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POSITIVE FIXED POINTS AND EIGENVECTORS OF NONCOMPACT DECRESING OPERATORS WITH APPLICATIONS TO NONLINEAR INTEGRAL EQUATIONS**

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Abstract

The author first establishes an existence and uniqueness theorem of positive fixed points and a theorem about the structure of positive eigenvectors for noncompact decreasing operators, and then, offers some applications to nonlinear integral equations on unbounded regions.

Keywords Noncompact decreasing operators, Positive fixed point theorems, Eigenvectors, Nonlinear integral equations.
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§1. Introduction

Let the real Banach space E be partially ordered by a cone P of E, i.e., $x \le y$ iff $y-x \in P$. Let $D \subset E$. Operator $A : D \to E$ is said to be decreasing if $x_1 \le x_2$ $(x_1, x_2 \in D)$ implies $Ax_1 \ge Ax_2$. Recall that cone P is said to be normal if there exists a positive constant N such that $\theta \le x \le y$ implies $\|x\| \le N\|y\|$, where θ denotes the zero element of E and N is called the normal constant of P. For details on cone theory, see [1].

In paper [2], an existence and uniqueness theorem was established for decreasing and completely continuous (i.e., continuous and compact) operator $A : P \to P$, which satisfies: for any $x > \theta$ and $0 < t < 1$, there exists $\eta = \eta(x,t) > 0$ such that

$$A(tx) \le [t(1+\eta)]^{-1}Ax. \tag{1.1}$$

This result was applied to a nonlinear integral equation on finite interval which is of interest in nuclear physics (see [2], Theorem 2 and Example 2).

In this paper, we will use a quite different method to drop the condition of complete continuity of A by strengthening the condition (1.1) in some sense. In addition to the existence and uniqueness theorem (see Theorem 2.1 in Section 2), we get more information about the structure of eigenvectors (see Theorem 2.2 in Section 2). Finally, we offer some applications of Theorems 2.1 and 2.2 to nonlinear integral equations on unbounded regions in which the corresponding integral operators are usually noncompact (see Theorem 3.1 in Section 3).

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§2. Main Theorem

Theorem 2.1. *Suppose that* (a) *cone P is normal and operator $A : P \to P$ is decreasing,* (b) $A\theta > \theta$, $A^2\theta \geq \varepsilon_0 A\theta$, *where* $\varepsilon_0 > 0$ *and* (c) *for any* $0 < a < b < 1$ *there exists* $\eta = \eta(a, b) > 0$ *such that*

$$A(tx) \leq [t(1+\eta)]^{-1}Ax, \qquad \forall t \in [a, b], \forall t \in [\theta, A\theta]. \tag{2.1}$$

Then A has exactly one fixed point x^* in P and $x^* > \theta$. Moreover, constructing successively sequence $x_n = Ax_{n-1}$ $(n = 1, 2, 3, \cdots)$ for any initial $x_0 \in P$, we have

$$\|x_n - x^*\| \to 0 \qquad (n \to \infty). \tag{2.2}$$

Proof. Letting

$$u_0 = \theta, u_n = Au_{n-1} \qquad (n = 1, 2, 3, \cdots)$$

and observing the decreasing property of A, it is easy to see that

$$\theta = u_0 \leq u_2 \leq \cdots \leq u_{2n} \leq \cdots \leq u_{2n+1} \leq \cdots \leq u_3 \leq u_1 = A\theta. \tag{2.4}$$

By condition (b),

$$u_2 \geq \varepsilon_0 u_1 > \theta, \tag{2.5}$$

so, (2.4) and (2.5) imply

$$u_{2n} \geq \varepsilon_0 u_{2n+1} \qquad (n = 1, 2, 3, \cdots). \tag{2.6}$$

Let $t_n = \sup\{t > 0 : u_{2n} \geq tu_{2n+1}\}$. Then

$$u_{2n} \geq t_n u_{2n+1} \qquad (n = 1, 2, 3, \cdots) \tag{2.7}$$

and, on account of (2.4) and (2.6) and the fact

$$u_{2n+2} \geq u_{2n} \geq t_n u_{2n+1} \geq t_n u_{2n+3},$$

we have

$$0 < \varepsilon_0 \leq t_1 \leq t_2 \leq \cdots \leq t_n \leq \cdots \leq 1, \tag{2.8}$$

which implies that

$$\lim_{n\to\infty} t_n = t^* \tag{2.9}$$

exists and $0 < t^* \leq 1$. We check

$$t^* = 1. \tag{2.10}$$

In fact, if $t^* < 1$, then $t_n \in [\varepsilon_0, t^*]$ $(n = 1, 2, 3, \cdots)$, and so, by virtue of (2.4), (2.7) and condition (c), there exists an $\eta > 0$ such that

$$\begin{aligned} u_{2n+1} &= Au_{2n} \leq A(t_n u_{2n+1}) \leq [t_n(1+\eta)]^{-1} Au_{2n+1} \\ &= [t_n(1+\eta)]^{-1} u_{2n+2}, \end{aligned}$$

i.e.,

$$u_{2n+2} \geq t_n(1+\eta)u_{2n+1} \geq t_n(1+\eta)u_{2n+3},$$

which implies

$$t_{n+1} \geq t_n(1+\eta) \qquad (n = 1, 2, 3, \cdots),$$

and therefore

$$t_{n+1} \geq t_1(1+\eta)^n \geq \varepsilon_0(1+\eta)^n \qquad (n = 1, 2, 3, \cdots).$$

Hence $t_n \to +\infty$, which contradicts (2.8), and so (2.10) is true. Now, (2.4) and (2.7) imply

$$\theta \leq u_{2n+2p} - u_{2n} \leq u_{2n+1} - u_{2n} \leq (1-t_n)u_{2n+1} \leq (1-t_n)A\theta,$$

and so

$$\|u_{2n+2p} - u_{2n}\| \leq N(1-t_n)\|A\theta\|, \tag{2.11}$$

where N is the normal constant of P. It follows from (2.11), (2.9) and (2.10) that $\lim\limits_{n\to\infty} u_{2n} = u^*$ exists. In the same way, we can prove that $\lim\limits_{n\to\infty} u_{2n+1} = v^*$ also exists. Since

$$u_{2n} \leq u^* \leq v^* \leq u_{2n+1},$$

we have

$$\theta \leq v^* - u^* \leq u_{2n+1} - u_{2n} \leq (1-t_n)A\theta,$$

so

$$\|v^* - u^*\| \leq N(1-t_n)\|A\theta\| \to 0 \qquad (n \to \infty).$$

Hence $v^* = u^*$. Let $x^* = u^* = v^*$. Then $x^* > \theta$ and

$$u_{2n} \leq x^* \leq u_{2n+1} \qquad (n = 1, 2, 3, \cdots).$$

Consequently,

$$u_{2n+1} = Au_{2n} \geq Ax^* \geq Au_{2n+1} = u_{2n+2} \qquad (n = 1, 2, 3, \cdots),$$

and, after taking limit,

$$x^* \geq Ax^* \geq x^*.$$

Hence $Ax^* = x^*$, i.e., x^* is a fixed point of A.

Let $\overline{x}$ be any fixed point of A in P. Then $\overline{x} \geq \theta$, and so

$$u_0 = \theta \leq \overline{x} = A\overline{x} \leq A\theta = u_1.$$

It is easy to show by induction that

$$u_{2n} \leq \overline{x} \leq u_{2n+1} \qquad (n = 1, 2, 3, \cdots),$$

which implies by taking limit that $\overline{x} = x^*$.

Finally, we prove that (2.2) is true. Let $x_0 \geq \theta$. Then

$$u_0 = \theta \leq x_1 = Ax_0 \leq A\theta = u_1$$

and $u_2 \leq x_2 \leq u_1$. By induction, we have

$$u_{2n} \leq x_{2n} \leq u_{2n-1},\ u_{2n} \leq x_{2n+1} \leq u_{2n+1}, \qquad (n = 1, 2, 3, \cdots). \tag{2.12}$$

Hence, (2.2) follows from (2.12) and the fact that P is normal and $u_{2n} \to x^*$ and $u_{2n+1} \to x^*$. The proof is complete.

Theorem 2.2. *Let the conditions* (a) *and* (b) *of Theorem* 2.1 *be satisfied. Suppose that* (c$'$) *for any* $0 < a < b < 1$ *and* $0 < s < 1$, *there exists an* $\eta = \eta(a, b, s) > 0$ *such that*

$$A(tx) \leq [t(1+\eta)]^{-1}Ax, \qquad \forall t \in [a, b], x \in [\theta, s^{-1}A\theta]. \tag{2.13}$$

Then, for any $\lambda > 0$, equation

$$Ax = \lambda x \tag{2.14}$$

has exactly one solution x_λ in P and $x_\lambda > \theta$. Moreover, we have

(i) *x_λ is strictly decreasing, i.e., $0 < \lambda_1 < \lambda_2$ implies $x_{\lambda_1} > x_{\lambda_2}$;*

(ii) *x_λ is continuous, i.e., $\lambda \to \lambda_0$ ($\lambda_0 > 0$) implies $\|x_\lambda - x_{\lambda_0}\| \to 0$;*

(iii) *$\|x_\lambda\| \to 0$ as $\lambda \to +\infty$.*

Proof. For $\lambda \geq 1$, we have

$$\frac{1}{\lambda}A\theta \leq A\theta, \qquad A\Big(\frac{1}{\lambda}A\theta\Big) \geq A^2\theta \geq \varepsilon_0 A\theta,$$
$$\frac{1}{\lambda}A\Big(\frac{1}{\lambda}A\theta\Big) \geq \varepsilon_0\frac{1}{\lambda}A\theta. \tag{2.15}$$

For $0 < \lambda < 1$, condition (c′) implies that there exists an $\eta_\lambda > 0$ such that

$$A^2\theta = A\Big(\lambda\cdot\frac{1}{\lambda}A\theta\Big) \leq [\lambda(1+\eta_\lambda)]^{-1}A\Big(\frac{1}{\lambda}A\theta\Big),$$

and so

$$\frac{1}{\lambda}A\Big(\frac{1}{\lambda}A\theta\Big) \geq (1+\eta_\lambda)A^2\theta \geq A^2\theta \geq \varepsilon_0 A\theta. \tag{2.16}$$

It follows from (2.15) and (2.16) that

$$\frac{1}{\lambda}A\Big(\frac{1}{\lambda}A\theta\Big) \geq \varepsilon_\lambda\frac{1}{\lambda}A\theta, \qquad \forall\lambda > 0, \tag{2.17}$$

where

$$\varepsilon_\lambda = \min\{\varepsilon_0, \lambda\varepsilon_0\} > 0. \tag{2.18}$$

Hence, operator $\frac{1}{\lambda}A$ satisfies all conditions of Theorem 2.1, and therefore, Theorem 2.1 implies that equation (2.14) has exactly one solution x_λ in P, $x_\lambda > \theta$, and, by (2.4),

$$\theta < \frac{1}{\lambda}A\Big(\frac{1}{\lambda}A\theta\Big) \leq x_\lambda \leq \frac{1}{\lambda}A\theta, \qquad \forall\lambda > 0. \tag{2.19}$$

Let $0 < \lambda_1 < \lambda_2$ and $t_0 = \sup\{t > 0 : x_{\lambda_1} \geq t x_{\lambda_2}\}$. Then $x_{\lambda_1} \geq t_0 x_{\lambda_2}$ and, by (2.19) and (2.17),

$$0 < \frac{\lambda_2\varepsilon_{\lambda_1}}{\lambda_1} \leq t_0 < +\infty.$$

We prove $t_0 \geq 1$. In fact, if $0 < t_0 < 1$, then condition (c′) implies that there are $\eta_1 > 0$ and $\eta_2 > 0$ such that

$$\begin{aligned} A(t_0x_{\lambda_2}) &\leq [t_0(1+\eta_1)]^{-1}Ax_{\lambda_2} \\ &= [t_0(1+\eta_1)]^{-1}\lambda_2 x_{\lambda_2}, \\ A\Big(\frac{\lambda_1}{\lambda_2}t_0x_{\lambda_1}\Big) &\leq \Big[\frac{\lambda_1}{\lambda_2}t_0(1+\eta_2)\Big]^{-1}Ax_{\lambda_1} \\ &= \lambda_2[t_0(1+\eta_2)]^{-1}x_{\lambda_1}. \end{aligned}$$

So,

$$x_{\lambda_2} \geq \frac{t_0}{\lambda_2}A(t_0x_{\lambda_2}) \geq \frac{t_0}{\lambda_2}Ax_{\lambda_1} = \frac{\lambda_1}{\lambda_2}t_0x_{\lambda_1},$$
$$\lambda_2x_{\lambda_2} = Ax_{\lambda_2} \leq A\Big(\frac{\lambda_1}{\lambda_2}t_0x_{\lambda_1}\Big) \leq \lambda_2[t_0(1+\eta_2)]^{-1}x_{\lambda_1}.$$

Consequently,

$$x_{\lambda_1} \geq t_0(1+\eta_2)x_{\lambda_2},$$

which contradicts the definition of t_0. Thus $t_0 \geq 1$, and $x_{\lambda_1} \geq x_{\lambda_2}$. If $x_{\lambda_1} = x_{\lambda_2}$, then $\lambda_1 x_{\lambda_1} = Ax_{\lambda_1} = Ax_{\lambda_2} = \lambda_2 x_{\lambda_2} = \lambda_2 x_{\lambda_1}$, which is impossible. Hence $x_{\lambda_1} > x_{\lambda_2}$ and conclusion (i) is proved.

Now, $0 < \lambda_1 < \lambda_2$ implies $x_{\lambda_1} > x_{\lambda_2}$, and so

$$\lambda_1 x_{\lambda_1} = Ax_{\lambda_1} \leq Ax_{\lambda_2} = \lambda_2 x_{\lambda_2}.$$

Hence, by (2.19),

$$\theta < x_{\lambda_1} - x_{\lambda_2} \leq \Big(\frac{\lambda_2}{\lambda_1} - 1\Big)x_{\lambda_2} \leq \Big(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\Big)A\theta,$$

which implies

$$\|x_{\lambda_1} - x_{\lambda_2}\| \leq N\Big(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\Big)\|A\theta\|, \qquad \forall 0 < \lambda_1 < \lambda_2.$$

Thus, $\|x_{\lambda_1} - x_{\lambda_2}\| \to 0$ as $\lambda_2 \to \lambda_1 + 0$ and also as $\lambda_1 \to \lambda_2 - 0$. This shows that conclusion (ii) is true.

Finally, conclusion (iii) follows from the following inequality

$$\|x_\lambda\| \leq \frac{N}{\lambda}\|A\theta\|, \qquad \forall \lambda > 0,$$

which is obtained by (2.19). The proof is complete.

Remark 2.1. It should be pointed out that in Theorems 2.1 and 2.2 we do not assume operator A to be continuous or compact.

§3. Applications

Consider the nonlinear integral equation in whole Euclidean space $\mathbb{R}^n$.

$$\lambda x(t) = \int_{\mathbb{R}^n} k(t,s)\Big\{\sum_{i=0}^{m} a_i(s)(x(s))^{\alpha_i}\Big\}^{-1} ds, \tag{3.1}$$

where

$$0 = \alpha_0 < \alpha_1 < \cdots < \alpha_{m-1} < \alpha_m = 1, \tag{3.2}$$

$k(t,s)$ is measurable and non-negative on $\mathbb{R}^{2n}$ and satisfies

$$0 < \sup_{t \in \mathbb{R}^n} \int_{\mathbb{R}^n} k(t,s)ds < +\infty \tag{3.3}$$

and

$$\lim_{t \to t_0} \int_{\mathbb{R}^n} |k(t,s) - k(t_0,s)|ds = 0, \qquad t_0 \in \mathbb{R}^n; \tag{3.4}$$

$a_i(t)$ $(i = 0, 1, \cdots, m)$ are non-negative bounded measurable functions on $\mathbb{R}^n$ and

$$\operatorname{ess} \cdot \inf_{t \in \mathbb{R}^n} a_0(t) > 0. \tag{3.5}$$

Here $\operatorname{ess} \cdot \inf_{t \in \mathbb{R}^n} a_0(t) = \sup\{m : a_0(t) \geq m, \text{ a.e. on } \mathbb{R}^n\}$.

Theorem 3.1. *Under the conditions mentioned above, for any $\lambda > 0$, equation (3.1) has exactly one non-negative bounded continuous (on $\mathbb{R}^n$) solution $x_\lambda(t)$ and the following conclusions hold:*

(a) $x_\lambda(t)\not\equiv 0$; *constructing successively sequence of functions*

$$x_{\lambda,j}(t)=\frac{1}{\lambda}\int_{\mathbb{R}^n}k(t,s)\Big\{\sum_{i=0}^{m}a_i(s)(x_{\lambda,j-1}(s))^{\alpha_i}\Big\}^{-1}ds\quad (j=1,2,3,\cdots)\tag{3.6}$$

for any initial non-negative bounded continuous (on $\mathbb{R}^n$*) function* $x_{\lambda,0}(t)$*, sequence* $\{x_{\lambda,j}(t)\}$ *converges to* $x_\lambda(t)$ *uniformly on* $\mathbb{R}^n$;

(b) $x_\lambda(t)$ *is strictly decreasing with respect to* λ*, i.e.,* $0<\lambda_1<\lambda_2$ *implies* $x_{\lambda_1}(t)\ge x_{\lambda_2}(t)$ $(t\in\mathbb{R}^n)$ *and* $x_{\lambda_1}(t)\not\equiv x_{\lambda_2}(t)$;

(c) $x_\lambda(t)$ *is continuous with respect to* λ*, i.e.,* $\lambda\to\lambda_0$ $(\lambda_0>0)$ *implies*

$$\sup_{t\in\mathbb{R}^n}|x_\lambda(t)-x_{\lambda_0}(t)|\to 0;$$

(d)

$$\lim_{\lambda\to+0}\sup_{t\in\mathbb{R}^n}x_\lambda(t)=+\infty,\tag{3.7}$$

$$\lim_{\lambda\to+\infty}\sup_{t\in\mathbb{R}^n}x_\lambda(t)=0.\tag{3.8}$$

Proof. Let E be the Banach space of all bounded continuous functions $x(t)$ on $\mathbb{R}^n$ with norm

$$\|x\|=\sup_{t\in\mathbb{R}^n}|x(t)|$$

and $P=\{x\in E: x(t)\ge 0, t\in\mathbb{R}^n\}$. Then, P is a normal cone in E. Define operator

$$Ax(t)=\int_{\mathbb{R}^n}k(t,s)\Big\{\sum_{i=0}^{m}a_i(s)(x(s))^{\alpha_i}\Big\}^{-1}ds.\tag{3.9}$$

It follows easily from (3.2)–(3.5) that A is a decreasing operator from P into P. Let

$$M=\sup_{t\in\mathbb{R}^n}\int_{\mathbb{R}^n}k(t,s)ds<+\infty,$$

$$\overline{a}_0=\operatorname{ess}\cdot\inf_{t\in\mathbb{R}.}a_0(t)>0$$

and

$$a_i^*=\operatorname{ess}\cdot\sup_{t\in\mathbb{R}.}a_i(t)<+\infty,$$

where $\operatorname{ess}\cdot\sup\limits_{t\in\mathbb{R}.}a_i(t)=\inf\{L: a_i(t)\le L,\ \text{a.e. on } \mathbb{R}^n\}$. Then, it is easy to see that $A\theta>\theta$ and $A^2\theta\ge\varepsilon_0A\theta$, where

$$\varepsilon_0=\overline{a}_0\Big\{\sum_{i=0}^{m}a_i^*M^{\alpha_i}(\overline{a}_0)^{-\alpha_i}\Big\}^{-1}>0.\tag{3.10}$$

So, conditions (a) and (b) of Theorem 2.1 are satisfied.

Now, let $0<a<b<1$ and $0<s<1$ be given and let

$$\eta=\overline{a}_0(1-b)\Big\{\overline{a}_0b+b\sum_{i=1}^{m}a_i^*M^{\alpha_i}(s\overline{a}_0)^{-\alpha_i}\Big\}^{-1}>0.\tag{3.11}$$

For any $t_1\in[a,b]$, $x\in[\theta,s^{-1}A\theta]$, we have

$$0\le x(t)\le\frac{1}{s}\int_{\mathbb{R}^n}k(t,s)(a_0(s))^{-1}ds\le M(s\overline{a}_0)^{-1},\qquad t\in\mathbb{R}^n\tag{3.12}$$

and

$$A(t_1x(t)) = \int_{\mathbb{R}^n} k(t,s)\Big\{a_0(s) + \sum_{i=1}^m a_i(s)t_1^{\alpha_i}(x(s))^{\alpha_i}\Big\}^{-1} ds$$
$$\leq t_1^{-1} \int_{\mathbb{R}^n} k(t,s)\Big\{\frac{1}{b}a_0(s) + \sum_{i=1}^m a_i(s)(x(s))^{\alpha_i}\Big\}^{-1} ds. \tag{3.13}$$

On the other hand, from (3.11) we know

$$\Big(\frac{1}{b} - 1 - \eta\Big)\overline{a}_0 = \eta \sum_{i=1}^m a_i^* M^{\alpha_i}(s\overline{a}_0)^{-\alpha_i},$$

and so, by (3.12),

$$\Big(\frac{1}{b} - 1 - \eta\Big)a_0(s) \geq \eta \sum_{i=1}^m a_i(s)(x(s))^{\alpha_i}, \qquad s \in \mathbb{R}^n,$$

which implies

$$\frac{1}{b}a_0(s) + \sum_{i=1}^m a_i(s)(x(s))^{\alpha_i} \geq (1+\eta)\Big\{a_0(s) + \sum_{i=1}^m a_i(s)(x(s))^{\alpha_i}\Big\},$$
$$s \in \mathbb{R}^n. \tag{3.14}$$

It follows from (3.13) and (3.14) that

$$A(t_1x(t)) \leq [t_1(1+\eta)]^{-1} \int_{\mathbb{R}^n} k(t,s)\Big\{a_0(s) + \sum_{i=1}^m a_i(s)(x(s))^{\alpha_i}\Big\}^{-1} ds$$
$$= [t_1(1+\eta)]^{-1} Ax(t), \qquad t \in \mathbb{R}^n. \tag{3.15}$$

So, condition (c′) of Theorem 2.2 is satisfied. Consequently, Theorems 2.1 and 2.2 imply that conclusions (a), (b), (c) and (3.8) are true. It remains to prove (3.7). Suppose that (3.7) is not true. Then, by conclusion (b),

$$\lim_{\lambda\to+0} \|x_\lambda\| = \lim_{\lambda\to+0} \sup_{t\in\mathbb{R}^n} x_\lambda(t) = M_0 < +\infty \tag{3.16}$$

and

$$0 \leq x_\lambda(t) \leq M_0, \qquad \forall \lambda > 0, t \in \mathbb{R}^n. \tag{3.17}$$

For $0 < \lambda < 1$, we define

$$\eta_\lambda = \overline{a}_0(1-\lambda)\Big\{\overline{a}_0\lambda + \lambda\sum_{i=1}^m a_i^* M_0^{\alpha_i}\Big\}^{-1}. \tag{3.18}$$

Similar to the establishment of (3.15) (using (3.17) instead of (3.12) and letting $b = \lambda$), we can prove that

$$A(\lambda x_\lambda) \leq [\lambda(1+\eta_\lambda)]^{-1} Ax_\lambda, \qquad 0 < \lambda < 1. \tag{3.19}$$

By virtue of (2.19), $\lambda x_\lambda \leq A\theta$, and so

$$A(\lambda x_\lambda) \geq A^2\theta \geq \varepsilon_0 A\theta, \tag{3.20}$$

where ε_0 is given by (3.10). It follows from (3.19) and (3.20) that

$$x_\lambda = \frac{1}{\lambda}Ax_\lambda \geq (1+\eta_\lambda)A(\lambda x_\lambda)$$
$$\geq \varepsilon_0(1+\eta_\lambda)A\theta, \qquad 0 < \lambda < 1,$$

and therefore

$$\|x_\lambda\| \geq \varepsilon_0(1+\eta_\lambda)\|A\theta\|, \quad 0<\lambda<1,$$

which implies $\|x_\lambda\| \to +\infty$ as $\lambda \to +0$ since, by (3.18), $\eta_\lambda \to +\infty$ as $\lambda \to +0$. This contradicts (3.16), and hence (3.7) is true. The Proof is complete.

Remark 3.1. It is easy to show that under the conditions of Theorem 3.1 operator A is continuous (from P into P), but usually A is noncompact.

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