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ON INTERIOR G-ALGEBRAS

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Abstract

In this paper, the main objects are interior G-algebras. We shall give some properties and relations between defect groups and vertices of interior G-algebras without the usual assumption that the residual field F is sufficiently large. Besides, we observe the behaviors of the defect groups under restrictions and inductions of interior algebras, and then obtain a theorem like extended Green Correspondence.

§ 0. Introduction

Let G be a finite group, p a prime number, and θ a p-adic complete discrete valuation ring with a maximal ideal (π) and residual field $F=\theta/(\pi)$ of characteristic p. All A-modules for a θ-algebra A are finitely generated and free as θ-modules.

In the paper, the main objects are interior G-algebrss and their defect groups. In [4], J. A. Green defined the concept of G-algebras and their defect groups and got some properties of the defect groups. M. Broué and L. Puig defined the interior G-algebras which are a kind of special G-algebras (see [2], [8] and [9]). Then T. Ikeda discussed epimorphic local interior G-algebras and obtained some properties of their vertices as $G\times G$-modules (see [5] and [6].)

In our section 1, we show that the assumption that F is sufficiently large is not necessary for Ikeda's main result. In fact, we extend the Ikeda's key lemma to a general ring-theoretic case. In section 2, we continue the discussion of relations between defect groups and vertices of interior G-algebras, and give a sufficent condition of another extreme case in Ikeda's theorem. In the last section, section 3, we study behaviors of the defect groups under restrictions and inductions of interior algebras, and obtain a theorem like Extended Green Correspondence, which is, as we show, a proper extension of the original Green Correspondence.

Throughout this paper, all the notations and results without explanations follow [7] or [3]. For example, if H, K are two subgroups of G, by $H\leqslant_G K$ we

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mean that H is G-conjugate to a subgroup of K, and for two G-modules M, N, by $N|M$ we mean that N is isomorphic to a direct summand of M.

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§ 1. The Extension of Ikeda' s Theorem

Before we start, let us have some definitions and notations as follows. An interior G-algebra (A, ρ) is a θ-algebra A with a θ-algebra homomorphism from $\theta[G]$ to A. Sometimes we also simply say A is an interior G-algebra instead of (A, ρ). For $a\in A$, $g\in G$, we write a. g and g. a instead of a. $\rho(g)$ and $\rho(g)$. a. When ρ is epimorphic, we call (A, ρ) an epimorphic interior G-algebra. For any $(g, h)\in G\times G$ and $a\in A$, defining $(g, h).a = g\cdot a\cdot h^{-1}$, we can make A an $G\times G$-module. If $H\leqslant G$, we write

$$H^{\Delta}: =\{(h, h)\,|\,h\in H\} \text{ and}$$
$$A^{H}: =\{a\in A\,|\,hah^{-1}=a \text{ for any } h\in H\}.$$

An interior G-algebra (A, ρ) is called local one if A^G is local. An interior G-algebra A can become a G-algebra with the action defined by $a^g: =g^{-1}ag$ for any $a\in A$ and $g\in G$, and therefore has defect groups when A is local (see [4]) which are called the defect groups of the interior G-algebra (A, ρ), and denoted by $D(A)$ for one of them usually. If (A, ρ) is epimorphic local, then as a $G\times G$-module A is indecomposable (see [5]) and has vertices. Denote by $Vtx_{G\times G}A$ one of the vertices of A. If B is a block of $\theta[G]$ and $\rho(B)\neq 0$, then we say the local interior G-algebra belongs to B. If V is a $\theta[G]$-module, P is a p-subgroup of G, we denote by $V(P)$ the θ-module $V^P/\sum_{Q<P} V^P_Q+(\pi)V^P$ where Q runs over the family of proper subgroups of P and V^P consists of the fixed points of V under the action of P.

In [5], T. Ikeda discussed the vertices of an interior G-algebra in the case of F being a splitting field for G. as follows.

Theorem 1. 1. *Let (A, ρ) be an epimorphic local interior G-algebra with a defect group D belonging to a block B of $\theta[G]$. Then we have*

$$D^{\Delta}{}_{G\times G} \leqslant Vtx\,{}_{G\times G}A_{G\times G}\leqslant D\times D$$

and $D^{\Delta}{}_{G\times G}=Vtx_{G\times G}A$ if and only if the restriction $\rho|_B: B\to A$ is an isomorphism of θ-algebra; in the case, D is a defect group of the block B.

The proof of this theorem depends mainly on the following essential lemma (see [5]).

Lemma 1. 2. *Let (A, ρ) be an epimorphic local interior G-algebra belonging to a block B. If A is a projective $\theta[G]$-module under left multiplication through ρ,* *then ρ induces an isomorphism between B and A.*

Now, we extend the lemma to a general ring-theoretic case, and prove it simply and straightly.

Lemma 1.3. *Let R be a ring with unit 1_R such that:*

i) *There is a primitive idempotent decomposition of 1_R in R: $1_R=\sum_{i=1}^{s} f_i$, where f_i is a primitive idempotent of R, $1\leqslant i\leqslant s$.*

ii) *Rf_i and Rf_j are linked for any $1\leqslant i, j\leqslant s$* (see [7] for the definition).

Let A be another ring with unit 1_A, ρ: $R\to A$ an epimorphism of ring with $\rho(1_R)=1_A$. Then ρ is isomorphic if and only if the left R-module ${}_RA$ induced by ρ is projective.

Proof Let ${}_RA$ be projective. If $\rho(f_i)\neq 0$ for some $i\in\{1, \cdots\cdots, s\}$, then the epimorphism

$$\rho|_{Rf_i}: Rf_i\to A\rho(f_i)$$

splits as $A\rho(f_i)$ is a projective R-module. Hence $\rho|_{Rf_i}$ is an isomorphism as f_i is a primitive idempotent. For any $j\in\{1, \cdots\cdots, s\}$ if $f_jRf_i\neq 0$, then there exists $\phi\in(Rf_j, AP(f_i))^R$ and $\phi\neq 0$ for $\varphi\in(Rf_j, AP(f_i))^R\simeq(Rf_j, Rf_i)\simeq^R f_jRf_i\neq 0$. It follows that $\varphi(f_j)\neq 0$. Hence $f_j\cdot AP(f_i)\neq 0$ as $\varphi(f_j)=f_j\varphi(f_j)\in f_jAP(f_i)$. Thus we have $\rho(f_j)\neq 0$. Therefore $\rho|_{Rfj}:Rf_j\to A\rho(f_j)$ is isomorphic for any $j\in\{1, \cdots\cdots, s\}$ by ii). And

$$\rho: R=\bigoplus_{j=1}^{s} Rf_j\to\bigoplus_{j=1}^{s} A\rho(f_j)=A \text{ is an ismorphism.}$$

Corollary 1.4. *Let A be a ring with unit 1_A, a d ρ: $\theta[G]\to A$ an epimorphism of ring with $\rho(1)=1_A$. If the center $Z(A)$ of A is local and B is a block of $\theta[G]$ such that $\rho(B)\neq 0$, then ρ induce an isomorphism between B and A if and only if ${}_GA$ is a projective $\theta[G]$-module.*

Proof It is easy to see that $\rho(B)=A$. Then the corollary is clear by Lemma 1.3.

Obviously, Corollary 1.4 implies Lemma 1.2 without any assumption on F. Therefore, as in [5], Theorem 1.1 holds in general case.

§ 2. Relations Between Defect Groups and Vertices

Definition 2.1. *Let (A, ρ), (A', ρ') be two interior G-algecras, φ: $A\to A'$ a θ-algebra homomorphism. If $\varphi(\rho(g).a)=\rho'(g)\varphi(a)$ and $\varphi(a.\rho(g))=\varphi(a)\cdot\rho'(g)$ for any $a\in A$ and $g\in G$, then φ is called a homomorpihsm of interior G-algebras from A to A'.*

If φ: $(A, \rho)\to(A', \rho')$ is an interior G-algebra homomorphism with $\varphi(1_A)=1_{A'}$, then $\rho'=\varphi\circ\rho$; conversely, if (A, ρ) is an interior G-algebra, A' an θ-algebra, and

φ: $A \to A'$ an θ-algebra homomorphism, then (A', ρ'), where $\rho' = \varphi \circ \rho$, is an interior G-algebra, and φ an interior G-algebra homomorphism from A to A'.

Proposition 2. 2. *Let (A, ρ), (A', ρ') be two local interior G-algebras, $\varphi: A \to A'$ an interior G-algebra homomorphism with $\varphi(1_A) = 1_{A'}$. Then $D(A')_G \leqslant D(A)$. Furthermore, if (A, ρ) and (A', ρ') are epimorphic, and $Vtx_{G\times G}A_{G\times G} = Vtx_{G\times G}A'$, then $D((A)_G = D(A')$.*

Proof Let $a \in A^{D(A)}$ such that $1_A = \mathrm{Tr}^G_{D(A)}(a)$. Then $1_{A'} = \varphi(1_A) = \mathrm{Tr}^G_{A(A)}(\varphi(a))$. Hence $D(A')_G \leqslant D(A)$.

Furthermore, if A, A' are epimorphic and $Vtx_{G\times G}A_{G\times G} = Vtx_{G\times G}A'$, then

$$D(A')^{\Delta}_{G\times G} \leqslant D(A)^{\Delta}_{G\times G} \leqslant Vtx_{G\times G}A_{G\times G} = Vtx_{G\times G}A'_{G\times G} \leqslant D(A') \times D(A')$$

by Theorem 1.1. Hence $D(A)_G \leqslant D(A')$.

Corollary 2.3. *Let (A, ρ) be a local interior G-algebra belonging to a block B of $\theta[G]$. Then $D(A)_G \leqslant D(B)$. Furthermore, if ρ is epimorphic, then $D(A)_G = D(B)$ if and only if $D(B)^{\Delta}{}_{G\times G} \leqslant Vtx_{G\times G}A$.*

Proof Clear by Propoistion 2.2 and Theorem 1.1.

Remark Corollary 2.3 shows that $Vtx_{G\times G}B_{G\times G} \leqslant Vtx_{G\times G}A$ implies $D(B)_G = D(A)$. If another incluison holds, namely $Vtx_{G\times G}B_{G\times G} \geqslant Vtx_{G\times G}A$, then $A \simeq B$ by Theorem 1.1. In fact,

$$D(A)^{\Delta}{}_{G\times G} \leqslant Vtx_{G\times G}A_{G\times G} \leqslant Vtx_{G\times G} = D(B)^{\Delta} \leqslant G^{\Delta} \text{ and}$$
$$Vtx_{G\times G}A_{G\times G} \leqslant D(A) \times D(A) \text{ imply that}$$

$D(A)^{\Delta}{}_{G\times G} \leqslant Vtx_{G\times G}A_{G\times G} \leqslant G^{\Delta} \cap (D(A)^x \times D(A)^y)$ for some x, $y \in G$. Thus $Vtx_{G\times G}A_{G\times G} = D(A)^{\Delta}$ and $A \simeq B$ by Theorem 1.1.

When we consider local interior G-algebra (A, ρ_V), where $A := \mathrm{End}_\theta(V)$, V is an indecomposable $\theta[G]$-module with a vertex D belonging to a block B of $\theta[G]$, and ρ_V is the representation of $\theta[G]$ induced by V, we can get a familiar result in representation theory by Proposition 2.2.

Corollary 2. 4. *Let V be an indecomposable $\theta[G]$-module belonging to B. Then V is $D(B)$-projective.*

In section 1, for an epimorphic local interior G-algebra A, an extreme case of $D(A)^{\Delta}{}_{G\times G} = Vtx_{G\times G}A$ was discussed. Now we shall discuss another extreme case of $Vtx_{G\times G}A_{G\times G} = D(A) \times D(A)$.

Theorem 2.5. *Let (A, ρ) be an epimorphic local interior G-algebra with D as a defect group. Let ${}_GA$ be the left G-module under left multiplication through ρ, then ${}_GA$ is D-projective. Moreover, if there exists an indecomposable summand of ${}_GA$ with D as a vertex, then $Vtx_{G\times G}A_{G\times G} = D \times D$.*

Proof Set $S := Vtx_{G\times G}A$. Then $S_{G\times G} \leqslant D \times D$. As the $G \times G$-module A is $D \times D$-projective, $A_{\downarrow G\times l} \mid (A_{\downarrow D\times D}{}^{\uparrow G\times G})_{\downarrow G\times l}$. Hence $A_{\downarrow G\times l}$ is $D \times l$-projective by Mackey Decomposition Theorem. Furthermore, if there exists an indecomposable summand of

${}_GA$ with D as a vertex, then as $A_{\downarrow G\times l}$ is $D\times l$-projective and $A_{\downarrow D\times l}{}^{\uparrow G\times l} | (A_{\downarrow s}{}^{\uparrow G\times G}{}_{\downarrow D\times l})^{\uparrow G\times l} \simeq \beta\in S\backslash\bigoplus^{\beta\in S/(G\times G)/(D\times l)} (\beta\otimes A)_{\downarrow \beta S\cap(D\times l)}{}^{\uparrow G\times l}$ by Mackey Theorem, there exists $\beta\in G\times G$ such that $D\times l_{G\times l}\leqslant S^{\beta}\cap(D\times l)\leqslant S^{\beta}$. By Theorem 1.1, $\leqslant D_{G\times G}\leqslant S_{G+G}\leqslant D\times D$. Hence there exist $\alpha\in G\times G$ such that $S_1: =S^{\alpha}\leqslant D\times D$ and $\alpha'=(g_1, g_2)\in G\times G$ such that $D^{\Delta}\leqslant S_1^{\alpha'}\leqslant D^{g_1}\times D^{g_2}$, which implies $g_1, g_2\in N_G(D)$ and $D^{\Delta}\leqslant S_1^{\alpha'}\leqslant D\times D$. Thus there is $g\in G$ such that $D^{\Delta}\leqslant S_1^{\alpha_1}\leqslant D\times D$. Thus there is $g\in G$ such that $S_1^{\alpha'}\geqslant(D\times 1)D^{\Delta}=D\times D$, that is, $S_{G\times G}\geqslant D\times D$.

Example 1 Let F be a splitting field for G, V a simple $F[G]$-module with a vertex D, ρ_V: $F[G]\to \mathrm{End}_F(V)$ the representation of $F[G]$ introduced by V. Then ρ_V is epimorphic by Burnside Theorem, and $(\mathrm{End}_F(V), \rho_V)$ is an epimorphic local interior G-algebra. Now by Theorem 2.5 $Vtx_{G\times G}$ $(\mathrm{End}_F(V))$ ${}_{G\times G}=D\times D$.

We may also consider the case of $(\mathrm{End}_\theta(V), \rho_V)$ for any indecomposable $\theta[G]$-module V. Let us have a lemma first.

Lemma 2. 6. *Let (A, ρ) be an interior G-algebra such that $1_A\in A_D^G$. Then the $G\times G$-module A induced by ρ is $D\times D$-projective.*

Proof As $1_A\in A_D^G$, by Higman's Criterion, A is $D\times G$-projective and $G\times D$-projective. Hence A is $D\times D$-projective by Mack-y Decomposition Theorem.

Proposition 2. 7. *Let $A=\mathrm{End}_\theta(V)$, where V is an indecomposable $\theta[G]$-module with a vertex D. Let $A=A_1\oplus\cdots\cdots\oplus A_n$ be an indecomposable direct summand decomposition of A as $G\times G$-module. Then A_i has $D\times D$ as a vertex for any $i\in\{1, \cdots\cdots, n\}$.*

Proof It is easy to see that any indecomposable summand of $A_i\downarrow_{G\times 1}$ has a vertex $D\times 1$ as V has a vertex, D. Set $S_i: =Vtx_{G\times G}A_i$. Then $S_{iG\times G}\leqslant D\times D$ by Lemma 2.6. With the same proof as Theorem 2.5, there exists $\gamma\in G\times G$ such that $S_i^{\gamma}\cap(D\times 1)_{G\times G}\geqslant D\times 1$. Hence $S_{iG\times G}\leqslant D\times D$ and we are done.

Remark. We know that $Vtx_{G\times G}B_{G\times G}=Vtx_{G\times G}A$ implies $B\simeq A$ when (A, ρ) is an epimorphic local interior G-algebra belonging to a block B. Does the condition of $D(B)=D(A)$ imply $B\simeq A$? The answer is "No" if $D(B)\neq 1$. We G only consider the case of $(A, \rho)=(\mathrm{End}_F(V), \rho_V)$ where V is a simple $F[G]$-module belonging to B with $D=D(B)$ as a vertex. Then $Vtx_{G\times G}A_{G\times G}=D\times D_{G\times G}\neq D^{\Delta}$ by Example 1. This shows that $B\neq A$.

Before the final proposition of this section which gives a characterization of defect groups of local interior G-algebras, let us introduce here a lemma from [1].

Lemma 2. 8. *Let H be a subgroup of G and P a p-subgroup of G. Let V be an H-projective $\theta[G]$-module. Then*

$$V(P)=V^P/(\sum_{Q<P}V_Q^P+(\pi)V^P)=0 \text{ unless } P_G\leqslant H.$$

Proof See [1] (1.3).

Lemma 2.9. *Let (A, ρ) be a local interior G-algebra with a defect group D.*

Then G^{Δ}-module $A_{\downarrow G^{\Delta}}$ is D^{Δ}-projective.

Proof By Lemma 2.6, $G\times G$-module A is $D\times D$-projective. Thus $A_{\downarrow G^{\Delta}} \mid (A_{\downarrow D\times D}\uparrow^{G\times G})_{\downarrow G^{\Delta}} \simeq \bigoplus_{\gamma\in(D\times D)\backslash(G\times G)/G^{\Delta}}(\gamma\otimes A)_{\downarrow {}^{\gamma}(D\times D)\cap G^{\Delta}}\uparrow^{G^{\Delta}}$ by Mackey Theorem. For any $\gamma=(x, y)\in G\times G$

$$(D^{x}\times D^{y})\cap G^{\Delta}=(D^{x}\cap D^{y})^{\Delta}\leqslant^{G^{\Delta}} D^{\Delta}.$$

Hence $A\downarrow_{G^{\Delta}}$ is D^{Δ}-projective.

Proposition 2. 10. *Let (A, ρ) be a local interior G-algebra with a defect group D. Then D is the maximal of vertices of indecomposable summands of $A_{\downarrow G^{\Delta}}$ up to conjugation in G, where $A_{\downarrow G^{\Delta}}$ is regarded as a $\theta[G]$-module.*

Proof Let $A_{\downarrow G^{\Delta}}=A_1\oplus A_2\oplus\cdots\cdots\oplus A_n$, A_i is indecomposable $\theta[G]$-module, $1\leqslant i\leqslant n$. Then

$$A(D)=A_{\downarrow G^{\Delta}}(D)=A_1(D)\oplus\cdots\cdots\oplus A_n(D)\neq 0.$$

Thus there is $i\in\{1, \cdots\cdots, n\}$ such that $A_i(D)\neq 0$, and then $D^{G}\leqslant Vtx_G\ A_i$ by Lemma 2.8. However, A_j is D-projective for any $j\in\{1, \cdots\cdots, n\}$ by Lemma 2.9. Thus $D^{G}\geqslant Vtx_G\ A_j$ for any $j\in\{1, \cdots\cdots, n\}$, and $D_G=Vtx_G\ A_i$ is the maximal of $\{Vtx_G A_j\}_1^n$.

§ 3. Restrictions and Inductions' Extended Green Correspondence

First of this section, we should introduce some notations and definitions we shall use, which are mainly from [8] or [9]. Let A be a θ-algebra. We denote by $\mathscr{P}(A)$ the set of A^*-conjugacy classes of primitive idempotents of A, where A^* is the group of invertible elements of A. An (G, A)-pointed group is a pair H_α where $H\leqslant G$, $\alpha\in\mathscr{P}_A(H):=\mathscr{P}(A^H)$ and is called a point of H on A. For two pointed groups K_α, H_β, we say K_α is contained in H_β, denoted by $K_\alpha\subseteq H_\beta$, if $K\leqslant H$ and for any $i\in\beta$ there exists $j\in\alpha$ such that $ij=j=ji$. For any pointed group P_γ, if $Br_P^A(\gamma)\neq 0$, i. e. $\gamma\nsubseteq A_Q^P$ for any proper snbgroup Q of P, then γ is called a local point of P on A, and P_γ a local pointed group on A. Let H_β be a pointed group on A. A defect pointed group P_γ of H_β is a local pointed group on A which is maximal fulfilling $P_\gamma\subseteq H_\beta$. A local pointed group P_γ contained in H_β is the maximal one if and only if $\beta\subseteq A_P^H$ (see [8] Theorem 1.2). Let A, B be two interior G-algebras, f: $A\to B$ an interior G-algebra homomorphism such that Ker $(f)=0$ and $\mathrm{Im}(f)=f(1_A)Bf(1_A)$. Then f is called interior G-algebra embedding or simply embedding from A to B, and denoted by f: $A\rightarrowtail B$. Let H_β be a pointed group on an interior G-algebra A, $j\in\beta$. We denote by A_β the interior H-algebra formed by θ-algebra jAj and the group homomorphism $x\mapsto x.j$, from H to $(jAj)^*$. Then the inclusion map $A_\beta\hookrightarrow \mathrm{Res}_H^G(A)$ is an interior H-algebra embedding, and A_β is independent of the choice of j up to interior H-algebra isomorphism. Let H be a subgroup of G, and B an interior H-algebra. The induced algebra $\mathrm{Ind}_H^G(B)$ is the interior G-algebra formed by the $(\theta[G], \theta[G])$-bimodule $\theta[G]\otimes_{\theta[H]}B\otimes_{\theta[H]}\theta[G]$ with the product defined by the formula

$$(x\otimes b\otimes y)(x'\otimes b'\otimes y')=\begin{cases}0 & \text{if } yx'\notin H,\\ x\otimes byx'b'\otimes y' & \text{if } yx'\in H.\end{cases}$$

for any x, x', y, $y'\in G$ and b, $b'\in B$.

and the map: $x\mapsto\sum_y xy^{-1}\otimes 1_B\otimes y$, where y runs over a left transversal to H in G, from $\theta[G]$ to the θ-algebra. There is a natural interior H-algebra mebedding from B to Res_H^G $(\mathrm{Ind}_H^G(B))$ mapping b into $1\otimes b\otimes 1$. We write $B_H\uparrow^G$ instead of $\mathrm{Ind}_H^G(B)$ and $A\downarrow_H$ instead of $\mathrm{Res}\,{}_H^G(A)$ for an interior G-algebra A.

Proposition 3. 1. *Let A be a local interior G-algebra. If A has a defect group D, then there exists a point β of D on A such that D_β is a defect pointed group of G_1 on A. Conversely, if D_β is a defect pointed group of G_1 on A, then D is a defect group of A.*

Proof Let D be a defect group of A. We may show that there is a point β of D such that D_β is local. Let $1=\sum_{k=1}^n e_k$ be a primitive idempotent decomposition of 1 on A^D. If for any $k\in\{1, \cdots\cdots, n\}$ there exists a proper subgroup Q_k of D such that $e_k\in A_{Q_k}^D$, then $1\in\sum_{Q<D}A_Q^D$, and thus $A(D)=0$. This is a contradiction. Therefore there is a point β of D on A such that D_β is a defect pointed group of G_1 as $1\in A_D^G$.

Conversely, if D_β is a defect pointed group of G_1, then $1\in A_D^G$. If $1\in A_P^G$ for some $P<D$, then there is $a\in A^P$ such that $1=\mathrm{Tr}_P^G(a)=\sum_{v\in P/G/D}\mathrm{Tr}_{P^x\cap D}^D(a^x)$. For any $j\in\beta$, we have $j=\sum_x\mathrm{Tr}_{P^x\cap D}^D\ (ja^xj)$ and then there exists $x\in G$ such that $j\in(jAj)_{P^x\cap D}^D$ by Rosenberg's Lemma. This is in contradiction to our assumption.

By Proposition 3.1 there is a sort of equivalence between defect groups and defect pointed groups. In the following, we shall concentrate our attention on defect groups.

Theorem 3. 2. *Let A be a local interior G-algebra with a defect group D, H a subgroup of G.*

i) For any pointed group H_β on A, $j\in\beta$, let $A_\beta=jAj$ be the interior H-algebra as we have mentioned. Then $D(A_\beta)_G\leqslant D$.

ii) If $1_A\in A_H^G$, then there exists a point α of H such that $D(A_\alpha)_G=D$. Moreover, if $D\leqslant H$ then there exists a point β of H such that $D(A_\beta)_H=D$.

Proof i) Let $a\in A^D$ such that

$1=\mathrm{Tr}_D^G(a)=\sum_{x\in D\backslash G/H}\mathrm{Tr}_{D^x\cap H}^H(a^x)$. Thus

$$j=\sum_{x}\mathrm{Tr}^{H}_{D^x\cap H}(ja^xj).$$

So there is $x\in G$ such that $j\in(jAj)^{H}_{D^x\cup H}$ by Rosenberg's Lemma. Hence $D(A_\beta)_H\leqslant D^x\cap H_G\leqslant D$.

ii) If $1\in A^G_H$, we may show that there exists $\alpha\in\mathscr{P}_A(H)$ such that $1\in A^D_G(A_\alpha)$ from which the first statement of ii) follows.

Let $1=\sum_{i\in I} i$ be a primitive idempotent decomposition of 1 on A^H. Then $A^H=\sum_{i\in I}A^HiA^H$ and

$$1\in\mathrm{Tr}^D_H(\sum_i A^HiA^H)\subseteq\sum_i\mathrm{Tr}^G_H(A^HiA^H).$$

Thus there exists an i such that $1\in\mathrm{Tr}^G_H(A^HiA^H)$ again by Rosenberg's Lemma. Let $\alpha\in\mathscr{P}_A(H)$ such that $i\in\alpha$, then $i\in(A\alpha)^H_{G(A\alpha)}\subseteq A^H_{D(A\alpha)}$. Therefore

$$1\in\mathrm{Tr}^G_H(A^H_{D(A\alpha)})=A^G_{D(A\alpha)}.$$

Moreover, let $D\leqslant H$ and let $1=\sum_k k$ be a primitive idempotent decomposition of 1 on A^D and $a\in A^D$ such that $1=\mathrm{Tr}^G_D(a)\in\mathrm{Tr}^G_D(\sum_k A^DkA^D)\subseteq\sum_k\mathrm{Tr}^G_D(A^DkA^D)$. Then there exists k such that $1\in\mathrm{Tr}^G_D(A^DkA^D)$. Let $\delta\in\mathscr{P}_A(D)$ such that $k\in\delta$. Then $1\in\mathrm{Tr}^G_D(A^D(A_\delta)^D_D, A^D)\subseteq\mathrm{Tr}^G_{D'}(A^{D'})$ where D' is a defect group of interior D-algebra A_δ. Hence $D=D'$. Let $\beta\in\mathscr{P}_A(H)$ such that $D_\delta\leqslant H_\beta$. Then $D=D'_H\leqslant D(A_\beta)_G\leqslant D$ by i), and we are done.

Theorem 3. 3. *Let A be a local interior G-algebra, D a p-subgroup of G, $N_G(D)\leqslant H\leqslant G$, β a point of H on A. If D is a defect group of the interior H-algebra A_β, then A has D as a defect group.*

Proof As $D_G\leqslant D(A)$ by Theorem 3.2.i), we should only prove ${}_GA=A^G_D$. Let $j\in\beta$, $j=\mathrm{Tr}^H_D(a)$ for some $a\in A^D$. Set $e=\mathrm{Tr}^G_D(a)$. Then

$$e-j=\mathrm{Tr}^G_D(a)-\mathrm{Tr}^H_D(a)=\sum_{x\in D\backslash G/H}\mathrm{Tr}^H_{D^x\cap H}(a^x)-\mathrm{Tr}^H_D(a)\in A^H_{\mathscr{Y}}$$

where $$\mathscr{Y}:=\{D^x\cap H\mid x\notin H\}.$$

$$e\equiv j \bmod (A^H_{\mathscr{Y}})\not\equiv 0 \bmod (A^H_{\mathscr{Y}}).$$

As A^G is local, $\bar{A}^G:=A^G/A^H_{\mathscr{Y}}\cap A^G$ is local, too. For any positive integer n, $e^n\equiv j^n \bmod (A^H_{\mathscr{Y}})$ and $j^n=j\notin A^H_{\mathscr{Y}}$, $e^n\notin A^H_{\mathscr{Y}}\cap A^G$, i. e. $\bar{e}\notin J(\bar{A}^G)$. Therefore e is invertible and $A^G=A^G\subseteq A^GA^G_D\subseteq A^G_D$.

Definition 3. 4. *Let A be an interior G-algebra, $H\leqslant G$. Then A is called quasi-projective relative to H, or H-quasi-projective, if $1_A\in A^G_H$.*

Set $$\mathscr{Q}(A):=\{H\leqslant G\mid 1_A\in A^G_H\}.$$

Then A is H-quasi projective if and only if $H\in\mathscr{Q}(A)$.

Let V be a $\theta[G]$-module and $A:=\mathrm{End}_\theta(V)$. Then the relative quasi-projectivety of A coincides with the relative projectivity of V by Higman's Criterion.

Now, we consider the relative quasi-projectivity in the viewpoint of inductions.

and restrictions of interior algebras.

Let us introduce a lemma from L. Puig in [8] first.

Lemma 3.5. *Let A be an interior G-algebra, $H\leqslant G$, $1_A\in A_H^G$, $B:=(A_{\downarrow H})^{\uparrow G}$. Then there exists $b\in(B^H)^*$ such that $f\colon A\to B$ mapping a into $(1\otimes a\otimes 1)^b$ is an interior G-algebra embedding.*

(See [8] Lemma 3.8 for the proof.)

Proposition 3.6. *Let A be an interior G-algebra. If there are an interior P-algebra B and an embedding $f\colon A\rightarrowtail B_P^{\uparrow G}$, then A is P-quasi-projective, namely $P\in\mathscr{Q}(A)$. Conversely, if $P\in\mathscr{Q}(A)$, then there exists an embedding from A to $(A_{\downarrow P})^{\uparrow G}$.*

Proof The second statement follows from Lemma 3.5. Set A': $B_P^{\uparrow G}$, $e=f(1_A)$. Then $e\in(A')^G$ and $e=e\cdot 1_{A'}\cdot e=e\cdot\mathrm{Tr}_P^G(1\otimes 1_{BG}\quad\otimes\otimes 1)\cdot e)=\mathrm{Tr}_G^P(e(1\otimes 1_B\otimes 1)e)\in(eA'e)_P^G$. Hence $1_A\in A_P^G$.

Lemma 3.7. *Let B be an interior H-algebra, $A:=B_H^{\uparrow G}$, $P\leqslant H$ such that $P\in\mathscr{Q}(B)$. Then $P\in\mathscr{Q}(A)$.*

Proof Let $b\in B^D$ such that $1_B=\mathrm{Tr}_P^H(b)$. Then

$$1\otimes 1_B\otimes 1=1\otimes\mathrm{Tr}_P^H(b)\otimes 1=\mathrm{Tr}_P^H(1\otimes b\otimes 1)\text{ and}$$
$$1_A=\mathrm{Tr}_H^G(1\otimes 1_B\otimes 1)=\mathrm{Tr}_P^G(1\otimes b\otimes 1)\in A_P^G.$$

Lemma 3.8. *Let A be an interior G-algebra, $P\in\mathscr{Q}(A)$, then $D(A_\alpha)^G\leqslant P$ for any $\alpha\in\mathscr{P}_A(G)$.*

Proof Let $a\in A^P$ such that $1=\mathrm{Tr}_P^G(a)$. Let $i\in\alpha$, $A_\alpha=iAi$. Then $i=\mathrm{Tr}_P^G(iai)\in(A_\alpha)_P^G$.

Corollary 3.9. *Let B be a local interior H-algebra with a defect group D, $A:=B_H{}^{\uparrow G}$. Then $D(A_\alpha)\leqslant^G D$ for any $\alpha\in\mathscr{P}_A(G)$.*

Proof. By Lemma 3.7 and Lemma 3.8.

Lemma 3.10. i) *Let A, B be two interior H-algebras, $f\colon B\rightarrowtail A$ an interior H-algebra embedding. Then there is an interior G-algebra embedding $f^{\uparrow G}\colon B_H^{\uparrow G}\rightarrowtail A_H^{\uparrow G}$ such that $x\otimes b\otimes y\mapsto x\otimes f(b)\otimes y$ for any $x, y\in G$, $b\in B$.*

ii) *Let $D\leqslant H\leqslant G$ and B be an interior D-algebra. Then $g\colon(B_D{}^{\uparrow H})^{\uparrow G}\to B_D{}^{\uparrow G}$ such that*

$$x\otimes_H(h\otimes_D b\otimes_D h')\otimes_H x'\mapsto xh\otimes_D b\otimes_D h'x'\ \text{for any } x, x'\in G,\ h, h'\in H,\ \text{and } b\in B.$$

is an interior G-algebra isomorphism.

iii) *Let B be an interior H-algebra, A an interior G-algebra, $f\colon B\rightarrowtail A_{\uparrow H}$ an embedding. Then there is a point α of G on A such that $f\colon B\rightarrowtail(A_\alpha)_{\downarrow H}$.*

Proof Trivial.

Lemma 3.11. *Let B be an interior H-algebra, $D\in\mathscr{Q}(B)$, $A:=B_H{}^{\uparrow G}$. Let β be the $(A^H)^*$-conjugacy class containing $1\otimes 1_B\otimes 1$ of primitive idempotents of A^H, and $1_A=1_H\otimes 1_{BH}\otimes 1+\sum_{j\in J} j$ primitive idempotent decomposition of 1_A in A^H. Then $B\simeq A_\beta$*

and, for any $j\in J$. *we have* $D^y\cap H\in\mathscr{Q}(A_\gamma)$ *for some* $y\in G\backslash H$ *where* $\gamma\in\mathscr{P}_A(H)$ *such that* $j\in\gamma$.

Proof Let $F: B\rightarrowtail(B_{\downarrow D})^{\uparrow H}$ be the interior H-algebra embedding in Lemma 4.5. Set $B':=(B_{\downarrow D})^{\uparrow H}$ and $A':=(B_{\downarrow D})^{\uparrow G}$. Then we have an interior G-algebra embedding $f:=g\cdot F^{\uparrow G}: A^{F\to G}\rightarrowtail g\ B'^{\uparrow G}_H\simeq A'$ by Lemma 3.10. i), ii). Let $1_{B'}=F(1_B)+e$ be an orthogonal idempotent decomposition of $1_{B'}$ in $(B')^H$. Then

$$\begin{aligned}1_{A'}&=g(Tr_H^G(1_H\otimes 1_{B'}\otimes^H 1))\\&=Tr_H^G(F(1_B))+Tr_H^G(e)\\&=F(1_B)+\sum_{x\in H\backslash G,x\in H}F(1_B)^x)+e+(\sum_{x\in H\backslash G,x\in H}e^x)\\&=F(1_B)+F(1_B)'+e+e'\end{aligned}$$

is an orthogonal idempotent decomposition of $1_{A'}$ in $(A')^H$, where $F(1_B)'=\sum_{x\in H\backslash G,x\in H}F(1_B)^x$ and $e'=\sum_{x\in H/G,x\in H}e^x$.

On the other hand,

$$\begin{aligned}1_{A'}&=Tr_D^G(1_D\otimes 1_{BD}\otimes 1)\\&=Tr_D^H(1_D\otimes 1_{BD}\otimes 1)+\sum_{y\in D\backslash G/H,y\notin H}Tr^H_{D^y\cap H}((1\otimes 1_B\otimes 1)^y)]\\&=F(1_B)+e+d,\text{ where } d=\sum_{y\in D^yG\backslash H,y\notin H}Tr^H_{D^y\cap H}((1\otimes 1_B\otimes 1)^y).\end{aligned}$$

Let $s=\sum_{j\in J'}aj=\sum_{x\in H\backslash G,x\notin H}(1\otimes^H 1_B 1)^x$. Then $f(s)=F(1_B)'$.

For any $j\in J$, we have $(1\otimes 1_B\otimes 1)\cdot j=j\cdot(1\otimes 1_B\otimes 1)=0$ and $s\cdot j=j\cdot s=j$. Then $f(j)F(1_B)=F(1_B)f(j)=0$ and

$$f(j)f(s)=f(s)f(j)=f(j).\ \text{Hence}$$

Therefore $$f(j)\cdot e=f(j)f(s)\cdot e=0=e\cdot f(j).$$

$$f(j)=f(j)1_{A'}f(j)=f(j)df(j)\in\sum_{y\notin H}(f(j)A'f(j))^H_{D^y\cap H}$$

and then there exists $y\notin H$ such that $f(j)\in(f(j)A'f(j))^H_{D^y\subset H}$ by Rosenberg's Lemma. This implies $j\in A^H_{D^y\cap H}$.

Corollary 3. 12. *Let B be a local interior H-algebra, $D\leqslant H$ such that $D\in\mathscr{Q}(B)$, $A:=B_H^{\uparrow G}$. Let $1_A=\sum_{i\in I}i$ be a primitive idempotent decomposition of 1_A in A^G. Then there exists $i'\in I$ and interior H-algebra embedding. $B\rightarrowtail(A_\alpha)_{\downarrow H}$ where $\alpha\in\mathscr{P}_A(G)$ such that $i'\in\alpha$ and $A_\alpha=i'Ai'$. And for any $i\in I\backslash\{i'\}$, $\beta\in\mathscr{P}_A(G)$ such that $i\in\beta$, there is $x\in G\backslash H$ such that $D^x\cap D\in\mathscr{Q}(A_\beta)$.*

Proof As we know, there is an interior H-algebra embedding $B\rightarrowtail(B_H^{\uparrow G})_{\downarrow H}=A_{\downarrow H}$. Let $\delta\in\mathscr{P}_A(H)$ such that $1\otimes 1_B\otimes 1\in\delta$. Let $\alpha\in\mathscr{P}_A(G)$ such that $H_\delta\leqslant G_\alpha$, and $i'\in I\cap\alpha$. Then we have an interior H-algebra embedding $B\rightarrowtail(A_\alpha)_{\downarrow H}$.

For any $i\in I\backslash\{i'\}$ and $\beta\in\mathscr{P}_A(G)$ such that $i\in\beta$, theer exists $y\notin H$ such that $D^y\cap H\in\mathscr{Q}(A_\gamma)$ for any $\gamma\in\mathscr{P}_A(H)$ such that $H_\gamma\leqslant G_\beta$ by Lemma 4.11. Let P be a defect group of A_β such that $P\leqslant D$ as $D\in\mathscr{Q}(A_\beta)$ by Lemma 3.7 and Lemma 3.8.

Then there exists $h\in H$ such that $p^h\leqslant D^y\cap H$ by Theorem 3.2. ii). Set $x=yh^{-1}$. Then $x\notin H$ and $P\leqslant D^x\cap D$.

Lemma 3.13. *Let A be a local ieterior G-algebra, B an interior H-algebra. Let f: $A\rightarrowtail\!\!\!\rightarrow B_H^{\uparrow G}$. Then there exists $\gamma\in\mathscr{P}_B(H)$ such that f: $A\rightarrowtail\!\!\!\rightarrow(B_\gamma)_H^{\uparrow G}$.*

Proof Trivial.

Theorem 3.14 (Extended Green Correspondence).

Let D be a p-subgroup of G, $N_G(D)\leqslant H\leqslant G$.

Set

$$\mathscr{X}:=\{P\leqslant G\,|\,P\leqslant D^g\wedge D, g\in G\backslash H\},$$

$$\mathscr{Y}:=\{P\leqslant G\,|\,P\leqslant D^g\cap H,\ g\in G\backslash H\},$$

$$\mathscr{S}_1:=\{A\,|\,A\text{ is a local interior }G\text{-algebra with a defect gorup }D\},$$

$$\mathscr{S}_2:=\{B\,|\,B\text{ is a local interior }H\text{-algebra with a defect group }D\}.$$

Then there is a one-to-one correspondence between $\mathscr{S}_1$ and $\mathscr{S}_2$ which is characterized as follows:

i) For any $A\in\mathscr{S}_1$, let $1_A=\sum_{j\in J}j$ be a primitive idempotent decomposition of 1_A in A^H, there is unique $f(A)\in\mathscr{S}_2$ such that $f(A)\simeq A_\delta=j_0Aj_0$ for a $j_0\in J$, where $\delta\in\mathscr{P}_A(H)$ such that $j_0\in\delta$, and for any $j\in J\backslash\{j_0\}$ the defect groups of A_β all lie in $\mathscr{Y}$, where $\beta\in\mathscr{P}_A(H)$ such that $j\in\beta$;

ii) For any $B\in\mathscr{S}_2$, let $1_{B_H^{\uparrow G}}=\sum_{i\in I}i$ be a primitive idempotent pecomposition of $1_{B_H^{\uparrow G}}$ in $(B_H^{\rightarrow G})^G$. there is unique $g(B)\in\mathscr{S}_1$ such that $g(B)\simeq(B_H^{\uparrow G})_\alpha$ for an $i_0\in I$, where $\alpha\in\mathscr{P}_{B_H^{\uparrow G}}(G)$ such that $i_0\in\alpha$, and for any $i\in I\backslash\{i_0\}$, $(B_H^{\uparrow G})_\beta$ has a defect group in $\mathscr{X}$ where $\beta\in\mathscr{P}_{B_H^{\uparrow G}}(G)$ such that $i\in\beta$.

iii) In particular, we have

$$g(f(A))\simeq A\text{ and }f(g(B))\simeq B$$

as interior G-or H-algebras.

Proof i) Let $A\in\mathscr{S}_1$. Then $A\rightarrowtail\!\!\!\rightarrow(A_{\downarrow D})^{\uparrow G}$ and there is $\gamma\in\mathscr{P}_A(D)$ such that $A\rightarrowtail\!\!\!\rightarrow(A_\gamma)_D{}^{\uparrow G}$ by Lemma 3.13. Set $B':=(A_\gamma)_D^{\uparrow H}$. Then there exists $\alpha\in\mathscr{P}_{B'}(H)$ such that $A\rightarrowtail\!\!\!\rightarrow(B'_\alpha)_{\rightarrow}^{HG}$. Set $B:=B'\alpha$. Then $D\in\mathscr{Q}(B)$. As $A\rightarrowtail\!\!\!\rightarrow B_H^{\uparrow G}$, B has D as a defect group. Thus $B\in\mathscr{S}_2$.By Theorem 3.2.ii), there is $\delta\in\mathscr{P}_A(H)$ such that D is a defect group of A_δ. Let $j_0\in J$ such that $j_0\in\delta$. As $A\rightarrowtail\!\!\!\rightarrow A\!\downarrow_H\rightarrowtail\!\!\!\rightarrow(B_{\rightarrow}^{HG})_{\downarrow H}$, we have $A_\delta\simeq B$ and for any $j\in J\backslash\{j_0\}$, all the defect groups of A_β lie in $\mathscr{Y}$ by Lemma 3.11 where $\beta\in\mathscr{P}_A(H)$ such that $j\in\beta$.

ii) For any $B\in\mathscr{S}_2$, set $A':=B_H^{\uparrow G}$. Then $B\rightarrowtail\!\!\!\rightarrow A'_{\downarrow H}$. There is $\alpha\in\mathscr{P}_{A'}(G)$ such that $B\rightarrowtail\!\!\!\rightarrow(A'_\alpha)_{\downarrow H}$ by Lemma 3.10. As $D_G\leqslant D(A'_\alpha)$ by Theorem 3.2.i) and $A'_\alpha\rightarrowtail\!\!\!\rightarrow B_H^{\uparrow G}$, A'_α has D as a defect group, and then $A'_\alpha\in\mathscr{S}_1$. Let $i_0\in I$ such that $i_0\in\alpha$. Then for any $i\in I/\{i_0\}$ and $\beta\in\mathscr{P}_{A'}(G)$ such that $i\in\beta$, A'_β has a defect group in $\mathscr{X}$ by Corollary 3.12.

iii) follows from i) and ii).

Theorem 3. 15. *Let A be a local interior G-algebra with D as a defect group. $D\leqslant H\leqslant G$. Then there is a local interior H-algebra B with D as a defect group such that $A \rightarrowtail\!\!\!\rightarrow B_H^{\updownarrow G}$ and $B \rightarrowtail\!\!\!\rightarrow A_{\downarrow H}$.*

Proof Set $K:=N_G(D)$. Let $f(A)$ be the extended Green correspondent of A in $A_{\downarrow K}$. As $f(A)$ has D as a defect group, $H\cap K\in\mathscr{Q}(f(A))$. by Proposition 3.6 and Lemma 3.13, there exists a local interior $K\cap H$-algebra C such that $C\rightarrowtail\!\!\!\rightarrow f(A)_{\downarrow K\cap H}$ and $f(A)\rightarrowtail\!\!\!\rightarrow C_{K\cap H}{}^{\uparrow K}$. Thus C has D as a defect group by Theorem 3.2 and Corollary 3.9. Let B be the extended Green correspondent of C in $C_{H\cap K}{}^{\uparrow H}$. Then

$$A\rightarrowtail\!\!\!\rightarrow f(A)_K{}^{\uparrow G}\rightarrowtail\!\!\!\rightarrow C_{K\cup H}{}^{\uparrow G}\simeq (C_{K\cap H}{}^{\uparrow H})^{\uparrow G}.$$

But $B\rightarrowtail\!\!\!\rightarrow C_{K\cap H}{}^{\uparrow H}$ and B has a defect group D. We have $A\rightarrowtail\!\!\!\rightarrow B_H^{\updownarrow G}$ by ExtendedGreen Correspondence Theorem 3.14.

Moreover $$C\rightarrowtail\!\!\!\rightarrow f(A)_{\downarrow K\cap H}\rightarrowtail\!\!\!\rightarrow (A_{\downarrow K})_{\downarrow K\cap H}=(A_{\downarrow H})_{\downarrow K\cap H}.$$
Then there exists $\gamma\in\mathscr{P}_A(H)$ such that $C\rightarrowtail\!\!\!\rightarrow (A_\gamma)_{\downarrow K\cap H}$ by Lemma 3.10. iii). As A and C both have D as a defect group, Theorem 3.2 asserts that A_γ has D as a defect group, too. Hence $B\simeq A_\gamma$ by Extended Green Correspondence.

Theorem 3. 16. *Let A be a local interior G-alcebra with D as a defect group, $N_G(D)\leqslant H\leqslant G$. Let $f(A)$ be the extended Green correspondent of A in $A_{\downarrow H}$.*

Let A' be an arbitrary interior G-algebra. Then the following are equivalent:

i) $A\rightarrowtail\!\!\!\rightarrow A'$;

ii) $f(A)\rightarrowtail\!\!\!\rightarrow A'_{\downarrow H}$.

Proof Obvious.

Lemma 3. 17. *Let M be a $\theta[G]$-module, N a $\theta[H]$-module. Then*

i) $\mathrm{End}_\theta(M_{\downarrow H})=(\mathrm{End}_\theta(M))_{\downarrow H}$;

ii) $\mathrm{End}_\theta(N_H^{\updownarrow G})\simeq(\mathrm{End}_\theta(N))_H{}^{\uparrow G}$ *as interior G-algebra, where $N_H^{\updownarrow G}$ denotes the usual induced $\theta[G]$-module.*

Proof i) Clear.

ii) Set Φ: $\mathrm{End}_\theta(N)_H^{\updownarrow G}\rightarrow\mathrm{End}_\theta(N_H^{\updownarrow G})$ such that for any $g, g'\in G$, $f\in\mathrm{End}_\theta(N)$

$$g\otimes f\otimes g'\mapsto\Phi(g\otimes f\otimes g'):x\otimes u\mapsto\begin{cases}0, & g'x\notin H,\\ g\otimes f(g'xu), & g x\in H\end{cases}$$

for any $x\in G$ and $u\in N$.

Then it is easy to check that Φ is an interior G-algebra isomorphism.

Now, for any indecomposable $\theta[G]$-module M with D as a vertex, $A:=\mathrm{End}_\theta(M)$ is a local interior G-algebra with D as a defect group. Then we have the extended Green correspondent of A in $A_{\downarrow H}=\mathrm{End}_\theta(M_{\downarrow H})$. Notice that there is a one-to-one correspondence between the primitive idempotent decomposition of 1_A in A^H and the indecomposable direct summand decomposition of $\theta[H]$-module $M_{\downarrow H}$. Hence by Theorem 3.14, if $M_{\downarrow H}=\bigoplus_{j\in J}N_j$, where N_j if indecomposable for all $j\in J$, there is $j_0 \in J$ such that N_{j0} has D as a vertex and all the vertices of N_j lie in $\mathscr{Y}$ for all $j \in J \setminus \{j_0\}$. Similarly, we may consider $N_H^{\uparrow G}$ for any indecomposable $\theta[H]$-module N with D as avertex. Thus by Theorem 3.14. we obtain the original Green Correspondence.

As we know in section 2, a block B of $\theta[G]$ with $D(\neq 1)$ as a defect group is are not $G\times G$-isomorphic to the endomorphism ring $\mathrm{End}_\theta(V)$ of V for some indecomposable $\theta[G]$-module V because they have different vertices as $G\times G$-modules. Hence we can not obtain Theorem 3.14 by simply using original Green Correspondence. This shows that Theorem 3.14 is a proper extension of the originrl Green Correspondence.

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