delta)$ such that (3.32) holds only for $0\leqslant t<\delta_0$, but fails at $t=\delta_0$:

$$s-r=s_+-r_+(\delta_0, x_2(\delta_0))>0, \tag{3.38}$$

ere (r, s) is the value of the solution at the point $(\delta_0, x_2(\delta_0))$. In this case, by 29) (which is equivalent to (2.17)), we have

$$r+s=r_+(\delta_0, x_2(\delta_0))+s_+, \tag{3.39}$$

nce

$$(r, s)=(r_+(\delta_0, x_2(\delta_0)), s_+), \tag{3.40}$$

mely, the shock must disappear at the point $(\delta_0, x_2(\delta_0))$. However, by the nclusion in the last paragraph, (3.34) holds for $0\leqslant t<\delta_0$, therefore the forward aracteristic passing through any point $(t, x)\in R(\delta_0)$ must intersect $x=x_1(t)$ at e and only one point. Since s must be a constant along every forward characteristic, holds on R (δ_0) that

$$s(t, x)\equiv s_-. \tag{3.41}$$

Moreover, since (r_0, s_-) can be connected with (r_+, s_+) by a forward shock, noting (3.10) and (2.35), we have

$$s_- > s_+, \tag{3.42}$$

which, together with (3.41), contradicts (3.40). This contradiction gives (3.32) and then (3.34). In the meantime, we get that this forward shock $x=x_2(t)$ never disappears and (3.41) holds on the existence domain of classical solution, that is the solution must be a backward simple wave.

Furthermore, by (3.34) the backward characteristic passing through any point $(t, x)\in R(T)$ must intersect $x=x_2(t)$ at one and only one point. Since r must constant along every backward characteristic, noting (3.41) and (3.32), it is to see that

$$s-r>0 \tag{3}$$

on the existence domain of classical solution, i. e., there never exists any vac state. Therefore, system (2.4) is actually a genuinely nonlinear and strictly hy bolic system, and this fact is also a basic assumption for using the results in [1

We turn now to the verification of hypothesis of monotonicity (H3). It is needed to prove that on $x=x_2(t)$,

$$\frac{\partial g}{\partial r_+}\left(\frac{\partial r_+(t, x)}{\partial t}+x_2'(t)\,\frac{\partial r_+(t, x)}{\partial x}\right)$$
$$=\frac{\partial g}{\partial r_+}(x_2'(t)-\lambda(r_+(t, x), s_+))\,\frac{\partial r_+(t, x)}{\partial x}\geqslant 0. \tag{3}$$

By (2.36), (3.23) and (3.37), however, (3.44) clearly holds.

Finally, we verify (H4), namely, we want to prove that on $x=x_2(t)$,

$$|\hat{G}(r_+(t, x), s_+, s)|\leqslant a_1(T_0, B)+a_2(T_0, B)|x|, \ \forall 0\leqslant t\leqslant T_0, \ \forall|s|\leqslant B, \tag{3}$$

where $a_1(T_0, B)$ and $a_2(T_0, B)$ are constants depending only on T_0 and B. By (3. we have

$$\hat{G}(r_+(t, x), s_+, s)=\hat{G}(r_+(t, x), s_+, s_-), \tag{3}$$

then, observing that $r_+(t, x)$ is bounded, we immediately obtain (3.45).

By the previous discussion, according to the results of Theorem 2 and Ren 3 in [1], on the augular domain R this typical free boundary problem characteristic bounbary (2.4), (3.26)-(3.32) admits a unique globally de classical solution

$$r=(t, x)\in C^1, \ s=s_- \tag{3}$$

and

$$x=x_2(t)\in C^2, \tag{3}$$

moreover, we have

$$\frac{\partial r(t, x)}{\partial x}\geqslant 0, \ \forall (t, x)\in R. \tag{3}$$

Therefore, the solution is a backward rarefaction wave with the forward s curve $x=x_2(t)$ as its right boundary. The shock $x=x_2(t)$ never disappear in the course of propagation and the solution on the right side of this shock is the backward rarefaction wave (3.20).

The proof of Theorem 1 is complete.

Remark 1. If $r_0^{\pm}(x)$ are only assumed to be bounded, piecewise continuous and ecewise smooth, lnondecreasing and satisfy the following condition

$$\begin{cases} s_+ - r_0^+(x\pm 0) > 0, & \forall x \geqslant 0, \\ s_- - r_0^-(x\pm 0) > 0, & \forall x \leqslant 0, \end{cases} \tag{3.50}$$

en the conclusion of Theorem 1 is still valid, only with a slight modification that e solution involved can also contain some backward centered rarefaction waves th point on the initial axis as center.

As an application of Theorem 1, we consider the interaction problem of a ward typical shock with a backward rarefaction wave. Suppose that a backward refaction wave (especially, a backward centered rarefaction wave)

$$r = r_+(t, x), \ s = s_+ \tag{3.51}$$

ets a forward typical shock

$$(r, s) = \begin{cases} (r_-, s_-), & x \leqslant Vt, \\ (r_+, s_+), & x \geqslant Vt \end{cases} \tag{3.52}$$

the origin, we want to determine the state produced by this interaction.

Here, in order that (3.51) denotes a backward rarefaction wave, $r = r_+(t, x)$ ist be a bounded, C^1 function and satisfies the following properties:

$$r_+(0, 0) = r_+, \tag{3.53}$$

$$\frac{\partial r_+}{\partial t} + \lambda(r_+, s_+)\frac{\partial r_+}{\partial x} = 0 \tag{3.54}$$

1

$$\frac{\partial r_+(t, x)}{\partial x} \geqslant 0, \tag{3.55}$$

$$s_+ - r_+(t, x) > 0. \tag{3.56}$$

the other hand, in order that (3.52) denotes a forward typical shock, it must d that

$$r_- = g(r_+, s_+, s_-), \tag{3.57}$$

$$V = G(r_+, s_+, r_-, s_-) \tag{3.58}$$

l

$$\begin{cases} V > \mu(r_+, s_+) > \lambda(r_+, s_+), \\ \mu(r_-, s_-) > V > \lambda(r_-, s_-), \end{cases} \tag{3.59}$$

$$s_- - r_- > s_+ - r_+ > 0. \tag{3.60}$$

Thus, the interaction problem asks us to solve the discontinuous initial value blem for system (2.4) with the following initial data

$$t = 0: \ r = \begin{cases} r_- \\ r_0^+(x) \end{cases}, \ s = \begin{cases} s_-, & x \leqslant 0, \\ s_+, & x \geqslant 0, \end{cases} \tag{3.61}$$

where

$$r_0^+(x) = r_+(0, x), \tag{3.62}$$

and then we have

$$r_0^+(x) \geqslant 0, \quad \forall x \geqslant 0, \tag{3.63}$$

$$s_+ - r_0^+(x) > 0, \quad \forall x \geqslant 0 \tag{3.64}$$

and

$$r_0^+(+0) = r_+, \tag{3.65}$$

moreover, (r_+, s_+) and (r_-, s_-) can be connected by the forward typical shock (3.52). This discontinuous initial value problem corresponds to a special case of Theorem 1, in which

$$r_0^-(x) \equiv r_- \qquad (:$$

and thesimilarity solution to the corresponding Riemann problem (2.4), (3.5 not contain any backward centered rarefaction wave, then from Theorem 1 w the following corollary.

Corollary 1. *For the system of isentropic flow* (2.4), *the interaction probl a forward typical shock with a backward rarefaction wave* $s \equiv s_+$ *admits a unique discontinuous solution containing only one forward shock* $x = x_2(t)$ *on* $t \geqslant 0$ *in a cl piecewise continuous and piecewise smooth functions. Moreover, after the collision waves, the original backward rarefaction wave* $s \equiv s_+$ *becomes another backward ra tion wave* $s \equiv s_-$, *and there never exists any vacuum state on* $t \geqslant 0$. *Using the notati Theorem* 1, *the global discontinuous solution to this discontinuous initial value pr* (2.4), (3.61) *can be explicitly expressed on* $t \geqslant 0$ *as*

$$r = \begin{cases} r_-, & x \leqslant x_1(t) \triangleq \lambda(r_-, s_-)t, \\ r(t, x), & x_1(t) \leqslant x \leqslant x_2(t), \\ r_+(t, x), & x \geqslant x_2(t), \end{cases} \qquad s = \begin{cases} s_-, & x \leqslant x_2(t), \\ s_+, & x \geqslant x_2(t). \end{cases} \qquad ($$

In particular, if there is a constant state on the right side of the or backward rarefaction wave, then after the interaction of the original fo typical shock with the backward rarefaction wave, the forward shock $x = x_2(t$ becomes a forward typical shock (i. e. with constant speed of propagation) well-known fact is simply denoted by

$$\vec{S}\overleftarrow{R} \Rightarrow \overleftarrow{R}\vec{S}.$$

§ 4. A Class of Discontinuous Initial Value Problems for the System of Isentropic Flow (continued)

We turn now to the discontinuous initial value problem for the syst isentropic flow (2.4) with the following initial data

$$t = 0: \; r = \begin{cases} r_0^-(x), \\ r_0^+(x), \end{cases} \quad s = \begin{cases} s_0^-(x), & x \leqslant 0, \\ s_0^+(x), & x \geqslant 0, \end{cases} \tag{4.1}$$

where $r_0^\pm(x)$ and $s_0^\pm(x)$ are all bounded, C^1 functions with the following properties:

$$\begin{cases} r_0^{+\prime}(x) \geqslant 0, & s_0^{+\prime}(x) \geqslant 0, \quad \forall x \geqslant 0, \\ r_0^{-\prime}(x) \geqslant 0, & s_0^{-\prime}(x) \geqslant 0, \quad \forall x \leqslant 0, \end{cases} \tag{4.2}$$

$$\begin{cases} s_0^{+}(x) - r_0^{+}(x) > 0, & \forall x \geqslant 0, \\ s_0^{-}(x) - r_0^{-}(x) > 0, & \forall x \leqslant 0, \end{cases} \tag{4.3}$$

, there is no vacuum state at the initial time.

Setting

$$r_{\pm} = r_0^{\pm}(\pm 0), \quad s_{\pm} = s_0^{\pm}(\pm 0), \tag{4.4}$$

till suppose that (3.3) holds and that the similarity solution to the correspond- Riemann problem (2.4), (3.5) is composed of constant states, a backward ered rarefaction wave and a forward typical shock, namely, there exists a state

$$(r_0, s_0) = (r_0, s_-) \tag{4.5}$$

that (3.6)-(3.10) and then (3.42) hold.

According to the corresponding local existence theorem (cf. [2] or Chapter 6 in , this discontinuous initial value problem (2.4), (4.1) admits a unique ntinuous solution in a class of piecewise continuous and piecewise smooth tions at least on a local domain $\{(t, x) \mid 0 \leqslant t \leqslant \delta_0, |x| < \infty\}$ ($\delta_0 > 0$ suitably small), this solution only contains a backward centered wave with the origin as its r and a forward shock $x = x_2(t)$ passing through the origin. We want to know her or not this problem admits a global discontinuous solution only containing kward centered wave and a forward shock on $t \geqslant 0$.

We first solve the corresponding Cauchy problem for system (2.4) by means of iitial data on $x \leqslant 0$. Using the results in [4] and [5], we can obtain a bounded, l C^1 solution $(r_-(t, x), s_-(t, x))$ on the maximum determinate domain

$$\hat{R}_- = \{(t, x) \mid t \geqslant 0, x \leqslant \hat{x}_1(t)\}, \tag{4.6}$$

e $x = \hat{x}_1(t)$ stands for the backward characteristic passing through the origin:

$$\frac{d\hat{x}_1(t)}{dt} = \lambda(r_-, s_-(t, \hat{x}_1(t))), \quad \hat{x}_1(0) = 0. \tag{4.7}$$

over, we have

$$(r_-(0, 0), s_-(0, 0)) = (r_-, s_-), \tag{4.8}$$

$$\frac{\partial r_-(t, x)}{\partial x} \geqslant 0, \quad \frac{\partial s_-(t, x)}{\partial x} \geqslant 0, \quad \forall (x, t) \in \hat{R}_-, \tag{4.9}$$

$$s_-(t, x) - r_-(t, x) > 0, \quad \forall (t, x) \in \hat{R}_-. \tag{4.10}$$

Next we solve a backward centered wave problem on the right side of $x = \hat{x}_1(t)$ t (r_-, s_-) and (r_0, s) can be connected at the origin by this centered wave. ding to the results in [6] or in Chapter 7 of [3], the solution of this problem e determined only by the value of the solution on the backward characteristic $x = \hat{x}_1(t)$: $(r_-, \hat{s}_1(t)) \triangleq (r_-, s_-(t, \hat{x}_1(t)))$ and the given value r_0. To show that this centered wave problem can be globally solved, it is only necessary to consider the following discontinuous initial value problem for system (2.4) with the initial data

$$t=0:\ r=\begin{cases} r_0^-(x) \\ r_0 \end{cases},\quad s=\begin{cases} s_0^-(x), & x\leqslant 0 \\ s_-, & x\geqslant 0. \end{cases} \tag{4.11}$$

Since the initial data are nondecreasing, this problem must admit a unique global continuous and piecewise smooth solution on $t>0$, and this solution contains a backward centered wave with the origin as its center, which realizes the connection of (r_-, s_-) with (r_0, s_-) at the origin (cf. [5]). Hence, the preceding centered problem really has a global solution on the domain

$$\hat{\hat{R}}_-=\{(t, x)\ |t\geqslant 0,\ \hat{x}_1(t)\leqslant x\leqslant x_1(t)\},\quad (\cdot$$

where $x=x_1(t)$ denotes the backward characteristic passing through the origin, : which the solution (r, s) has the limit value (r_0, s_-) as $\to 0$. Moreover, the rela similar to (4.9)–(4.10) still hold on $\hat{\hat{R}}_-$.

Thus, we finally get a global solution $(r_-(t, x), s_-(t, x))$ on the domain

$$R_-=\hat{R}_-\cup\hat{\hat{R}}_-=\{(t, x)\,|t\geqslant 0,\ x\leqslant x_1(t)\}.\quad (\cdot$$

This solution is continuous and piecewise smooth for $t>0$, and it holds that

$$\frac{\partial r_-(t, x)}{\partial x}\geqslant 0,\quad \frac{\partial s_-(t, x)}{\partial x}\geqslant 0,\quad \forall (t, x)\in R_-\backslash\{(0, 0)\}\quad (\cdot$$

and

$$s_-(t, x)-r_-(t, x)>0,\quad \forall (t, x)\in R_-.\quad ($$

Setting

$$s_1(t)=s_-(t, x_1(t)),\quad ($$

it is easy to see that $s_1(t)$ is bounded

$$s_1'(t)\leqslant 0,\quad \forall t\geqslant 0\quad ($$

and

$$s_1(0)=s_-.\quad ($$

Moreover, on $x=x_1(t)$ we have

$$s=s_1(t),\quad ($$

$$r=r_0\quad ($$

and

$$x_1'(t)=\lambda(r_0, s_1(t)),\quad x_1(0)=0.\quad ($$

We nowsolve the Cauchy problem for system (2.4) according to the initia on $x\geqslant 0$. This problem admits a unique bounded, global C^1 solution $(r_+(t, x),$ $x))$ on the corresponding maximum determinate domain

$$\hat{R}_+=\{(t, x)\,|t\geqslant 0,\ x\geqslant\hat{x}_2(t)\},$$

where $x=\hat{x}_2(t)$ denotes the forward characteristic passing through the c Furthermore, we have

$$\frac{\partial r_+(t, x)}{\partial x}\geqslant 0,\quad \frac{\partial s_+(t, x)}{\partial x}\geqslant 0,\quad \forall (t, x)\in\hat{R}_+, \tag{4.23}$$

$$s_+(t, x)-r_+(t, x)>0,\quad \forall (t, x)\in\hat{R}_+. \tag{4.24}$$

According to the entropy condition (2.20) on a ofrward shock, for the solution the original discontinuous initial value problem (2.4), (4.1), it is easy to see at the forward shock $x=x_2(t)$ passing through the origin must lie in the interior the domain $\hat{R}_+$ and on the right side of the characteristic $x=x_1(t)$. Therefore, e solution on the right side of $x=x_2(t)$ should be furnished by $(r_+(t, x), s_+(t, x))$, d in order to construct a global discontinuous solution only containing a forward ock on the domain $\overline{\{t\geqslant 0\}\backslash R_-}$, it is only necessary to solve the following typical ee boundary problem with characteristic boundary for system (2.4) on the gular domain

$$R=\{(t, x)\,|\,t\geqslant 0,\ x_1(t)\leqslant x\leqslant x_2(t)\}: \tag{4.25}$$

On the given backward characteristic $x=x_1(t)$, we prescribe the boundary ndition (4.19), where $s_1(t)$ is bounded and satisfies (4.17)—(4.18). Moreover, $x=x_1(t)$ we have (4.20)-(4.21).

On the free boundary $x=x_2(t)$,

$$r=g(r_+(t, x), s_+(t, x), s), \tag{4.26}$$

$$\frac{dx}{dt}=\hat{G}(r_+t, x), s_+(t, x), s), \tag{4.27}$$

ere $\hat{G}$ is still defined by (3.31). Moreover, the solution should satisfy the lowing property:

$$\text{on}\quad x=x_2(t),\quad s-r>s_+(t, x)-r_+(t, x)>0. \tag{4.28}$$

Since $s_1(t)$, $r_+(t, x)$ and $s_+(t, x)$ are all bounded, by (4.19) and (4.26), it is ily seen that the classical solution $(r(t, x), s(t, x))$ to this free boundary problem, any, must be bounded.

Now we compute the values of the first derivatives of the solution $(r(t, x)$, $s(t,$) at the origin: $\partial r/\partial x(0, 0)$ and $\partial s/\partial x(0, 0)$.

We first prove that

$$\frac{\partial s}{\partial x}(0, 0)=s_0^{-\prime}(0)\geqslant 0. \tag{4.29}$$

fact, the forward characteristic passing through any point $(t, x_1(t))$ on $x=x_1(t)$ >0) must intersect the initial axis at one and only one point $(0, \bar{x})$ $(\bar{x}<0)$. Noting 18), we have

$$\begin{aligned} s_1'(0)&=\lim_{t\to 0}\frac{s_1(t)-s_-}{t}=\lim_{t\to 0}\frac{s_0^-(\bar{x})-s_-}{(\mu-\lambda)t}(\mu-\lambda)\\ &=(\lambda(r_0, s_-)-\mu(r_0, s_-))s_0^{-\prime}(0). \end{aligned} \tag{4.30}$$

system (2.4), however, we have

$$\begin{aligned} s_1'(0)&=\frac{\partial s}{\partial t}(0, 0)+x_1'(0)\frac{\partial s}{\partial x}(0, 0)\\ &=(\lambda(r_0, s_-)-\mu(r_0, s_-))\frac{\partial s}{\partial x}(0, 0). \end{aligned} \tag{4.31}$$

Putting (4.30) and (4.31) together, we get (4.29).

Differentiating (4.26) with respect to t and using system (2.4), we have

$$\text{on}\quad x=x_2(t),\ (x_2'(t)-\lambda(r,s))\frac{\partial r}{\partial x}$$
$$=\frac{\partial g}{\partial r_+}(x_2'(t)-\lambda(r_+(t,x),s_+(t,x)))\frac{\partial r_+(t,x)}{\partial x}$$
$$+\frac{\partial g}{\partial s_+}(x_2'(t)-\mu(r_+(t,x),s_+(t,x)))\frac{\partial s_+(t,x)}{\partial x}$$
$$+\frac{\partial g}{\partial s}(x_2'(t)-\mu(r,s))\frac{\partial s}{\partial x}. \quad (4$$

Setting $t=0$ in (4.32) and noting (4.29), it comes that

$$(V-\lambda(r_0,s_-))\frac{\partial r}{\partial x}(0,0)=\frac{\partial g}{\partial r_+}(r_+,s_+,s_-)(V-\lambda(r_+,s_+))r_0^{+\prime}(0)$$
$$+\frac{\partial g}{\partial s_+}(r_+,s_+,s_-)(V-\mu(r_+,s_+))s_0^{+\prime}(0)$$
$$+\frac{\partial g}{\partial s}(r_+,s_+,s_-)(V-\mu(r_0,s_-))s_0^{-\prime}(0), \quad (4$$

where

$$V=\hat{G}(r_+,s_+,s_-)=G(r_+,s_+,r_0,s_-) \quad (4$$

and (3.9) (in which s_0 is replaced by s_-) holds.

Then, we have the following theorem.

Theorem 2. *Under the previous assumptions, if*

$$\frac{\partial g}{\partial r_+}(r_+,s_+,s_-)(V-\lambda(r_+,s_+))r_0^{+\prime}(0)$$
$$+\frac{\partial g}{\partial s_+}(r_+,s_+,s_-)(V-\mu(r_+,s_+))s_0^{+\prime}(0)$$
$$+\frac{\partial g}{\partial s}(r_+,s_+,s_-)(V-\mu(r_0,s_-))s_0^{-\prime}(0)<0, \quad (4.$$

then the discontinuous initial value problem (2.4), (4.1) *never admits a gl discontinuous solutiond only containing a backward centered wave and a forward s on* $t\geqslant 0$.

Proof By (4.33), (4.35) and (3.9), we have

$$\frac{\partial r}{\partial x}(0,0)<0. \quad (4.$$

Observing that the solution is bounded, this theorem follows directly from the e in Remark 4 of [1].

As applications of Theoremr 2, we discuss now the interaction problem forward typical shock with a forward rarefaction wave.

We first consider the interaction problem of a forward rarefaction wave

$$r=r_-,\quad s=s_-(t,x) \quad (4.37)$$

catching up wrth a forward typical shock (3.52) at the origin. In order to determine the state produced by this interaction it is only needed to solve the following liscontinuous nitial value problem for system (2.4) with the initial data

$$t=0:\ r=\begin{cases} r_- \\ r_+ \end{cases},\quad s=\begin{cases} s_0^-(x), & x\leqslant 0 \\ s_+(x), & x\geqslant 0, \end{cases} \tag{4.38}$$

vhere

$$s_0^-(x)=s_-(0,\ x) \tag{4.39}$$

vith

$$s_0^{-\prime}(x)\geqslant 0,\quad \forall x\leqslant 0, \tag{4.40}$$

$$s_0^-(x)-r_->0,\quad \forall x\leqslant 0 \tag{4.41}$$

ınd

$$s_0^-(0)=s_-. \tag{4.42}$$

Moreover, $(r_+,\ s_+)$ can be connected with $(r_-,\ s_-)$ by the forward typical shock (3.52). This problem can be regarded as a specal case in Theorem 2, in which

$$r_0^\pm(x)\equiv r_\pm,\quad s_0^+(x)\equiv s_+, \tag{4.43}$$

ınd the similarity solution to the corresponding Riemann problem (2.4), (3.5) loes not contain any backward centered rarefaction wave, then we have the 'ollowing corollary.

Corollary 2. *For the system of isentropic flow, the interaction problem of a forward rarefaction wave catching up with a forward typical shock never admits a global discontinuous solution containing only one forward shock on* $t\geqslant 0$.

Proof In the case

$$s_0^{-\prime}(0)>0, \tag{4.44}$$

ıoticing (4.43), (2.35) and (3.9), this corollary is a direct consequence of Theorem 2.

We now prove that even if (4.44) fails, the conclusion of thes corollary is still rue. As a matter of fact, since, as we have assumed, the interactioh bagins at the rigin, there exists $x_0<0$ such that $|x_0|$ is small enough and

$$s_0^{-\prime}(x_0)>0. \tag{4.45}$$

Observe that the typical free boundary problem with characteristic boundary, orresponding to the discontinuous initial value problem (2.4), (4.38), always dmits a locally defined classical solution, and then on the free boundary $x=x_2(t)$ here exists a point $(t_1,\ x_2(t_1))$ sufficiently close to the origin such that the forward haracteristic passing through this point intersects the initial axis at the point $(0,\ x_0)$. Hence, by means of the method in [1], it is easy to prove that

$$v(t_1,\ x_2(t_1)) = \frac{e^{k(r_-,\ s_0^-(x_0))}\, s_0^{-\prime}(x_0)}{1+\int_0^{t_1} \frac{\partial\mu}{\partial s}\left(r(\tau,\ x(\tau,\ x_0)),\ s_0^-(x_0)\right) s_0^{-\prime}(x_0)\, e^{k(r_-,\ s_0^-(x_0))-k(r(\tau,\ x(\tau,\ x_0)),\ s_0^-(x_0))}\,d\tau}, \tag{4.46}$$

where $x=x(\tau,\ x_0)$ denotes the forward characteristic passing through the point $(0,\ x_0)$,

$$v=e^{k(r,s)}\frac{\partial s}{\partial x} \tag{4.47}$$

and $k(r, s)$ is determined by

$$\frac{\partial k}{\partial r}=\frac{\dfrac{\partial \mu}{\partial r}}{\mu-\lambda}. \tag{4.48}$$

Thus, by (4.45) and the genuinely nonlinear hypothesis (2.11), we get

$$\frac{\partial s}{\partial x}(t_1, x_2(t_1))>0. \qquad (4.4$$

Noting (2.35), (2.20) and

$$r_+(t, x)\equiv r_+, \quad s_+(t, x)\equiv s_+, \qquad (4.5$$

it follows from (4.32) (at the point $(t_1, x_2(t_1))$) that

$$\bar{r}_0'(t_1)=(x_2'(t_1)-\lambda(r, s))\frac{\partial r}{\partial x}(t_1, x_2(t_1))<0, \qquad (4.5$$

where

$$\bar{r}_0(t)=r(t, x_2(t)). \qquad (4.5$$

Thus, Remark 4 in [1] directly gives the desired conclusion. The proof of Corolla 2 is then completed.

We next consider the interaction problem of a forward typical shock (3.5 catching up with a forward rarefaction wave

$$r=r_+, \quad s=s_+(t, x) \qquad (4.5$$

at the origin. Similarly, in order to determine the state produced by this interactic it is only necessary to solve the following discontinuous inital value problem system (2.4) with the initial data

$$t=0: \; r=\begin{cases} r_-, \\ r_+, \end{cases} \quad s=\begin{cases} s_-, & x\leqslant 0, \\ s_0^+(x), & x\geqslant 0, \end{cases} \qquad (4.5$$

where

$$s_0^+(x)=s_+(0, x) \qquad (4.5$$

with

$$s_0^{+\prime}(x)\geqslant 0, \quad \forall x\geqslant 0, \qquad (4.5$$

$$s_0^+(x)-r_+>0, \quad \forall x\geqslant 0 \qquad (4.5$$

and

$$s_0^+(0)=s_+, \qquad (4.5$$

moreover, (r_+, s_+) can be connected with (r_-, s_-) by the forward typical sh (3.52). This problem can be also regarded as a special case in Theorem 2, in whi

$$r_0^{\pm}(x)\equiv r_{\pm}, \quad s_0^-(x)\equiv s_- \qquad (4.$$

and the similarity solution to the corresponding Riemann problem (2.4), (3.5) d not contain any backward centered rarefaction wave. We have the follbowing corollary.

Corollary 3. *For the system of isentropic flow, the interaction problem of a* *ward typical shock catching up with a forward rarefaction wave never admits a global scontinuous solution containing only one forward shock on* $t\geqslant 0$.

Proof The proof is similar to that of Corollary 2. If

$$s_0^{+\prime}(0)>0, \tag{4.60}$$

is corollary follows directly from Theorem 2. If (4.60) fails, there must exist a fficiently small number $x_0>0$ such that

$$s_0^{+\prime}(x_0)>0, \tag{4.61}$$

ıen we can similarly prove that on the free boundary $x=x_2(t)$ there exists a point $(t_1, x_2(t_1))$ sufficiently close to the origin such that

$$\frac{\partial s_+}{\partial x}(t_1, x_2(t_1))>0. \tag{4.62}$$

oting (2.37), (2.20) and

$$r_+(t, x)\equiv r_+, \quad s(t, x)\equiv s_-, \tag{4.63}$$

y (4.32) we can get (4.51) and then the desired conclusion.

Corollary 2 and Corollary 3 tell us that for the system of isentropic flow, new ngularities must occur in a finite time in the solution to the interaction problem : a forward (resp. backward) typical shook with a forward (resp. backward) ırefaction wave. Therefore, in these cases we should seek the global discontinuous lution in a wider class of functions instead of in a class of piecewise continuous ıd piecewise smooth functions.

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