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ON INNERπ'-CLOSED GROUPS AND NORMAL π-COMPLEMENTS

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Abstract

In this paper, the author classifies the finite inner π'-closed groups, and proves the following results

1. If each proper subgroup K of a group G is weak π-homogeneous and weak π'-homogeneous, then G is a Schmidt group, or a direct product of two Hall subgroups.

2. If G is a weak π-homogeneous group, then G is π'-closed if one of the following statements is true: (1) Each π-subgroup of G is 2-closed. (2) Each π-subgroup of G is 2'-closed.

3. Let G be a group and π be a set of odd primes. If $N_G(Z(J(P)))$ has a normal π-completement for a Sylow p-subgroup of G with prime p in π, then so does G.

§ 1. Introduction

All groups in this paper are finite. Let π be a set of primes and let π' be complementary set of π. Let G be a group. If G has a normal Hall π'-subgro then G is called a π'-closed group, or in other words, G has a normal π-compleme A group G is said to be π-homogeneous if $N_G(K)/C_G(K)$ is a π-group for each subgroup K of G. $K\neq 1$, G is said to be weak π-homogeneous if $N_G(P)/C_G(P)$ i π-group for each p-subgroup P of G with prime p in π. It is easy to prove that G π-homogeneous if G has a normal π-complement. A group G is called a D_π-grou any two of its maximal π-subgroups are conjugate in G. Let Σ be an abstr group theoretical property. A group G is called an inner Σ-group if every pro subgroup of G is a Σ-group, but G itself is not. According to this definition Schmidt group is an inner nilpotent group.

If $\pi=\{p\}$, then a p-homogeneous gronp has a normal p-complement Frobenius' Theorem ([1], Theorem 9.12). However, there are groups (see Theor 2.4) which are π-homogeneous, but not π'-closed. Z. Arad and D. Chillag pro the following theorem in [2].

Theorem A. *If π is a set of odd primes and a group G is π-homo* geneous, *then*

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has a normal π-complement.

Z. Arad proved the following theorem in [3].

Theorem B. *Le π be a set of primes which includes 2. Assume all π-subgroups G are π'-closed. Then G is π'-closed if and only if G is π-homogeneous.*

In [4], M. Enguehard and L. Puig proved the following theorem.

Theorem C. *Let $\pi=\{p, q\}$, where p and q are odd primes and the order of mod p) is even. If G is weak π-homogeneous, hen G has a normal π-complement.*

In [5], the author suggested the following question (Problem 2.11, p. 17).

Suppose every proper subgroup of a group G is the direct product of a Hall π-bgroup and a Hall π'-subgroup, but G itsself is not of this form. Is G a Schmidt oup?

Using Theorem A, we know the answer is affirmative. we obtain a stronger sult as follows:

Theorem 3.3. *If each proper subgroup K of a group G is weak π-homogeneous d weak π'-homogeneous, then G is an inner nilpotent group (Schmidt group), or a rect product of a Hall π-subgroup and a Hall π'-subgroup.*

It is easy to prove that each subgroup and any homomorphic image of a weak -homogeneous group are also weak π-homogeneons (see Lemma 3.1). In this paper follow the approach developed in [6] to derive a sufficient condition for a weak -homogeneous group to have a normal π-complement. First, we has a classfication eorem for inner π'-closed groups:

Therem 2.1. *If a group G is inner π'-closed, then one of the following tements holds.*

(1) *G is an inner nilpotent group of order $p^\alpha q^\beta$ where $p\in\pi'$, $q\in\pi$, and the Sylow -subgroup of G is normal.*

(2) *$C/\Phi(G)$ is a nonabelian simple group, and the following statements are true:*

(a) *$\phi(G)=O_\pi(G)\times O_{\pi'}(G)$, and $O_\pi(G)\subseteq Z(G)$;*

(b) *$G/\phi(G)$ is also an inner π'-closed group;*

(c) *G is π-homogeneous;*

(d) *$2\in\pi\cap\pi(G)$, and he nonabelian simple proper subgroups of $G/\Phi(G)$ are π-roups.*

From this theorem, we obtain a theorem whicn generalizes Theorems A–C:

Theorem 3.4. *Assume G is a weak π-homogeneous group. Then G has a normal -complement if one of the following statements is satisfied:*

(1) *Each π-subgroup of G is 2-closed.*

(2) *Each π-subgroup of G is 2′-closed.*

Using this theorem, we can generalize Glauberman's Theorem ([7], Theorem 8.3.1.) as follows

Theorem 3.5. *Let G be a group and π be a set of odd primes. If $N_G(Z(J(P)))$ has a normal π-complement for a Sylow p-subgroup P of G with prime p in π, then so dose G.*

Finally, we obtain the following result:

Theorem 3.6. *Let G be a group. Then G is weak π-homogeneous if and only if G is π-homogeneous.*

The notation in this paper is standard and follows [7].

§2. Inner π'-Closed Groups

We now prove Theorem 2.1, which generalizes Theorem 1 of [8].

Proof of Theorem 2.1. Let N be a maximal normal subgroup of G. G/N simple. Since N is a proper subgroup of G, N is π'-closed by the hypothesis of t theorem. Now we show that G/N is not a π-group. If G/N is a π-group and let be the normal Hall π'-subgroup of N, then H char N and H is a normal Hall subgroup of G. This is a contradiction. Thus G/N is not a π-group. We have t cases.

Case 1. $N \not\subseteq \Phi(G)$.

By the definition of $\Phi(G)$, there exists a maximal subgroup M of G, $N \not\subseteq$ such that $G=MN$. Thus $G/N \cong N/M \cap N$. Since M has a normal π-complement the hypothesis, $M/M \cap N$ and G/N have also normal π-complements. Since G/N a simple non-π-group, G/N is a π'-group. Furthermore, N is π'-closed, so G is separable. Theorem 6.8 of [1] implies there is a Hall π'-subgroup in G, say Set $H=N \cap H_1$ then H is a Hall π'-subgroup of N and H char N; so $H \trianglelefteq G$. Si $N \trianglelefteq G$, H_1 acts on N by conjugation and H is an H_1-invariant normal subgroup N. Thus π'-group H_1 acts on π-group N/H. From Theorem 7.6 of [1], there ex a Sylow q-subgroup Q of N for each q in $\pi(N/H)$ such that QH/H is an invariant Sylow q-subgroup of N/H. Thus $QHH_1=QH_1 \leqslant G$. If $H_1Q<G$, then is π'-closed by the hypothesis. So $H_1<H_1Q$ and $Q \subseteq N_G(H_1)$. Since G/N is a group, it follows that $\pi \cap \pi(N)=\pi \cap \pi(G)$ and Q is a Sylow q-subgroup of G. Si $H_1Q<G$ and $\pi \cap \pi(N)=\pi \cap \pi(G)$, there exists a Sylow r-subgroup R of G such t RH/H is H_1-invariant and $H_1R<G$ for each $r \in \pi \cap \pi(G)$. This yields $N_G(H_1)=$ This is a contradiction and we have proved that $G=H_1Q$.

Now we shall prove that $Q \trianglelefteq G$. First, there exists a nonidentical subgroup of Q such that $Q_1 \trianglelefteq G$. If not, then $N_G(Q_0)<G$ for every nonidentical subgroup Q Q. This forces $N_G(Q_0)$ to have a normal π-complement by the hypothesis. Fr Theorem 9.12 of [1], G has a normal π-complement, a contradiction. If $Q=Q_1$, then $Q \trianglelefteq G$. If $Q_1<Q$, we investigate $\bar{G}=G/Q_1$. Obviously, each proper subgroup of

$\overline{G}$ has a normal π-complement. If $\overline{G}$ is π'-closed, then H_1Q_1/Q_1 is the normal Hall $'$-subgroup of $\overline{G}$. Since $Q_1 \leqslant Q$, H_1Q_1 is a nontrivial normal subgroup of G and $I_1Q_1 = H_1 \times Q_1$, H_1 char H_1Q_1. Thus $H_1 \trianglelefteq G$, a contradication. So $\overline{G}$ is also inner $'$-closed. Furthermore, we claim that $Q_1 \subseteq \Phi(G)$. Indeed, if $Q \not\subseteq \Phi(G)$, then there xists a maximal subgroup M_1 of G such that $G = M_1Q_1$. But $G/Q_1 \cong M_1/M_1 \cap Q_1$ has normal π-complement if M_1 dose. This is a contradiction. So $Q_1 \subseteq \Phi(G)$ and learly $\Phi(G/Q_1) = \Phi(G)/Q_1$. Thus we have $N/Q_1 \subseteq \Phi(G/Q_1)$ and N/Q_1 is a maximal ormal subgroup of G/Q_1. Hence G/Q_1 has a normol Sylow q-subgroup Q/Q_1 by nduction, then so dose G and we have proved $Q \trianglelefteq G$.

Finally, we show that H_1 is a p-group. Let $p \in \pi(H_1)$, P be a Sylow p-group of G. Then $PQ \leqslant G$. If $PQ < G$, then $PQ = P \times Q$ and $Q \subseteq N_G(P)$. This forces $\subseteq N_G(H_1)$ by the choice of p, and so $H_1 \leqslant G$. This contradiction proves that $H_1 =$ $'$ and $G = PQ$. Hence G is an inner nilpotent group.

Case 2. $N \subseteq \Phi(G)$.

Clearly, $N = \Phi(G) = O_\pi(G) \times O_{\pi'}(G)$ and $G/\Phi(G) = G/N$ is a nonabelian simple roup Let y be any π'-element of G, then $O_\pi(G)\langle y\rangle \leqslant G$, and $O_\pi(G)\langle y\rangle = O_\pi(G) \times \langle y\rangle$ y the hypothesis of the theorem. This implies that $O_\pi(G) \leqslant C_G(y)$ for each π'-ement y of G. We claim that $C_G(x) = G$ for any x in $O_\pi(G)$. If not, then there xists an element x of $O_\pi(G)$ such that $C_G(x) < G$. However, G is inner π'-closed, so $_G(x)$ has a normal Hall π'-subgroup H. Since each π'-subgroup of G centralizes x, I is the unique Hall π'-subgroup of G, a contradiction. Thus $C_G(x) = G$, and onsequently $O_\pi(G) \subseteq Z(G)$. So (a) holds.

If $G/\Phi(G)$ is π'-closed, then $G/\Phi(G)$ is a π'-group, for it is simple non-π-group. hus $O_\pi(G)$ is a Hall π-subgroup of G. But $O_\pi(G) \subseteq Z(G)$ by (a), this implies that has a normal π-complement. This is a contradiction. Hence $G/\Phi(G)$ is also inner $'$-closed and (b) holds.

Let Q beany nonidentical π-subgroup of G. If $Q \trianglelefteq G$, then $P \subseteq O_\pi(G)$ and the $'$-elements of G centralize Q. So we conclude that $N_G(Q)/C_G(Q)$ is a π-group. On e other hand, if Q is not normal in G, then $N_G(Q) < G$ and $N_G(Q)$ has a normal -complement. Set $M = N_G(Q)$, Then $N_M(Q) = N_G(Q)$, $C_M(Q) = C_G(Q)$ and $N_G(Q)/$ $_G(Q) = N_M(Q)/C_M(Q)$ is a π-group. Hence (c) holds.

From Theorem of [2] and (c), we obtain $2 \in \pi \cap \pi(G)$. Moreover, if there exists nonabelian simple proper subgroup of $G/\Phi(G)$, say $L/\Phi(G)$, then $L/\Phi(G)$ is π'-osed by the hypothesis. Since groups of odd order are solvable, $2 \in \pi(L/\Phi(G))$ and $/\Phi(G)$ is a π-group. So (d) holds and the proof of the theorem is complete.

From Theorem 2.1 and Theorem of [9], we have

Corollary 2.2. *If finite group G is a D_π-group and every proper subgroup of G has a normal π-complement, then one of the following statements holds:*

(1) *G is inner nilpotent; or*

(2) *G has a normal π-complement.*

Itö ([10], Theorem 3.3) proved that an inner p'-closed group is an inner nilpotent group. Clearly, Corollary 2.2 generalizes Itô's result.

A group G is perfect if $G'=G$. A group X is qusisimple if X is perfect and $X/Z(X)$ is simple, where X is called a covering group of simple $X/Z(X)$. Schur[11] showed that a nonabelian simple group X has a universal covering grpup $\hat{X}$ such that every covering group of X is a homomorphic image of $\hat{X}$, and $Z(\hat{X})$ is ca Schur multiplier of X. By the table 4.1 of [12], we get the following lemma.

Lemma 2.3. (1) *If $\hat{X}/Z(\hat{X})\cong PSL_2(r)$ where $r>3$ and $r\neq 9$ is an odd pr power, then $\hat{X}\cong SL_2(r)$ and $|Z(\hat{X})|=2$.*

(2) *If $\hat{X}/Z(\hat{X})\cong PSL_2(2^r)$ where $r\geqslant 3$, then $\hat{X}\cong PSL_2(2^r)$.*

(3) *If $\hat{X}/Z(\hat{X})\cong PSL_3(r)$ where r is an odd prime power, hen $\hat{X}\cong PSL_3($*

(4) *If $\hat{X}/Z(\hat{X})\cong S_z(2^r)$ where $r>3$, then $\hat{X}\cong S_z(2^r)$; if $\hat{X}/Z(\hat{X})\cong S_z(2^3)$, $Z(\hat{X})$ is an elementary abelian group of order 4.*

Now we can state

Theorem 2.4. *If group G is inner π'-closed and each π-subgroup of solvable, then either G is inner nilpotent, or one of the following statements holds.*

(1) *$G/\Phi(G)\cong PSL_2(r)$, where $r>3$ is a prime, $r^2\not\equiv 1 \pmod 5$, and 2, 3 $O_\pi(G)\subseteq Z(G)$, $|O_\pi(G)|\leqslant 2$, and if r is a π-number, then $r-1$ is a π-number, $(r+1)/2$ is not.*

(2) *$G/\Phi(G)\cong PSL_2(2^r)$, where $r\geqslant 3$ is a prime, $2\in\pi$, 2^r-1 is a π-number $O_\pi(G)=1$.*

(3) *$G/\Phi(G)\cong PSL_2(3^r)$, where $r\geqslant 3$ is a prime, 2, $3\in\pi$, 3^r-1 is a π-nu and $O_\pi(G)\subseteq Z(G)$, $|O_\pi(G)|\leqslant 2$.*

(4) *$G/\Phi(G)\cong PSL_3(3)$, where 2, $3\in\pi$ and $O_\pi(G)=1$.*

(5) *$G/\Phi(G)\cong S_z(2^r)$, where $r\geqslant 3$ is a prime, $2\in\pi$, 2^r-1 is a π-number and e $O_\pi(G)=1$ if $r>3$, or $O_\pi(G)\subseteq Z(G)$ is an elementary abelian group of order 2^e $e\leqslant 2$ if $r=3$.*

Proof Let G be an inner π'-closed group and each π-subgroup of G is solv If G is not inner nilpotent, then $G/\Phi(G)$ is a minimal nonabelian simple grou Theorem 2.1 and the assumption of the theorem. From the theorem of [13] $\Phi(G)$ is isomorphic to one of the followsing

1. $PSL_2(r)$, prime $r>3$, $r^2\not\equiv 1 \pmod 5$;
2. $PSL_2(2^r)$, prime $r\geqslant 3$;
3. $PSL_2(3^r)$, prime $r\geqslant 3$;
4. $PSL_3(3)$;
5. $S_z(2^r)$, prime $r\geqslant 3$.

Let $G/\Phi(G)\cong PSL_2(q)$, $q=r$, 2^r, or 3^r with prime $r\geqslant 3$. Set $\bar{G}=G/\Phi(G)$. By ckson's results ([14], p. 414), any maximal subgroup of $\bar{G}$ is isomorphic to one of e following groups:

(a) A dihedral group of order $2(q+1)/d$, where $d=(2, q-1)$;

(b) A Frobenius group $\bar{M}$ whose kernel is a Sylow q-subgroup $\bar{S}_q$ of $\bar{G}$, and $/\bar{S}_q$ is a cyclic group of order $(q-1)/d$, $d=(2, q-1)$;

(c) Alternating group A_4, or symmetric group Σ_4, if $q\neq 2^r$.

Assume $G/\Phi(G)\cong PSL_2(r)$ with prime $r>3$, then $2, 3\in\pi$ by (c) and Theorem 1. If $r\in\pi$, then $r-1$ is a π-number by (b). Since $G/\Phi(G)$ is inner π'-closed, $+1)/2$ is not a π-number. Conversely if $r\notin\pi$ or $r\in\pi$ and $(r+1)/2$ is not a π-mber, $PSL_2(r)$ is obviously an inner π'-closed group under the assumption that $3\in\pi$. By Theorem 2.1, $O_\pi(G)\subseteq Z(G)$ and $Z(G/O_{\pi'}(G))\cong(O_\pi(G)\times O_{\pi'}(G))/O_{\pi'}(G)$. erefore $G/\Phi(G)\cong(G/O_{\pi'}(G))/Z(G/O_{\pi'}(G))\cong PSL_2(r)$. Hence $|Z(G)/O_{\pi'}(G)|=$ $_\pi(G)|\leqslant 2$ by Lemma 2.3, and the statement (1) of the theorem follows. Similarly, can prove (2) and (3).

As above, we can prove (4) and (5) by using the results of [15] and [16], pectively. This completes the proof of the theorem.

From Burnside's $p^a q^b$-Theorem([7], p.131) and Theorem 2.4 we obtain a classifica-n of inner $\{p, q\}'$-closed groups.

Corollary 2.5. *If group G is an inner $\{p, q\}'$-closed group with $p<q$, $p, q\in$ G) then one of the following statements holds.*

(1) $G/\Phi(G)\cong PSL_2(r)$, where $r>2$ is a prime, $r^2\not\equiv 1 \pmod 5$, $\{2, 3\}=\{p, q\}$, $(G)\subseteq Z(G)$ and $|O_\pi(G)|\leqslant 2$.

(2) $G/\Phi(G)\cong PSL_2(2^r)$, where $r\geqslant 3$ is a prime, $p=2$, $q=2^r-1$ is a Mersenne ime, and $O_\pi(G)=1$.

(3) $G/\Phi(G)\cong PSL_3(3)$, $p=2$, $q=8$, and $O_\pi(G)=1$.

(4) $G/\Phi(G)\cong S_z(2^r)$, where $r\geqslant 3$ is a prime, $p=2$, $q=2^r-1$ is a Mersenne ime. Moreover, either $O_\pi(G)=1$ if $r>3$, or $O_\pi(G)\subseteq Z(G)$ is an elementary abelian oup of order 2^e with $e\leqslant 2$ if $r=3$.

§ 3. Normal π-Complement

Clearly we have the following lemma.

Lemma 3.1. *Any subgroup and any homomorphic image of a weak π-nogeneous group are weak π-homogeneous.*

Lemma 3.2. *A group G has a normal π-complement if and only if G is a π-separable weak π-homogeneous group.*

Proof. The necessity of the lemma is obvious. We should only prove the sufficiency of the lemma. Suppose the conclusion is false and let G be a counterexample of the smallest order. From the minimality of G, we know that each proper subgroup of G has a normal π-complement; in other words, G is an inner π'-closed group. However, G is a D_π-group by Theorem 6.8 of [1], so G is inner nilpotent by Corollary 2.2. From Frobenius' theorem, we know this is contrary to the minimality of G, and the lemma is proved.

Proof of Theorem 3.3. Assume G is a minimal counterexample to the theorem. Then G is neither π-group nor π'-group, and for each maximal subgroup M of G, M is either a direct product of a Hall π-subgroup and a Hall π'-group or an inner nilpotent group of order $p^\alpha q^\beta$. However, M is both weak π-homogeneous and weak π'-homogeneous, so M is not an inner nilpotent group of order $p^\alpha q^\beta$ with $p\in\pi'$ and $q\in\pi$. Hence each proper subgroup of G is both π-closed and π'-closed. By Theorem 3.1, G is either π-closed or π'-closed. We assume G is π-closed. By the minimality of G, G is inner π'-closed and is not inner nilpotent. Hence G is π-homogeneous by Theorem 2.1. Since G is π-separable, G is π'-closed by Lemma 3.2, a contradiction. So the theorem holds.

Proof of Theorem 3.4 We shall prove the theorem by a reduction to absurdity. Let G be a minimal counterexample to the theorem. Then G is inner π'-closed by the minimality of G. Theorem 2.1 implies G is either inner nilpotent or nonsolvable and $G/\Phi(G)$ is a nonabelian simple group. By Lemma 3.2, G is not inner nilpotent. On the other hand, $G/\Phi(G)$ is also weak π-homogeneous and each π-subgroup of $G/\Phi(G)$ is also 2-closed, or each π-subgroup of $G/\Phi(G)$ is also 2'-closed. So $\Phi(G)=1$ by Theorem 2.1 and the minimality of G. Theorem 2.1 now implies G is π-homogeneous. If each π-subgroup of G is 2-closed, then Theorem A and Theorem B imply G is π'-closed, a contradiction. We now assume each π-subgroup of G is 2'-closed. Since a 2'-closed group is solvable, each π-subgroup of G is solvable. From Theorem 2.4, G possesses non-2'-closed π-subgroups. This is a contradiction and the proof is complete.

Let p be a prime, P be a p-group, $\mathscr{A}(P)$ be the set of abelian subgroups of P of maximal order. Define

$$J(P)=\langle A \mid A\in\mathscr{A}(P)\rangle.$$

Let p be an odd prime and let P be a Sylow p-subgroup of a group G. A normal subgroup T of P is said to be a control strong fusion in P with respect to G, if it has the following property:

If $W\subseteq P$, $g\in G$, and $W^g\subseteq P$, then there exist $c\in C_G(W)$ and $h\in N_G(T)$ such that $ch=g$.

Proof of Theorem 3.5 Let $p\in\pi\cap\pi(G)$ and P be a Sylow p-subgroup of G. Denote $M=N_G(Z(J(P)))$. Since M has a normal π-complement, $N_M(K)/C_M(K)$ is a π-

group for any subgroup K of $Z(J(P))$. By the Corollary 2 of [17], $Z(J(P))$ controls strong fusion in P with respect to G. Let W be a subgroup of P and $g\in N_G(W)$. Then there exist $c\in C_G(W)$ and $h\in N_G(Z(J(P)))$ such that $g=ch$ by the definition stated above. Furthermore, $h\in N_G(W)\cap M$ and $W\subseteq M$, so $h\in N_M(W)$. Since M has a normal π-complement, M is π-homogeneous and there exist $c_1\in C_M(W)$ and a π-element $h_1\in N_M(W)$ such that $h=c_1h_1$. Hence $g=cc_1h_1$ with $c, c_1\in C_G(W)$ and $h_1\in N_G(W)$. This forces $N_G(W)/C_G(W)$ to be a π-group; in other words, G is weak π-homogeneous. By Theorem 3.4, G has a normal π-complement, and the theorem is proved.

Proof of Theorem 3.6 The necessity is clear. We should only prove the sufficiency of the theorem. Assume G is a minimal counterexample for the theorem. Then every proper subgroup of G is π-homogeneous, and there exists a π-subgroup W of G such that $N_G(W)/C_G(W)$ is a non-π-group. Set $N=N_G(W)$. Then $N_N(W)=N_G(W)$ and $C_N(W)=C_G(W)$. If $N<G$, then $N_N(W)/C_N(W)=N_G(W)/C_G(W)$ is a π-group, for N is π-homogeneous. This is a contradiction. So $N_G(W)=G$, and $W\trianglelefteq G$. Assume x is a π'-element of G, then $\langle x\rangle W$ is a subgroup of G. If $\langle x\rangle W=G$, then W is a normal Hall π-subgroup, and G is π'-closed by Lemma 3.2, hence $G=\langle x\rangle W$ is π-homogeneous, a contradictiction. Therefore $\langle x\rangle W$ is a proper subgroup of G, and $\langle x\rangle W$ is a π-homogeneous group. Thus $x\in C_G(W)$, and each π'-element of G centralizes W by the choice of x. So $N_G(W)/C_G(W)$ must be a π-group. This is a contradiction and the theorem is proved.

From Corallary 2.5, we can prove

Theorem 3.7. *Let $\pi=\{2, p\}$, where p is not a Mersenne prime. Assume a group G is weak π-homogeneous. Then G has a normal π-complement.*

Remark. From Theorem 3.6, the argument of Theorem 3.4 under the assumption of (1) is a direct consequence of Theorem A and Theorem B.

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References

[1] Kurzweil, H. Endliche Gruppen, Springer, Berlin, 1977.
[2] Arad, Z. & Chillag, D., A criterion for the existence of normal π-compltements in finite groups, *J. Alg.*, **87** (1984), 472—482.
[3] Arad, Z., π-Homogeneity and π'-closure of finite groups, *Pacific J. Math.*, **51** (1974), 1—9.
[4] Enguehard, M. & Puig, L. Un critère déx'stence de $\{p, q\}$-complement normal, *J. Alg.*, **55** (1978), 281—292.
[5] Johnson, D. J., Unsolved problems in group theory, Amer. Math. Soc. Trans., Series 2, Vol. 121, Amer. Math. Soc., Providence, R. I., 1983.
[6] Chen Zongmu, Inner groups, *Acta Math. Sinica*, **23**: 2(1980), 239—243; **24**: 4(1981), 331—335.
[7] Gorenstein, D., Finite groups, Harper and Row, New York, 1968.

[8] Chen Zongmu, Inner p-closed groups, *Adv. Math. Sinica*, **15**:4(1986), 385—388.
[9] Ferguson, P., On π'-closure of π-homogeneous groups, *Proc. Amer. Math. Soc*, **66**(1977), 9—12.
[10] Rboinson, D, J. S., A course in the theory of groups, Springer, Berlin, New York, 1980.
[11] Schur, I., Untersuchen Über die Darstellung der endlichen Gruppen durch gebrochenen linearen Substitutionen, *Crell J.*, **182**(1907), 85—137.
[12] Gorenstein, D., Finite simple groups, Plenum, NewYork, 1982.
[13] Thompson, J. G., Nonsolvable finite groups all of whose local subgroups are solvable, I-VI, *Bull. Amer. Math. Soc.*, **74**(1968), 383—437; *Pacific J. Math.*, **33**(1970), 451—536; **39**(1971), 483—534; **48**(1973), 511—592; **50**(1974), 215—297; **51**(1974), 573—630.
[14] Suzuki, M., Group theory I, Springer, Berlin, New York, 1982.
[15] Bloom, D. M., The subgroups of PSL(3, q) for odd q, *Trans. Amr..Math. Soc.*, **127**(1967), 150—178.
[16] Suzuki, M., On a class of double transtive gaoups, I-II, *Ann. of Math.*, **75**(1962), 105—145; **79**(1964 514—589.
[17] Glauberman, G., A sufficient condition for p-stability, *Proc. London Math. Soc.*, **25**(1972), 253—287.