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ADMISSIBLE ESTIMATES IN THE IMPORTANT CLASS OF ESTIMATES OF THE COVARIANCE MATRIX**

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Abstract

Let $X_1, \cdots, X_N$ (where $N > m$) be independent $N_m(\mu, \Sigma)$ random vectors, and put

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \quad \text{and} \quad T'T = A = \sum_{i=1}^{N}(X_i - \overline{X})(X_i - \overline{X})',$$

where T is upper- triangular with positive diagonal elements. The author considers the problem of estimating Σ ,and restricts his attention to the class of estimates $\mathbb{D} = \{T'\Delta^* T + Nb^*\overline{X}\,\overline{X}' : \Delta^*$ is any diagonal matrix and b^* is any nonnegative constant$\}$ because it has the following attractive features:

(a) Its elements are all quadratic forms of the sufficient and complete statistics $(\overline{X}, T)$.

(b) It contains all estimates of the form $aA + Nb\overline{X}\,\overline{X}'$ ($a \geq 0$ and $b \geq 0$), which construct a complete subclass of the class of nonnegative quadratic estimates $\mathbb{D}^* = \{X'BX : B \geq 0\}$ (where $X = (X_1, \cdots, X_N)'$) for any strict convex loss function.

(c) It contains all invariant estimates under the transformation group of upper-triangular matrices.

The author obtains the characteristics for an estimate of the form

$$T'\Delta T + Nb\overline{X}\,\overline{X}' \, (\Delta = \text{diag}\{\delta_1, \cdots, \delta_m\} \geq 0 \text{ and } b \geq 0)$$

of Σ to be admissible in $\mathbb{D}$ when the loss function is chosen as $\text{tr}(\Sigma^{-1}\widehat{\Sigma} - I)^2$, and shows, by an example, that $aA + Nb\overline{X}\,\overline{X}'$ ($a \geq 0$ and $b \geq 0$) is admissible in $\mathbb{D}^*$ can not imply its admissibility in $\mathbb{D}$.

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Let $X_1, \cdots, X_N$ be independent $N_m(\mu, \Sigma)$ random vectors, where $N > m$, $\mu \in R^m$ and $\Sigma > 0$ are parameters, and put

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \qquad \text{and} \qquad T'T = A = \sum_{i=1}^{N}(X_i - \overline{X})(X_i - \overline{X})',$$

where T is the Bartlett's decomposition of A, that is, T is upper-triangular with positive diagonal elements. In this paper, we consider the problem of admissibility of the estimate

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of the form $T'\Delta T + Nb\overline{X}\,\overline{X}'$ (where $\Delta = \mathrm{diag}\{\delta_1, \cdots, \delta_m\} \geq 0$ and $b \geq 0$) on Σ using the loss function

$$L(\Sigma, \widehat{\Sigma}) = \mathrm{tr}(\Sigma^{-1}\widehat{\Sigma} - I)^2, \tag{1}$$

considered by Olkim and Selliah[1] and Haff[2], and restrict our attention to the class of estimates $\mathbb{D} = \{T'\Delta^* T + Nb^*\overline{X}\,\overline{X}' : \Delta^*$ is any diagonal matrix and b^* is any nonnegative constant$\}$.

The class of estimates $\mathbb{D}$ has the following attractive features:

(a) Any element in $\mathbb{D}$ is a quadratic form of the sufficient and complete statistics $(\overline{X}, T)$.

(b) $\mathbb{D}$ contains all estimates of the form $aA + Nb\overline{X}\,\overline{X}'$ (where a and b are nonnegative constants),and the latter constructs a complete subclass of the class of nonnegative quadratic estimates $\mathbb{D}^* = \{X'BX : B \geq 0\}$ (where $X = (X_1, \cdots, X_N)')$ for any strict covex loss function $L_1(\Sigma, \widehat{\Sigma})$.

It is clear that $\mathbb{D}$ contains all estimates of the form $aA + Nb\overline{X}\,\overline{X}'$. What remains is only to show that the latter constructs a complete subclass of $\mathbb{D}^*$. For this purpose, let

$$Z = (Z_1, \cdots, Z_N)' = HX,$$

where H is an orthogonal matrix with the first row

$$\Big(\frac{1}{\sqrt{N}}, \frac{1}{\sqrt{N}}, \cdots, \frac{1}{\sqrt{N}}\Big).$$

Then $Z_1, Z_2, \cdots, Z_N$ are independent,

$$Z_1 \sim N(\sqrt{N}\mu, \Sigma), \quad Z_k \sim N(0, \Sigma), \quad k = 2, \cdots, N,$$

$$\overline{X} = \frac{1}{\sqrt{N}} Z_1, \quad A = \sum_{k=2}^{N} Z_k Z_k',$$

$$E(Z_1 Z_1' | \overline{X}, A) = N\overline{X}\,\overline{X}' \quad \text{and} \quad E(Z_k Z_k' | \overline{X}, A) = \frac{1}{N-1} A, \quad k = 2, \cdots, N. \tag{2}$$

Now write $C = (c_{ij})_{N\times N} = HBH' \geq 0$. Then

$$\begin{aligned} & E_{(\mu,\Sigma)}[L_1(\Sigma, X'BX)] \\ &= E_{(\mu,\Sigma)}[L_1(\Sigma, Z'CZ)] \\ &= E_{(\mu,\Sigma)}\{E[L_1(\Sigma, Z'CZ)|\overline{X}, A]\} \\ &\geq E_{(\mu,\Sigma)}\{L_1[\Sigma, E(Z'CZ|\overline{X}, A)]\} \quad (\text{ by Jensen's inequality }) \\ &= E_{(\mu,\Sigma)}[L_1(\Sigma, aA + Nb\overline{X}\,\overline{X}')] \qquad (\text{ by (2) }) \end{aligned}$$

for every (μ, Σ), and a necessary and sufficient condition under which the above equality holds for every (μ, Σ) is that $X'BX = aA + Nb\overline{X}\,\overline{X}'$, where $b = c_{11} \geq 0$ and

$$a = \frac{1}{N-1}\sum_{k=2}^{N} c_{kk} \geq 0.$$

(c) $\mathbb{D}$ contains all invariant estimates $\Phi(A)$ under the transformation group of upper-triangular matrices Q, that is,

$$\Phi(Q'AQ) = Q'\Phi(A)Q \tag{3}$$

for all upper-triangular matrices Q.

To show (c), let $A = I$ in (3). Then

$$\Phi(Q'Q) = Q'\Phi(I)Q. \tag{4}$$

Take $Q = \text{diag}\{\pm 1, \pm 1, \cdots, \pm 1\}$ in (4). Then $Q'Q = I$ and (4) becomes

$$\Phi(I) = Q'\Phi(I)Q \tag{5}$$

for all such matrices Q. It follows from (5) that $\Phi(I)$ is a diagonal matrix. Now write $A = T'T$. Then

$$\Phi(A) = \Phi(T'T) = T'\Phi(I)T \in \mathbb{D},$$

namely, the proof of (c) is completed.

Now let us compute the risk fuction of $T'\Delta T + Nb\overline{X}\,\overline{X}'$

$$R(\mu, \Sigma, \Delta, b) = E_{(\mu,\Sigma)}\{\text{tr}[\Sigma^{-1}(T'\Delta T + Nb\overline{X}\,\overline{X}') - I]^2\}.$$

If we write Σ^{-1} as $\Sigma^{-1} = LL'$, where L is upper-triangular with positive diagonal elements, then

$$L(\Sigma, \widehat{\Sigma}) = \text{tr}(\Sigma^{-1}\widehat{\Sigma} - I)^2 = \text{tr}(L'\widehat{\Sigma}L - I)^2, \tag{6}$$

$$L'\overline{X} \sim N_m(L'\mu, N^{-1}I) \quad \text{and} \quad (TL)'(TL) = L'AL \sim W_m(n, I) \quad \text{with n=N-1.} \tag{7}$$

Note that TL is upper-triangular with positive diagonal elements. Then

$$\begin{aligned} R(\mu, \Sigma, \Delta, b) &= E_{(\mu,\Sigma)}\text{tr}[(TL)'\Delta(TL) + Nb(L'\overline{X})(L'\overline{X})' - I]^2 \quad (\text{ by (6) }) \\ &= R(L'\mu, I, \Delta, b) \qquad\qquad (\text{ by (7) }). \end{aligned} \tag{8}$$

From (8), without loss of generality, we may assume that $\Sigma = I$. Right now, from independence of T and $\overline{X}$, one has

$$\begin{aligned} &R(\mu, I, \Delta, b) \\ &= E\text{tr}(T'\Delta T)^2 + 2\text{tr}[E(T'\Delta T)E(Nb\overline{X}\,\overline{X}' - I)] + E\text{tr}(Nb\overline{X}\,\overline{X}' - I)^2, \end{aligned} \tag{9}$$

where all expectations are computed with (μ, I). From Bartlett's decomposition theorem, the elements t_{ij} of T are all independent,

$$t_{ii}^2 \sim \chi^2_{n-i+1} \qquad (1 \le i \le m) \quad \text{and} \quad t_{ij} \sim N(0,1) \qquad (1 \le i < j \le m),$$

where χ^2_k is the random variable which has the central χ^2 distribution with k degrees of freedom. Hence

$$\begin{aligned} &E\text{tr}(T'\Delta T)^2 \\ &= \sum_{i=1}^m \delta_i^2(n+m-2i+1)(n+m-2i+3) + \sum_{i<j}\delta_i\delta_j(n+m-2j+1) \end{aligned} \tag{10}$$

and

$$\begin{aligned} &E(T'\Delta T) \\ &= \text{diag}\{\delta_1 n, \delta_1 + \delta_2(n-1), \cdots, \delta_1 +, \cdots, +\delta_{m-1} + \delta_m(n-m+1)\}. \end{aligned} \tag{11}$$

Since $\sqrt{N}\overline{X} \sim N_m(\sqrt{N}\mu, I)$, we have

$$E(Nb\overline{X}\,\overline{X}' - I) = b(I + N\mu\mu') - I = Nb\mu\mu' - (1-b)I \tag{12}$$

and

$$\begin{aligned} & E\mathrm{tr}(Nb\overline{X}\,\overline{X}' - I)^2 \\ &= b^2 E[\chi_m^2(N\|\mu\|^2)]^2 - 2bE[\chi_m^2(N\|\mu\|^2)] + m \\ &= b^2[(m+N\|\mu\|^2)^2 + 2m + 4N\|\mu\|^2] - 2b(m+N\|\mu\|^2) + m, \end{aligned} \tag{13}$$

where $\chi_m^2(N\|\mu\|^2)$ is the random variable which has the noncentral χ^2 distribution with m degrees of freedom and noncentral parameter $N\|\mu\|^2$. Putting

$$\delta = (\delta_1, \cdots, \delta_m)', \qquad \beta = N(\mu_1^2, \cdots, \mu_m^2)',$$

$$h = (n+m-1, \cdots, n+m-2k+1, \cdots, n-m+1)', \tag{14}$$

$$B = (b_{ij})_{m\times m} \quad \text{and} \quad C = (c_{ij})_{m\times m}, \tag{15}$$

where μ_k is the k-th element of μ,

$$b_{ij} = \begin{cases} (n+m-2j+1), & i<j, \\ (n+m-2j+1)(n+m-2j+3), & i=j, \\ (n+m-2i+1), & i>j, \end{cases}$$

and

$$c_{ij} = \begin{cases} 1, & i<j, \\ (n-i+1), & i=j, \\ 0, & i>j, \end{cases}$$

and substituting (10), (11), (12) and (13) in (9), we then have

$$\begin{aligned} R(\mu, I, \Delta, b) &= \delta' B\delta - 2(1-b)\delta' h + m[b^2(m+2) - 2b + 1] \\ &\quad + 2b[\delta' C + (bm+2b-1)\mathbf{1}']\beta + b^2(\mathbf{1}'\beta)^2 \\ &\doteq g(\beta, \delta, b), \end{aligned} \tag{16}$$

where $\mathbf{1}$ is the m-vector with all elements being 1.

If we put

$$f(t,\beta) = g[\beta, \delta + t(\delta^* - \delta), b + t(b^* - b)], 0 \le t \le 1, \delta^* \in R^m \text{ and } b^* \ge 0,$$

from (16), we then have

Lemma. *A necessary and sufficient condition for the estimate $T'\Delta T + Nb\overline{X}\,\overline{X}'$ of Σ to be inadmissible in $\mathbb{D}$ is that there are $\delta^* \in R^m$ and $0 \le b^* \le b$ such that*

$$f_t'(0,\beta) < 0 \tag{17}$$

for every $\beta \in R_+^m = \{\beta = (\beta_1, \cdots, \beta_m)' : \beta_i \ge 0, 1 \le i \le m\}$, where

$$f_t'(t_0, \beta) = \frac{\partial f(t,\beta)}{\partial t}\Big|_{t=t_0}.$$

Proof. Necessity. Let $T'\Delta^* T + Nb^*\overline{X}\,\overline{X}'$ $(\in \mathbb{D})$ beat $T'\Delta T + Nb\overline{X}\,\overline{X}'$. Then from (16),

$$g(\beta, \delta, b) \ge g(\beta, \delta^*, b^*)$$

for all $\beta \in R_+^m$ with the strict inequality for at least some β, which implies $b^* \le b$ and

$$f(0,\beta) \ge f(1,\beta) \tag{18}$$

for all $\beta \in R_+^m$. Noting that $f(t,\beta)$ is a strict convex function of t for every $\beta \in R_+^m$, and from (18), we have (17).

Sufficiency. Note that

$$\begin{aligned}
&f_t'(t,\beta)\\
&= 2(\delta^*-\delta)'B[\delta+t(\delta^*-\delta)]+2(b^*-b)[\delta+t(\delta^*-\delta)]'h\\
&\quad -2[1-b-t(b^*-b)](\delta^*-\delta)'h+2m(b^*-b)\{(m+2)[b+t(b^*-b)]-1\}\\
&\quad +2(b^*-b)\{[\delta+t(\delta^*-\delta)]'C+[(b+t(b^*-b))(m+2)-1]\mathbf{1}'\}\beta\\
&\quad +2[b+t(b^*-b)][(\delta^*-\delta)'C+(m+2)(b^*-b)\mathbf{1}']\beta\\
&\quad +2(b^*-b)[b+t(b^*-b)](\mathbf{1}'\beta)^2\\
&= f_t'(0,\beta)+2t\{(\delta^*-\delta)'B(\delta^*-\delta)+2(b^*-b)(\delta^*-\delta)'h+m(m+2)(b^*-b)^2\\
&\quad +2(b^*-b)[(\delta^*-\delta)'C+(m+2)(b^*-b)\mathbf{1}']\beta+(b^*-b)^2(\mathbf{1}'\beta)^2\},
\end{aligned} \tag{19}$$

and

$$\begin{aligned}
f_t'(0,\beta) = 2(\delta^*-\delta)'[B\delta-(1-b)h+bC\beta]+2(b^*-b)\{m(m+2)b\\
-m+\delta'h+\delta'C\beta+[2(m+2)b-1]\mathbf{1}'\beta+b(\mathbf{1}'\beta)^2\}.
\end{aligned} \tag{20}$$

When $b^*<b$, taking $t_1\,(0<t_1<1)$ such that

$$(b^*-b)[b+t_1(b^*-b)]<0,$$

and noting (19), we see that there is a sufficient large positive constant M such that

$$f_t'(t_1,\beta)<0 \tag{21}$$

for all $\mathbf{1}'\beta>M$. On the other hand, from (17), (19) and the continuity of $f_t'(t,\beta)$ in the closed interval $\{(t,\beta): 0\le t\le 1, \mathbf{1}'\beta\le M\}$, there is a sufficient small $t_2\,(0<t_2<1)$ such that

$$f_t'(t_2,\beta)<0 \tag{22}$$

for all $\mathbf{1}'\beta\le M$. Now put $t_0=\min\{t_1,t_2\}$. From (21) and (22), and noting that $f_t'(t,\beta)$ is an increasing function of t for every $\beta\in R_+^m$, we have

$$f_t'(t_0,\beta)<0 \tag{23}$$

for every $\beta\in R_+^m$. (17) and (23) imply that

$$T'[\Delta+t_0(\Delta^*-\Delta)]T+N[b+t_0(b^*-b)]\overline{X}\,\overline{X}'$$

beats $T'\Delta T+Nb\overline{X}\,\overline{X}'$.

When $b^*=b$, (19) and (20) become

$$f_t'(t,\beta)=f_t'(0,\beta)+2t(\delta^*-\delta)'B(\delta^*-\delta). \tag{24}$$

and

$$f_t'(0,\beta)=2(\delta^*-\delta)'[B\delta-(1-b)h+bC\beta], \tag{25}$$

respectively. From (17) and (25), we have

$$\overline{\lim_{\|\beta\|\to\infty}} f_t'(0,\beta)<0. \tag{26}$$

(17) and (26) imply that there is a constant $d > 0$ such that

$$f_t'(0, \beta) < -d \tag{27}$$

for all $\beta \in R_+^m$. From (24) and (27), there is a sufficient small $t_0 > 0$ such that (23) holds. So

$$T'[\Delta + t_0(\Delta^* - \Delta)]T + N[b + t_0(b^* - b)]\overline{X}\,\overline{X}'$$

beats $T'\Delta T + Nb\overline{X}\,\overline{X}'$.

In the light of the above statements, the proof of the Lemma is completed.

Now put

$$D = (d_1, \cdots, d_m) = B\delta - (1 - b)h$$

and

$$e_k = \begin{cases} d_m, & k = m, \\ d_k - \displaystyle\sum_{i=k+1}^{m} \frac{e_i}{n - i + 1}, & 1 \le k < m. \end{cases} \tag{28}$$

By using the above lemma, we can obtain a necessary and sufficient condition for the estimate $T'\Delta T + Nb\overline{X}\,\overline{X}'$ of Σ to be admissible in $\mathbb{D}$, as the following result shows.

Theorem. *A necessary and sufficient condition for the estimate $T'\Delta T + Nb\overline{X}\,\overline{X}'$ of Σ to be admissible in $\mathbb{D}$ is that*

(i) *when $b = 0, \delta = B^{-1}h$; or*

(ii) *when $b > 0$,*

$$e_k \le 0, \quad k = 1, 2, \cdots, m, \tag{29}$$

and

$$\begin{aligned} &m(m+2)b^2 - mb + \delta'h - \delta'B\delta + [2(m+2)b - 1][(1-b)m - n^{-1}\mathbf{1}'B\delta] \\ &+ [(1-b)m - n^{-1}\mathbf{1}'B\delta]^2 \le 0. \end{aligned} \tag{30}$$

Proof. We first prove the case of $b = 0$. Right now $b^* = 0$ and $f_t'(0, \beta) = 2(\delta^* - \delta)'(B\delta - h)$ by (20). Hence $f_t'(0, \beta) \ge 0$ for all $\delta^* \in R^m$ if and only if $\delta = B^{-1}h$. From the above lemma, the proof of the case is completed.

We next consider the case of $b > 0$. From (29), we can take

$$\beta = \beta^0 = (\beta_1^0, \beta_2^0, \cdots, \beta_m^0),$$

where $\beta_k^0 = -\frac{1}{b(n-k+1)} e_k \ge 0$, $k = 1, 2, \cdots, m$. Hence,

$$B\delta - (1 - b)h + bC\beta^0 = D - D = 0 \quad (\text{ by (28) }) \tag{31}$$

and

$$\begin{aligned} \mathbf{1}'\beta^0 &= n^{-1}\mathbf{1}'C\beta^0 = n^{-1}b^{-1}\mathbf{1}'[(1-b)h - B\delta] \\ &= b^{-1}[(1-b)m - n^{-1}\mathbf{1}'B\delta] \quad (\text{ by (31) }). \end{aligned} \tag{32}$$

Substituting (31) and (32) into (20), and from (30), we then have

$$\begin{aligned} f_t'(0,\beta^0) &= 2(b^*-b)\{m(m+2)b-m+\delta'h+b^{-1}\delta'[(1-b)h-B\delta] \\ &\quad +[2(m+2)b-1]b^{-1}[(1-b)m-n^{-1}\mathbf{1}'B\delta]+b^{-1}[(1-b)m-n^{-1}\mathbf{1}'B\delta]^2\} \\ &= 2b^{-1}(b^*-b)\{m(m+2)b^2-mb+\delta'h-\delta'B\delta \\ &\quad +[2(m+2)b-1][(1-b)m-n^{-1}\mathbf{1}'B\delta]+[(1-b)m-n^{-1}\mathbf{1}'B\delta]^2\} \\ &\geq 0 \end{aligned}$$

for all $\delta^* \in R^m$ and all $b^* \leq b$. From the above lemma, the proof of the sufficiency of the case is completed.

Necessity. If there is a k_0 such that $e_{k_0} > 0$, taking

$$b^* = b, \quad \delta^*_{k_O} < \delta_{k_0}$$

and

$$\delta_k^* = \begin{cases} \delta_k, & k < k_0, \\ \delta_k - \frac{1}{n-k+1}\sum\limits_{j=k_0}^{k-1}(\delta_j^*-\delta_j), & k > k_0 \end{cases} \tag{33}$$

we have

$$\begin{aligned} f_t'(0,\beta) &= 2(\delta^*-\delta)'[B\delta-(1-b)h+bC\beta] && (\text{ by (20) and (33) }) \\ &= 2(\delta^*-\delta)'C\Big[\Big(\frac{e_1}{n},\frac{e_2}{n-1},\cdots,\frac{e_m}{n-m+1}\Big)'+b\beta\Big] && (\text{ by (31) }) \\ &= 2(\delta^*_{k_0}-\delta_{k_0})[e_{k_0}+b(n-k_0+1)\beta_{k_0}] && (\text{ by (33) }) \\ &< 0. \end{aligned}$$

for all $\beta \in R^m_+$. It follows from the above lemma that $T'\Delta T + Nb\overline{X}\,\overline{X}'$ is inadmissible in $\mathbb{D}$. This is contradictory to the assumption of the necessity. Hence (29) holds.

If (30) does not hold, taking $b^* < b$ and

$$\delta^* = \delta - b^{-1}(b^*-b)\{\delta+n^{-1}[2(m+2)b-1]\mathbf{1}+2n^{-1}[(1-b)m-n^{-1}\mathbf{1}'B\delta]\mathbf{1}\}$$

and substitiuting for them in (20), then we have

$$\begin{aligned} f_t'(0,\beta) &= 2(b^*-b)\{m(m+2)b-m+\delta'h+\delta'C\beta+[2(m+2)b-1]\mathbf{1}'\beta \\ &\quad +b(\mathbf{1}'\beta)^2\}-2b^{-1}(b^*-b)\{\delta'+n^{-1}[2(m+2)b-1]\mathbf{1}' \\ &\quad +2n^{-1}[(1-b)m-n^{-1}\mathbf{1}'B\delta]\mathbf{1}'\}[B\delta-(1-b)h+bC\beta] \\ &= 2b^{-1}(b^*-b)\{m(m+2)b^2-mb+\delta'h-\delta'B\delta+[2(m+2)b-1][(1-b)m \\ &\quad -n^{-1}\mathbf{1}'B\delta]+[(1-b)m-n^{-1}\mathbf{1}'B\delta]^2+[(1-b)m-n^{-1}\mathbf{1}'B\delta-b\mathbf{1}'\beta]^2\} \\ &\leq 2b^{-1}(b^*-b)\{m(m+2)b^2-mb+\delta'h-\delta'B\delta+[2(m+2)b-1][(1-b)m \\ &\quad -n^{-1}\mathbf{1}'B\delta]+[(1-b)m-n^{-1}\mathbf{1}'B\delta]^2\} \\ &< 0 \end{aligned}$$

for all $\beta \in R^m_+$. It follows from the above lemma that $T'\Delta T + Nb\overline{X}\,\overline{X}'$ is inadmissible in $\mathbb{D}$. This is constradictory to the assumption of the necessity, so (30) holds.

In the light of the above statements, the proof of the Theorem is completed.

Remark. From (b), $\mathbb{D}$ contains all estimates of the form $aA + Nb\overline{X}\,\overline{X}'$ ($a \geq 0$ and $b \geq 0$), which construct a complete subclass of $\mathbb{D}^*$. Therefore, that $aA + Nb\overline{X}\,\overline{X}'$ ($a \geq 0$ and $b \geq 0$) is admissible in $\mathbb{D}$ can imply its admissibility in $\mathbb{D}^*$.But its converse is not correct. For example, $(n+m)^{-1}A$ is admissible in $\mathbb{D}^*$ from Theorem 2 in [3], but it is inadmissible in $\mathbb{D}$ from the above theorem.

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