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ON THE HARISH-CHANDRA HOMOMORPHISM FOR THE CHEVALLEY GROUPS OVER p-ADIC FIELD**

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Abstract

The Harish-Chandra homomorphism for the higher congruence spherical functions algebra of Chevalley groups over p-adic fiealds is given in the case of the Levi-component of a (rational) parabolic subgroup. It is a generalization for the Harish-Chandra homomorphism for the higher congruence spherical functions algebra of the groups GL_n over p-adic field in the same case.

Keywords Harish-Chandra homomorohism, Chevalley group, p-adic field.
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§1. Introduction

There are many parallels between the representation theory on real simple groups and the representation theory of the p-adic groups. R. Howe[4] and A. Moy gave a partial analogue for the groups GL_n over p-adic fields of Harish-Chandra homomorphism.

In this paper, the Harish-Chandra homomrphism for higher congruence spherical functions algebra of Chevalley groups over p-adic fields is given in the case of the Levi-component of a (rational) parabolic subgroup. It is a generalization of R. Howe's works[4] for the groups GL_n over p-adic fields in the same case.

Let $\langle S\rangle$ be the subgroup generated by the elements in the subset S of a group. For any set S, let $\#(S)$ denote the number of the elements in S and let ϕ denote the vacuous set. We write $\mathbf{Z}$ for the set of all integers and $\mathbf{N}$ the set of all natural numbers. For any pair $p,q\in\mathbf{Z}, p<q$, let $[p,q]=\{p,p+1,\cdots,q\}$.

Clearly, the p-adic field F is a field equipped with a non-trivial, non-Archimedean discrete valuation $|\,.\,|_F$. We define

$$R=\{t\mid t\in F;\mid t\mid_F\leq 1\}.$$

Then R is the ring of the integers of F and $P=\pi R$ is the maximal ideal of R where π is a prime element of F. The complement R^* of P in R is the group of units of R. We denote by $\mathbf{F}$ the residue class field R/P which is a finite field. It is well known that F is locally compact, and R and P are open, compact subsets of F. For each $i\in\mathbf{Z}$, let $P_i=\pi^iR$ and $R_i^*=\{1+p;p\in P_i\}$. Let $\mathbf{R}$ be the representative system of $\mathbf{F}$ in R.

Lemma 1.1. *If $f(x)=x^m+a_1x^{m-1}+\cdots+a_{m-1}x+a_m\in R[x]$ and λ is a root of $f(x)$, then there is a finite dimensional extension F' of F with a non-trivial, non-Archimedean discrete valuation $|\cdot|_{F'}$ such that $\mid\lambda\mid_{F'}\leq 1$.*

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Proof. Let F' be the splitting field over F of $f(x)$. Then $[F':F]<\infty$. So F' is a p-adic field and the restriction of $|\cdot|_{F'}$ on F is just $|\cdot|_F$. Thus, we have

$$f(x)=(x-b_1)(x-b_2)\cdots(x-b_m),$$

$b_i\in F', i\in[1,m]$. Clearly,

$$R\subset R'=\{t\mid t\in F';|\,t\,|_{F'}\le 1\}.$$

It is easily shown that $b_1b_2\cdots b_m=a_m$. Hence, we have

$$|\,b_1\,|_{F'}|\,b_2\,|_{F'}\cdots|\,b_m\,|_{F'}=|\,a_m\,|_{F'}.$$

Thus, there exists at least a root $b_j, j\in[1,m]$ such that $|\,b_j\,|_{F'}\le 1$. Let $b=b_j$. It follows that

$$f(x)=(x-b)g(x),\quad g(x)=x^{m-1}+c_1x^{m-2}+\cdots+c_{m-1},$$

where $c_i\in F',\ i\in[1,m-1]$ satisfy

$$c_1-b=a_1, c_2-c_1b=a_2,\cdots,c_{m-1}-c_{m-2}b=a_{m-1}, c_{m-1}b=a_m.$$

Hence, since $a_1,a_2,\cdots,a_m\in R$, we have $c_1,c_2,\cdots,c_{m-1}\in R'$. It follows that $g(x)\in R'[x]$. Therefore, the lemma can be shown by induction.

Let L be a simple Lie algebra over $\mathbf{C}$ ($L\not\cong G_2$) and Φ the root system of L with respect to a Cartan subalgebra of L. Let $\Pi=\{\alpha_1,\alpha_2,\cdots,\alpha_n\}$ be a fundamental root system of Φ and Φ^+ the positive root system containing Π. Clearly, $\Phi^-=-\Phi^+$ is a negative root system in Φ. Let $C_L=\{h_\alpha,\alpha\in\Pi;e_r,r\in\Phi\}$ be a Chevalley basis of L and $G_F=L(F)$ the Chevalley group of type L over F. It follows from [1] that $G_F=\langle exptade_r, t\in F, r\in\Phi\rangle$.

Hereinbelow, we shall use the notations and terminology given by [1] directly.

§2. Some Subgroups of G

For each $r\in\Phi, q\in\mathbf{N}$, we put

$$x_{r,q}=\{x_r(t);t\in P_q\},\quad U_q=\langle x_{r,q};r\in\Phi^+\rangle,$$
$$V_q=\langle x_{r,q};r\in\Phi^-\rangle,\quad H_q=\langle h_\alpha(d);d\in R_q^*,\alpha\in\Phi\rangle.$$

Let $K_q=H_qV_qU_q, q\in\mathbf{N}$.

Lemma 2.1. *Let $s\in\Phi$ and let $a,b\in F^*=F\setminus 0$ such that $e=1+ab\neq 0$. Then*

$$x_{-s}(b)x_s(a)=h_s(e^{-1})x_s(a')x_{-s}(b'),\quad a'=ae,\quad b'=be^{-1}.$$

Proof. Let $c=(ae)^{-1}$. Clearly we have

$$-c+a^{-1}=c(-1+e)=c(-1+1+ab)=cab=be^{-1}.$$

Thus, by 5.2.2 and 8.1.4 in [1], we have

$$\begin{aligned}x_{-s}(b)x_s(a)&=x_{-s}(b)x_{-s}(a^{-1})h_{-s}(-a^{-1})n_{-s}x_{-s}(a^{-1})\\&=h_{-s}(-a^{-1})x_s(c)h_s(-c)n_sx_s(c)n_{-s}x_{-s}(a^{-1})\\&=h_s(e^{-1})x_s(ae)x_{-s}(be^{-1}).\end{aligned}$$

Hence the lemma is proved.

Lemma 2.2. (i) *Every element u of $U_q, q\in\mathbf{N}$ has a unique expression*

$$u=x_{r_1}(a_1)x_{r_2}(a_2)\cdots x_{r_h}(a_h),\quad a_i\in P_q,\quad r_i\in\Phi^+,\quad 1\le i\le h,\quad r_1\prec r_2\prec\cdots\prec r_h.$$

(ii) *Every element* v *of* $V_q, q \in \mathbf{N}$, *has a unique expression*

$$v = x_{s_1}(b_1)x_{s_2}(b_2)\cdots x_{s_k}(b_k), \quad b_1 \in P_q, \quad r_i \in \Phi^-, \quad 1 \le i \le k, \quad s_1 \prec r_2 \prec \cdots \prec r_k.$$

(iii) *Every element* h *of* $H_q, q \in \mathbf{N}$, *has a unique expression*

$$h = h_{\alpha_1}(d_1)h_{\alpha_2}(d_2)\cdots h_{\alpha_n}(d_n), \quad d_i \in R_q^*, \quad \alpha_i \in \Pi, \quad \alpha_1 \prec \alpha_2 \prec \cdots \prec \alpha_n.$$

Proof. Clearly, if $t, t' \in P_q$, then $t + t', t^i t'^j \in P_q$ for any $i, j \in \mathbf{N}$. Thus using an argument similar to that used in 5.3.3 in [1], by the Chevalley's commutator formula, (i) can be shown. Similarly, (ii) can be proved also.

Let $q_1, q_2, \cdots, q_n$ be the dual basis of the basis $h_{\alpha_1}, h_{\alpha_2}, \cdots, h_{\alpha_n}, h_{\alpha_i} \in C_L$, $\alpha_i \in \Pi$, $1 \le i \le n$. It follows from 7.1.1 in [1] that $h(\chi_{r,d}) = h_r(d) \in H_q$, for any $r \in \Phi, d \in R_q^*, q \in \mathbf{N}$, and we have $\chi_{r,d}(q_i) \in R_q^*, 1 \le i \le n$ since $d^j \in R_q^*$ for any $j \in \mathbf{Z}$. Clearly, if $d, d' \in R_q^*$, then $dd' \in R_q^*$. Hence $\chi(q_i) \in R_q^*, 1 \le i \le n$ for each $h(\chi) \in H_q$. By 7.1.1 in [1], every element $h(\chi)$ of H_q has a unique expression

$$h(\chi) = h_{\alpha_1}(d_1)h_{\alpha_2}(d_2)\cdots h_{\alpha_n}(d_n), \quad d_i \in F^*, \quad 1 \le i \le n.$$

It is easily shown that $\chi(q_i) = d_i \in R_q^*$, $1 \le i \le n$, so, (iii) follows.

Let $r(u) = \{r_i \in \Phi^+, a_i \ne 0\}$ and $r(v) = \{s_i \in \Phi^-, b_i \ne 0\}$. For any $s \in \Phi^+$, let $h(s)$ be the height of s. Let $[x, y] = xyx^{-1}y^{-1}, x, y \in G$.

Lemma 2.3. *For any* $s \in \Phi^+, h(s) > 1, b \in P_q, q \in \mathbf{N}$, *there exists* $\alpha \in \Pi$ *and* $r \in \Phi^+, h(r) = h(s) - 1$ *such that* $s = r + \alpha$ *and*

$$x_s(b) = \prod_{i=1}^{t}[x_\alpha(c_i), x_r(d_i b)], \quad d_i, c_i \in R_q^*, \quad 1 \le i \le t, \quad i = 1, 2.$$

Proof. Clearly, there exists α in Π and r in Φ^+ with $h(r) = h(s) - 1$ such that $s = r + \alpha$. Let $A_{r,\alpha} = \frac{(r,r)}{(\alpha,\alpha)}$. If $A_{r,\alpha} = -1$, then by the Chevalley's commutator formula, the lemma is easily shown with $t = 1, c_1 = -C_{11\alpha r}, d_1 = 1$.

Similarly, the lemma is easily verified for $A_{r,\alpha} = -2$ or $-\frac{1}{2}$.

Lemma 2.4. (i) *If* $\alpha \in \Pi$, $d \in R^*$ *and* $q \in \mathbf{N}$, *then* $x_\alpha(d)U_q \subset U_q x_\alpha(d)$.

(ii) *If* $\alpha \in \Pi, d \in R^*$ *and* $q \in \mathbf{N}$, *then*

$$x_\alpha(d)V_q \subset H_q V_q x_\alpha(d'), \quad d' = td, \quad t \in R_q^*.$$

Proof. Clearly, if $i\alpha + jr \in \Phi$ for any $r \in \Phi$, $i, j \in \mathbf{N}$, then $i\alpha + jr \in \Phi^+$, so, (i) can be shown by (i) in Lemma 2.2 and Chevalley's commutator formula.

We shall show (ii). If $s \in \Phi^-, s \ne -\alpha$ and $i\alpha + jr \in \Phi$ for some $i, j \in \mathbf{N}$, then $i\alpha + jr \in \Phi^-$ and $d^i a^j \in P_q$ for any $a \in P_q$. So we have $[x_\alpha(d), x_s(a)] \in V_q$.

If $s = -\alpha$ and $a \in P_q$, then by Lemma 2.1, we have $e = 1 + da \in R_q^*$ and

$$x_\alpha(d)x_s(a) = h_s(e^{-1})x_s(a')x_\alpha(d'), a' = ae, d' = de^{-1}. \tag{2.a}$$

Clearly, $h_s(e^{-1}) \in H_q$ and $x_s(a') \in V_q$ and $d' = td, t = e^{-1} \in R_q^*$. By (2.a), (ii) can be proved immediately from Lemma 2.2.

Lemma 2.5. (i) *Let* $s \in \Phi^+$ *and* $0 \ne b \in P_q, q \in \mathbf{N}$. *Then* $x_s(b)V_q \subset V_q U_q H_q$.

(ii) *For each* $q \in \mathbf{N}$, $H_q U_q = U_q H_q$, $H_q V_q = V_q H_q$.

(iii) *If* $h(\chi) \in H_q, q \in \mathbf{N}$ *and* $r \in \Phi$, *then* $\chi(r) \in R_q^*$.

Proof. We first show (ii). By the argument used in (iii) in Lemma 2.2, R_q^* is a subgroup of R^*. Clearly,for any $d \in R_q^*, t \in P_q$,we have $dt \in P_q$. It is easy to see that if $h(\chi) \in H_q$ and $s \in \Phi^-$, then for any $t \in P_q$ we have

$$h(\chi)x_s(t)h(\chi)^{-1} = x_s(\chi(s)t) \in x_{s,q},$$

because $\chi(s) \in R_q^*$ by (iii) in Lemma 2.2. Thus, by (ii) in Lemma 2.2, we have $H_qV_q = V_qH_q$. Similarly, it is easily shown that $H_qU_q = U_qH_q$. (ii) is proved. Now, we show (i) by induction on the height $h(s)$ of s.

1. $h(s) = 1$: Clearly, we have $s = \alpha \in \Pi$ and $tb \in P_q$ for any $t \in R_q^*$. Hence by (ii) in Lemma 2.4, (i) can be shown immediately.

2. $h(s) = c > 1$: Let $x = [x_\alpha(d), x_r(fb)], d, f \in R^*, \alpha \in \Pi, r \in \Phi$. Clearly, we have $h(r) = c - 1$. Let $s = r + \alpha$. Then we have

$$\begin{aligned} xV_q &= x_\alpha(d)x_r(fb)x_\alpha(-d)x_r(-fb)V_q \\ &\subset x_\alpha(d)x_r(fb)x_\alpha(-d)V_qU_qH_q \quad \text{(by hypothesis of induction)} \\ &\subset x_\alpha(d)x_r(fb)V_qx_\alpha(-td)U_qH_q \quad \text{(by Lemma 2.4)} \\ &\subset x_\alpha(d)H_qV_qU_qx_\alpha(-td)U_qH_q \quad \text{(by hypothesis of induction)} \\ &\subset V_qU_qH_qx_\alpha(t'd)x_\alpha(-td)H_q \quad \text{(by Lemma2.4)}, \end{aligned}$$

where t and t' are the elements in R_q^*. Thus, we have $t'd - td = d(t' - t) \in P_q$. It follows that $x_\alpha(t'd)x_\alpha(-td) = x_\alpha(d(t' - t)) \in U_q$. Hence. by Lemma 2.3, (i) follows.

(iii) is clear.

The proof is complete.

Lemma 2.6. *For any $q \in \mathbb{N}$, $K_q = H_qV_qU_q$ is an open compact subgroup of $G = G_F$.*

Proof. By (i) in Lemma 2.2, for each $u \in U_q$ we have

$$\begin{aligned} uH_qV_qU_q &= x_{r_1}(a_1)x_{r_2}(a_2)\cdots x_{r_h}(a_h)H_qV_qU_q \\ &\subset x_{r_1}(a_1)x_{r_2}(a_2)\cdots x_{r_{h-1}}(a_{h-1})H_qV_qU_q \quad \text{(by Lemma2.5)} \\ &\subset \cdots \subset H_qV_qU_q. \end{aligned}$$

Thus, we have $U_qV_qU_q \subset H_qV_qU_q$. So, by (ii) in Lemma 2.5, we obtain

$$K_qK_q = H_qV_qU_qH_qV_qU_q \subset H_qV_qU_qV_qU_q \subset H_qV_qU_q = K_q.$$

Clearly, $H_q^{-1} \subset H_q, V_q^{-1} \subset V_q$ and $U_q^{-1} \subset U_q$. Hence, by Lemma 2.5, we have

$$K_q^{-1} = U_q^{-1}V_q^{-1}H_q^{-1} \subset U_qV_qH_q \subset H_qV_qU_qH_q = H_qV_qU_q = K_q.$$

Therefore, K_q is a subgroup of G.

For each $k \in \mathbb{N}$, let $S^k = S \times S \times \cdots \times S$ (k times), $S \subset F$. Since R is an open compact subset of F, $P_q = \pi^q R$ is open compact for any $q \in Z$. For each $r \in \Phi$, there is a homeomorphism ρ_r from P_q to $x_{r,q}$ defined by $\rho_r(t) = x_r(t)$, $t \in P_q$. Hence, $x_{r,q}$ is open compact. Thus, by Lemma 2.2, U_q, V_q and H_q are homeomorphic to open compact set. So U_q, V_q and H_q are open compact subgroups of G. It is easily shown that for any $q \in \mathbb{N}$, $K_q = H_qV_qU_q$ is homeomorphic to $H_q \times V_q \times U_q$. Hence K_q is an open compact subgroup of G. The proof is complete.

Clearly, $K = HVU = H_0V_0U_0 = K_0$ is the maximal compact subgroup of G and for each $q \in \mathbb{N}$, K_q is called the q-congruence maximal compact subgroup of G.

For any $q \in \mathbf{N}$, $B_q = H_q V_{q+1} U_q$ is a subgroup of G.

Clearly, $B = HV_1U = H_0V_1U_0$ is the Iwahori subgroup of G and $B_q, q \in \mathbf{N}$, is called the q-congruence Iwahori subgroup of G.

For each $a \in P_q, q \in \mathbf{N}$, there exists a representative element $\mathbf{a}$ in $\pi^q\mathbf{R}$ such that $a \equiv \mathbf{a}(\mathrm{mod} P_{q+1})$.

Corollary 2.1. *Each element k in $K_q, q \in \mathbf{N}$, has a unique expression*

$$k = h(\chi)x_{s_1}(b_1)x_{s_2}(b_2)\cdots x_{s_h}(b_h)x_{r_1}(a_1)x_{r_2}(a_2)\cdots x_{r_h}(a_h),$$

where $a_i, b_i \in P_q, 1 \le i \le h$ and $h(\chi)$ is an element of H_q which has a unique expression

$$h(\chi) = h_{\alpha_1}(d_1)h_{\alpha_2}(d_2)\cdots h_{\alpha_n}(d_n), d_i = 1 + c_i \in R_q^*, \quad c_i \in P_q, \quad 1 \le i \le n.$$

Put $T_k = \pi^q(T' + T'')$, where

$$T' = \mathbf{c}_1 h_{\alpha_1} + \mathbf{c}_2 h_{\alpha_2} + \cdots + \mathbf{c}_n h_{\alpha_n},$$

and

$$T'' = \mathbf{b}_1 e_{s_1} + \mathbf{b}_2 e_{s_2} + \cdots + \mathbf{b}_h e_{s_h} + \mathbf{a}_1 e_{r_1} + \mathbf{a}_2 e_{r_2} + \cdots + \mathbf{a}_h e_{r_h}.$$

Corollary 2.2. *If $k, k' \in K_q, q \in \mathbf{N}$, then $T_{kk'} = T_k + T_{k'}$.*

For each $q \in \mathbf{N}$, let $P_q^\star = R \setminus P_q$, $R_q^\star = R^* \setminus R_q^*$ and for each $r \in \Phi$, let

$$x_{r,q}^\star = \{x_r(t); t \in P_q^\star\}, \quad h_{r,q}^\star = \{h_r(t), t \in R_q^\star\}.$$

For each $q \in \mathbf{N}$, we put

$$H_q^\star = h_{\alpha_1,q}^\star h_{\alpha_2,q}^\star \cdots h_{\alpha_n,q}^\star, \quad U_q^\star = x_{r_1,q}^\star x_{r_2,q}^\star \cdots x_{r_h,q}^\star, \quad V_q^\star = x_{s_1,q}^\star x_{s_2,q}^\star \cdots x_{s_h,q}^\star.$$

For each $q \in \mathbf{N}$, we define

$$K_q^\star = H_q^\star V_q^\star U_q^\star.$$

By an argument similar to that used in Corollary 2.1, we have

Lemma 2.7. *For each $q \in \mathbf{N}, K = K_q K_q^\star = K_q^\star K_q$.*

Let $D = \{h(\chi); \chi \in \mathrm{Hom}(Z\Phi, \pi^i), i \in Z\}$. By 2.17 in [5], we have $G = KDK$ (K was denoted by U in [5]). Thus by 2.16 in [5] and Lemma 2.7 we obtain

Lemma 2.8. $G = K_q D_q^\star K_q, q \in \mathbf{N}$, *where* $D_q^\star = K_q^\star D K_q^\star$.

§3. Higher Congruence Spherical Functions Algebra

Let V be a finite dimensional Hilbert space, and $\mathrm{End}(V)$ the C^*-algebra of the linear transformations from V to itself. Let $C_c(G)$ be the set of the continous, complex-valued functions of compact support on G. Let $C_c(G : \mathrm{End}V)$ be the set of continous compactly supported functions with values in $\mathrm{End}V$.

Let J be an open compact subgroup of G. Suppose that there exists a decomposition of G into the double cosets of J. Let σ be an irreducible unitary representation of J on finite dimensional Hilbert space V. Consider a space $H(G//J, \sigma)$ of functions in $C_c(G : \mathrm{End}V)$ such that

$$f(k_1 g k_2) = \sigma(k_1)f(g)\sigma(k_2), \quad k_1, k_2 \in J, \quad g \in G.$$

It is easy to check that $H(G//J, \sigma)$ is a convolution subalgebra of $C_c(G : \mathrm{End}V)$. We call it the σ-spherical Hecke algebra (cf. Appendix I in [4]).

Clearly, for any $q \in \mathbf{N}, K_q$ is an open compact subgroup of G by Lemma 2.6 and there is a decomposition of G into the double cosets of K_q by Lemma 2.8. Thus we are able to consider the algebra $H(G//K_q, \sigma)$, where σ is some irreducible unitary representation of K_q.

Let $C_L^0 = \{h_\alpha; \alpha \in \Pi\}$ and $C_{\mathbf{R}} = C_L^0 \otimes \mathbf{R}$. The elements in $C_{\mathbf{R}}$ are called the nondegenerate elements. Let $L_R = C_L \otimes R$ and $L_{\mathbf{R}} = C_L \otimes \mathbf{R}$. Set $L_F = C_L \otimes F$.

Let χ_0 be an additive character of F with conductor R. For a given nondegenerate element $x \in C_{\mathbf{R}}$ and for any $i, \ell \in \mathbf{N}$, we define

$$\psi_x = \chi_0(\pi^{-h}(\mathrm{tr}(adx adT_k)), \quad h = \ell + i, \quad k \in K_i. \tag{3a}$$

Hence, by Corollary 2.2, we have $\psi_x \in (K_i/K_{i+\ell})^\wedge$. It is easy to see that ψ_x is an irreducible unitary representation of K_i with $\ker\psi_x = K_{i+\ell}$. By Lemma 2.8, there exists a decomposition of G into the double cosets of K_i. Thus we shall be able to consider the ψ_x-spherical Hecke algebra $H(G//K_i, \psi_x)$.

Lemma 3.1. *For any $i \in \mathbf{N}$, $g \in \mathrm{supp}H(G//K_i, \psi_x)$ if and only if there exist $y_1, y_2 \in L_R$ such that*

$$adz_1 = Adg(adz_2) = g(adz_2)g^{-1}, \quad z_s = x + \pi^\ell y_s, \quad s = 1, 2.$$

Proof. By [3], it is easy to see that $g \in \mathrm{supp}H(G//K_i, \psi_x)$ if and only if the character $gkg^{-1} \to \psi_x(k), k \in K_i$, agrees with ψ_x on $gK_ig^{-1} \cap K_i$, that is to say, for each $k \in gK_ig^{-1} \cap K_i$,

$$\chi_0(\pi^{-h}\mathrm{tr}((adx)(adT_k))) = \chi_0(\pi^{-h}\mathrm{tr}((adx)adT_{g^{-1}kg})).$$

It is easy to see that $adT_{g^{-1}kg} = g^{-1}(adT_k)g$, so we have

$$\chi_0(\pi^{-h}\mathrm{tr}((adx)(adT_k))) - \chi_0(\pi^{-h}\mathrm{tr}((adx)g^{-1}(adT_k)g)) = 0.$$

It follows that

$$\chi_0(\pi^{-h}\mathrm{tr}((adx - g(adx)g^{-1})(adT_k)) = 0.$$

For each $k \in K_i$ we have $T_k \in \pi^i L_R$. Hence

$$\pi^{-\ell}(adx - g(adx)g^{-1}) \in ad(L_R).$$

For each $k \in gK_ig^{-1}$, let $k' = g^{-1}kg$. If $k \in gK_ig^{-1} \cap K_i$, then we have $k' \in K_i$ and

$$adT_k = adT_{gk'g^{-1}} = g(adT_{k'})g^{-1}.$$

Hence

$$\pi^{-l}g^{-1}(adx - g(adx)g^{-1})g \in ad(L_R).$$

Summarizing the results above, we have

$$adx - g(adx)g^{-1} \in \pi^l(ad(L_R) + gad(L_R)g^{-1}).$$

Therfore, the proof is complete.

For a given nondegenerate element x, let

$$L_{F,x}^0 = \{y \mid y \in L_F, adx(y) = 0\},$$
$$L_{F,x}^* = \{z \mid z \in L_F, (z, y) = 0, y \in L_{F,x}^0\}.$$

Let $L_{\mathbf{R},x}^0 = L_{F,x}^0 \cap L_{\mathbf{R}}$ and $L_{\mathbf{R},x}^* = L_{F,x}^* \cap L_{\mathbf{R}}$. It is clear that

$$L_F = L_{F,x}^0 \oplus L_{F,x}^* \quad \text{and} \quad L_{\mathbf{R}} = L_{\mathbf{R},x}^0 \oplus L_{\mathbf{R},x}^*.$$

For any $y, z \in L_F, (adx(y), z) + (y, adx(z)) = 0$, so $adx(L^*_{\mathbf{R},x}) \subset L^*_{\mathbf{R},x}$.

Thus, it is easy to see that the restriction of adx on $L^*_{\mathbf{R},x}$ denoted by $(adx)^*$ is an invertible linear transformation on $L^*_{\mathbf{R},x}$, so $(adx)^*$ is bijective. Hence we have

(I) For each nondegenerate element x, $(adx)^*$ is surjective on $L^*_{\mathbf{R},x}$.

Lemma 3.2. *For any $i \in \mathbf{N}$ and each nondegenerate element x, let $u \in L_R$ such that*

$$x \equiv u(\mathrm{mod} L_i), u = u_0 + \pi^i u^*, u_0 \in L^0_{\mathbf{R},x}, u^* \in L^*_{\mathbf{R},x}, \quad L_i = \pi^i L_R.$$

Then adu is conjugate to adu_0, $u_0 \in L^0_{\mathbf{R},x}$ under K_i.

Proof. By (I), there is a $u_1^* \in L^*_{\mathbf{R},x}$ such that $adx(u_1^*) \equiv u^*(\mathrm{mod} L_1)$.

So we have $adu(u_1^*) \equiv u^*(\mathrm{mod} L_1)$. Let $k_1 = \exp(\pi^i adu_1^*)$. Then we have

$$\begin{aligned} Adk_1(adu) &\equiv (I + \pi^i adu_1^*) adu (I + \pi^i adu_1^*)^{-1}, \ (\mathrm{mod} L_i) \\ &\equiv (I + \pi^i adu_1^*) adu (I - \pi^i ad_1^*), \ (\mathrm{mod} L_i) \\ &\equiv adu + \pi^i adu_1^* adu - \pi^i aduadu_1^*, \ (\mathrm{mod} L_i) \\ &\equiv adu + \pi^i [adu_1^*, adu], \ (\mathrm{mod} L_i) \\ &\equiv adu - \pi^i (ad[u, u_1^*]), \ (\mathrm{mod} L_i) \\ &\equiv adu - \pi^i adu^*, \ (\mathrm{mod} L_{i+1}) \\ &= adu_0, \ (\mathrm{mod} L_{i+1}). \end{aligned}$$

Similarly, for any $p \in \mathbf{N}$, there is a $k_p \in K_i$ such that

$$adk_p(adu) \equiv adu_0, \ (\mathrm{mod} L_{i+p}).$$

Therefore, the lemma follows by taking $p \to \infty$.

Let α_0 be the maximal positive root. Then the root α_0 can be expressed in the form

$$\alpha_0 = m_1\alpha_1 + m_2\alpha_2 + \cdots + m_n\alpha_n,$$

where $m_i \in \mathbf{N}, \ 1 \le i \le n$. Let

$$N(\alpha_0) = \{k \in [1, n] \mid m_k \le 2\}.$$

Let $A(\alpha_0)$ be the subset consisting of the integers k in $N(\alpha_0)$ which satisfy the condition:

$k \ne \frac{1}{2}(n+1)$ if n is odd and $L \simeq A_n$, $k \ne 7$ if $L \simeq E_7$;

$k \ne \frac{1}{2}n$ if n is even and $L \simeq B_n$ or C_n.

For each $k \in A(\alpha_0)$, set $\Pi_k^1 = \Pi \setminus \alpha_k$, and put $\Pi_k = \Pi_k^1$ if $m_k = 1$, $\Pi_k = \Pi_k^1 \cup \alpha_0$ if $m_k \ne 1$.

For each $k \in A(\alpha_0)$, let

$$\Phi_k^s = \{\pm r \mid r = m_{r,1}\alpha_1 + m_{r,2}\alpha_2 + \cdots + m_{r,n}\alpha_n \in \Phi^+, \quad m_{r,k} = s\}, \quad s = 0, 2.$$

Let $\Phi_k = \Phi_k^0 \cup \Phi_k^2$ and $\Phi_k^* = \Phi \setminus \Phi_k$. For any $\ell \in \mathbf{N}, k \in A(\alpha_0)$, we define

$$\begin{aligned} V^*_{\ell,k} &= \{v \in V_\ell \mid r(v) \in \Phi_k^{*-}\}, \quad U^*_{\ell,k} = \{u \in U_\ell \mid r(u) \in \Phi_k^{*+}\}, \\ V_{\ell,k} &= \{v \in V_\ell \mid r(v) \in \Phi_k^-\}, \quad U_{\ell,k} = \{u \in U_\ell \mid r(u) \in \Phi_k^+\}, \end{aligned}$$

where $\Phi_k^\pm = \Phi_k \cap \Phi^\pm$ and $\Phi_k^{*\pm} = \Phi_k^* \cap \Phi^\pm$. For any $i \in \mathbf{N}, k \in A(\alpha_0)$; if i is odd, then we say $j = \frac{1}{2}(i+1)$ and set $J^*_{j,k} = U^*_{j,k} U^*_{j,k}$, if i is even, then we say $j = \frac{1}{2}i$ and set $J^*_{j,k} = V^*_{j+1,k} U^*_{j,k}$.

Let x be a nondegenerate element and let Σ_x be the set of the eigenvalues of adx. If

$$\Sigma_x = \Sigma_x^0 \cup \Sigma_x^*, \quad \Sigma_x^0 \cap \Sigma_x^* = \phi; \quad 0 \in \Sigma_0,$$

then (Σ_x^0, Σ_x^*) is called a decomposition of Σ_x. For any $\mu \in \Sigma_x$, let

$$\Phi_x^\mu = \{r \in \Phi \mid adx(e_r) = \mu(e_r)\} \\ = \{r \in \Phi \mid (r, x) = r(x) = \mu\}.$$

For each nondegenerate element x, if there is $a \in A(\alpha_0)$ and there is a decomposition (Σ_x^0, Σ_x^*) of Σ_x such that $\Phi_a = \bigcup\limits_{\mu \in \Sigma_x^0} \Phi_x^\mu$, then (x, a) is called a compatible pair (or separable pair). For each $r \in \Phi$, let $M_r = \{te_r, t \in F\}$. For each $\mu \in \Sigma_x$, let $M_\mu = \sum\limits_{r \in \Phi_x^\mu} M_r$.

Let $C_F = C_L^0 \otimes F$. For any $\mu \in \Sigma_x$, we define $L_x^\mu = M_\mu + C_F$ if $\mu = 0$, $L_x^\mu = M_\mu$ if $\mu \neq 0$.

For each $a \in A(\alpha_0)$ let χ_a be the character of $Z\Phi$ such that

$$\chi_a(\alpha_i) = 1, 1 \le i \le n, \quad i \neq a, \quad \chi_a(\alpha_a) = -1, \quad \alpha_i \in \Pi, \quad 1 \le i \le n.$$

Clearly, $\chi_a(r) = 1$ if $r \in \Phi_a$, $\chi_a(r) = -1$ if $r \in \Phi_a^*$.

For any $a \in A(\alpha_0)$, let $G_a = \{g \mid g \in G, gh(\chi_a) = h(\chi_a)g\}$. For any nondegenerate element x, let $G_x = \{g \mid g \in G, g(adx) = (adx)g\}$.

Lemma 3.3. (i) *For any nondegenerate element x and any $i \in \mathbf{N}$, $i < l$,*

$$\mathrm{supp} H(G//K_i, \psi_x) \subset K_i G_x K_i.$$

(ii) *If (x, a) is a compatible pair, then for any $i \in \mathbf{N}$, $i < l$,*

$$\mathrm{supp} H(G//K_i, \psi_x) \subset K_i G_a K_i.$$

Proof. For $\ell \in \mathbf{N}, i \le l$, let $y \in L_{\mathbf{R},x}^0$ and $z = x + \pi^\ell y$. Since $y \in L_{\mathbf{R},x}^0$, for each $\mu \in \Sigma_x$, there exists $0 \neq v \in (L_x^\mu)' = L_x^\mu \otimes F'$ such that $ady(v) = \xi v, \xi \in F'$, where F' is some extension of F, and the eigenvalue ξ of ady is called the eigenvalue of ady associated with μ. We denote by Σ_y^μ the set of all eigenvalues of ady associated with μ. It is easily shown that $\Sigma_y = \bigcup\limits_{\mu \in \Sigma_x} \Sigma_y^\mu$, where Σ_y is the set of all eigenvalues of ady. Hence we have the following statement.

(A) If λ is an eigenvalue of $adx + \pi^\ell ady, y \in L_{\mathbf{R},x}^0$, then there exist $\mu \in \Sigma_x$ and $\xi \in \Sigma_y$ such that $\lambda = \mu + \pi^\ell \xi$.

By an obvious modification of Lemma 3.1, for each $g \in \mathrm{supp} H(G//K_i, \psi_x)$, there exist $y_s, s = 1, 2$ in $L_{\mathbf{R}}$ such that

$$Adg(adu_1) = adu_2, \quad u_s = x + \pi^\ell y_s, \quad s = 1, 2.$$

By Lemma 3.2, we can find $y_s' \in L_{F,x}^0 \cap L_R$, and $k_s \in K_i, s = 1, 2$, such that

$$Adk_s(adu_s) = adz_s, \quad z_s = x + \pi^\ell y_s', \quad s = 1, 2.$$

Thus, we have $Adg'(adz_1) = adz_2, g' = k_2 g k_1^{-1}$. This implies that

(B) $g'(adz_1) = (adz_2)g'$.

Let $f_s(\lambda), s = 1, 2$ be the characteristic polynomial of $adz_s, s = 1, 2$, and let $f(\lambda) = f_1(\lambda) f_2(\lambda)$. Let F' be the splitting field of $f(\lambda)$ over F.

Set $L_{F'} = C_L \otimes F'$. If $adz_1(v) = \lambda v, \lambda \in F', 0 \neq v \in L_{F'}$, then by (A), it follows that $\lambda = \mu + \pi^\ell \xi, \mu \in \Sigma_x, \xi \in \Sigma_{y_1}^\mu$. Thus, by (B), we have

$$adz_2(v') = \lambda v', v' = g'(v).$$

Hence it follows from (A) that $\lambda = \mu' + \pi^\ell \xi', \mu' \in \Sigma_x, \xi' \in \Sigma_{y_2}^{\mu'}$. By Lemma 1.1, it is easy to see that $\mu = \mu'$ and $\xi = \xi'$. Thus, by (B), we have $g'((L_x^\mu)') = (L_x^\mu)'$ for any $\mu \in \Sigma_x$.

Clearly, v and v' have the following expression

$$v = \sum_{r \in \Phi_x^\mu} t_r e_r + h, \quad v' = \sum_{r \in \Phi_x^\mu} t'_r e_r + h',$$

where $t_r, t'_r \in F', e_r \in C_L, r \in \Phi_x^\mu$ and $h, h' \in C_{F'}^0, h = h' = 0$ if $\mu \neq 0$.

Since (x, a) is compatible pair, we have

$$\begin{aligned} h(\chi_a)(v) &= \sum_{r \in \Phi_x^\mu} t_r \chi_a(r) e_r + h(\chi_a) h \\ &= \sum_{r \in \Phi_x^\mu} d t_r e_r + h, \end{aligned}$$

where $d = 1$ if $\Phi_x^\mu \subset \Phi_a$, $d = -1, h = 0$ if $\Phi_x^\mu \subset \Phi_a^*$. Thus, it follows that $h(\chi_a)(v) = dv$, so $g' h(\chi_a)(v) = dg'(v)$. Similarly, we have $h(\chi_a) g'(v) = dg'(v)$. Hence, for any $v \in (L_x^\mu)', \mu \in \Sigma_x$, it follows that

$$g' h(\chi_a)(v) = h(\chi_a) g'(v).$$

It is clear that

$$L_{F'} = \sum_{\mu \in \Sigma_x} (L_x^\mu)'.$$

Thus we have $g' h(\chi_a) = h(\chi_a) g'$, so (ii) follows.

Similarly, for any $v \in (L_x^\mu)', \mu \in \Sigma_x$, we have

$$g'(adx)(v) = (adx) g'(v) = \mu g'(v).$$

Therefore, (i) can be shown immediately.

Hereafter, we shall fix an positive integer i. Moreover, we shall fix a nondegenerate element x and an integer $a \in A(\alpha_0)$ such that (x, a) is compitable. Let $J^* = J_{j,a}^*$, $J' = H_i V_{i,a} U_{i,a}$ and $J = J^* J'$. Clearly, J is a subgroup of G and J' is a subgroup of K_i.

Let ψ'_x be the restriction of ψ_x on J', where x is a nondegenerate element and ψ_x is a character of K_i defined in Section 2 with $\ker\psi_x \subset K_{i+1}$. Then we can extend ψ'_x in a unique fashion to a representation σ of J by letting it be trivial on J^* (if $k = 0$, then σ is a cuspidal representation). We may assume that (x, a) is a compatible pair. By an obvious modification of the argument used in (ii) in Lemma 3.3, we can show the following statement immediately (hereafter, let $l = i = j$ in (3a)).

(II) Let $G' = G_a$. Then $\mathrm{supp} H(G//J, \sigma) \subset JG'J$.

Let $U = \langle x_r; r \in \Phi^+ \rangle$ and $V = \langle x_s; s \in \Phi^- \rangle$. Let W be the Weyl group of L. Clearly $W = \langle w_r; r \in \Phi \rangle$.

Lemma 3.4. *Let $G_a^\times = \langle x_r, r \in \Phi_a; H \rangle$, $H = \langle h_\alpha(t), \alpha \in \Pi, t \in F^* \rangle$. Then $G_a = G_a^\times$.*

Proof. Clearly, if $g \in G_a$, then $g = g^*, g^* = h_a g h_a^{-1}$, $h_a = h(\chi_a)$. By 8.4.4 in [1], g has a unique expression $g = uhn_w u'$, where $w \in W, u \in U, h \in H$ and

$$u' \in U_w^- = \{u \mid u \in U, \quad n_w x n_w^{-1} \in V\}.$$

By 8.4.4 in [1], $g^* = u^* h^* n_w u'^*$, where

$$u^* = h_a u h_a^{-1}, \quad u'^* = h_a u' h_a^{-1} \text{ and } h^* = h_a h {h'_a}^{-1}, \quad h'_a = n_w h_a n_w^{-1}.$$

Assume that $u \notin U_a = U \cap G_a^\times$. It is easy to see that $u^* \neq u$. So, it follows from 8.4.4 in [1] that $g \neq g^*$, a contradiction. Thus we have $u \in U_a$. Similarly, we can show that $u' \in U_a \cap U_w^-$ and $n_w \in N_a$. It follows that $g \in G_a^\times$, so $G_a \subset G_a^\times$. Clearly, we have $G_a^\times \subset G_a$. So $G_a = G_a^\times$. The proof is complete.

Clearly, if $s \in \Phi_a$, $r \in \Phi_a^*$ and $is + jr \in \Phi$ for some positive integers i and j, then $is + jr \in \Phi_a^*$. Hence, by Lemmas 2.1 and 2.2, using an argument similar to that used in Lemma 2.5 and Corollary 2.1, we have

(III) Let $K' = K \cap G'$. Then K' normalizes the subgroup J.

For each $r \in \Phi, \ell \in Z$, let $\mathbf{x}_{r,\ell} = \{x_r(t), t \in \pi^\ell \mathbf{R}\}$.

Let $g_0 = h(\chi_0)$ such that $\chi_0(\alpha_i) = 1, 1 \leq i \leq n, i \neq a$ and $\chi_0(\alpha_a) = \pi$. It follows from [5] that each element g' in G' has a unique expression

$$g' = k_1 d' k_2, \quad k_1, k_2 \in K', \quad d' \in D' = G' \cap D.$$

It is clear that if $d' \in D'$, then $d' = h(\chi')$, χ' satisfying $\chi'(r) = t\pi^{\ell(r)}, t \in \mathbf{R}, \ell(r) \in Z$. For each $d' \in D'$ we define

$$\Phi_+^{*+}(\text{resp.}\Phi_-^{*+}, \Phi_0^{*+}) = \{r \mid r \in \Phi_a^{*+}, \ell(r) > 0(\text{resp.} < 0, = 0)\},$$
$$\Phi_+^{*-}(\text{resp.}\Phi_-^{*-}, \Phi_0^{*-}) = \{s \mid s \in \Phi_a^{*-}, \ell(s) > 0(\text{resp.} < 0, = 0)\}.$$

The following lemma is easily verified.

(IV) (i) $\Phi_a^{*+} = \Phi_+^{*+} \cup \Phi_-^{*+} \cup \Phi_0^{*+}$, $\quad \Phi_a^{*-} = \Phi_+^{*-} \cup \Phi_-^{*-} \cup \Phi_0^{*-}$.

(ii) $\Phi_-^{*+} = -\Phi_+^{*-}$, $\Phi_+^{*+} = -\Phi_-^{*-}$ and $\Phi_0^{*+} = -\Phi_0^{*-}$.

We define

$$J_+^{*+}(\text{resp.}J_-^{*+}, J_0^{*+}) = \{u \mid u \in U \cap J^*, r(u) \in \Phi_+^{*+}(\text{resp.}\Phi_-^{*+}, \Phi_0^{*+})\},$$
$$J_+^{*-}(\text{resp.}J_-^{*-}, J_0^{*-}) = \{v \mid v \in V \cap J^*, r(v) \in \Phi_+^{*-}(\text{resp.}\Phi_-^{*-}, \Phi_0^{*-}).$$

Let $\mathbf{U}_q = \langle x_r(t); r \in \Phi^+, t \in \pi^q \mathbf{R}\rangle$ and $\mathbf{V}_q = \langle x_s(t); s \in \Phi^-, t \in \pi^q \mathbf{R}\rangle, q \in Z$.

Let G'^+ be the set of the elements g' in G' such that

$$Adg'(\mathbf{U}_t) \subset \mathbf{U}_{t-t'}, \quad Adg'(\mathbf{V}_t) \subset \mathbf{V}_{t+t''}, \quad t', t'' \in \mathbf{N} \text{ for any } t \in Z.$$

The following statement is easily verified.

(V) With the notations given above,

(i) $g_0^{-1} \in G'^+$;

(ii) if $g' \in G'$, then $g_0^{-m} g' \in G'^+$ for some m sufficiently large;

(iii) $g_0 \in \text{supp} H(G'//J', \psi_x') = \text{supp} H'$ and $f_{g_0}' * f_{g'}' = f_{g_0 g'}' = f_{g'}' * f_{g_0}'$ for any $g' \in \text{supp} H'$;

(iv) $\{f_g', g' \in G'^+ \cap \text{supp} H'\}$ and f_{g_0}' generate $H' = H(G'//J', \psi_x')$.

By **(III)** and Lemma 2.8 we have

Lemma 3.5. (i) *Let $J_\kappa^* = J_\kappa^{*+} J_\kappa^{*-}, \kappa = +, -, 0$. Then for each $g' \in G'$,*

$$Jg'J = J'J_+^* d' J_0^* J_-^* J',$$

where $d' \in D'$ satisfying $g' = k_1 d' k_2, k_1, k_2 \in K'$ as above.

(ii) *For each $g' \in G'$, $Jg'J \cap G' = J'g'J'$.*

Let η be the map from H' to $H = H(G//J, \sigma)$ defined by

$$\eta(f_{g'}') = f_{g'}(\text{vol}(J'g'J')/\text{vol}(Jg'J))^{\frac{1}{2}}, \quad g' \in \text{supp} H'. \tag{3.η}$$

We shall show that the map η defined above is an isomorphism of algebra.

Clearly J' is a subgroup of J. Therefore, if $g' \in G'$ is in supp$H$, then it certainly is in suppH'. Therefore, (II) guarantees that the map η is surjective. It follows from (ii) in Lemma 3.5 that the map η is injective.

Therefore, to show that the map η is an algebra homomorphism, it suffices to show that for any $g', g'' \in G'^{+} \cap$ suppH',

$$\eta(f'_{g'}) * \eta(f'_{g''}) = \eta(f'_{g'} * f'_{g''}), \tag{3.i}$$

and for any $g' \in G' \cap$ suppH',

$$\eta(f'_{g_0}) * \eta(f'_{g'}) = \eta(f'_{g_0 g'}). \tag{3.ii}$$

Now, we shall show (3.i). For each $g' \in G'^{+}$, we have

$$Adg'(J^* \cap V) \subset J^* \cap V, \quad \text{and} \quad Adg'^{-1}(J^* \cap U) \subset J^* \cap U.$$

Write $J^{*-} = J^* \cap V$ and $J^{*+} = J^* \cap U$. By (III), it is easy to see that J' normalizes the subgroup J^*. Thus for any $g' \in G'^{+}$, we have

$$Jg'J = J'J^*g'J^*J' = J^{*-}J'g'J'J^{*+},$$
$$\mathrm{vol}(Jg'J) = \#\{J^{*-}/(J^{*-} \cap Adg'(J^{*-}))\}\mathrm{vol}(J'g'J').$$

By the formulas given above, using the arguments which are analogous to them used in [4] for the separated case, (3.i) can be established.

For each $X \subset G, d' \in D'$, let $C(X) = X \cap Add'X$. By Lemma 3.5, it is easily shown that

$$\mathrm{vol}(Jd'J) = [\#C(J_+^{*+})][\#C(J_+^{*-})]\mathrm{vol}(J'd'J').$$

Let $q = \#(\mathbb{F})$. Then we have

$$\log_q \mathrm{vol}(Jg_0J) = \#(\Phi_a^{*-}),$$

$$\log_q(\mathrm{vol}(Jd'J)/\mathrm{vol}(J'd'J')) = \sum_{r \in S} \ell(r),$$

where $S = \Phi_+^{*+} \cup \Phi_+^{*-}$. Simimlarly, we have

$$\mathrm{vol}(Jg_0d'J) = [A_1A_2A_3]\mathrm{vol}(J'd'J'),$$

where $A_1 = \#(C(J_+^{*+}))$, $A_2 = \#(C(J_+^{*-}))$ and $A_3 = \#(C(J_0^{*+}))$.

For each $r \in \Phi_a^*$, it follows that $m_{r,a} = 1$ if $r \in \Phi_a^{*+}$, $m_{r,a} = -1$ if $r \in \Phi_a^{*-}$. Therefore, we have $\ell(r) = \ell(r) + m_{r,a}$. Thus, it follows that

$$\log_q(A_1) = \sum_{r \in S_1} (\ell(r) + 1), \quad \log_q(A_2) = \sum_{r \in S_2} (\ell(r) - 1),$$

$$\log_q(A_3) = \sum_{r \in S_3} (\ell(r) + 1),$$

where $S_1 = \Phi_+^{*+}, S_2 = \Phi_+^{*-}$ and $S_3 = \Phi_0^{*+}$.

Thus, we obtain

$$\log_q(\mathrm{vol}(Jg_0d'J)/\mathrm{vol}(Jd'J)) = \#(\Phi_+^{*+}) + \#(\Phi_0^{*+}) - \#(\Phi_+^{*-}).$$

The following formula is easily verified:

$$\log_q(\mathrm{vol}(Jg_0J)) = \#(\Phi^{*+}).$$

Using the formulas given above and the arguments which are analogous to them used in [4] (cf. p.25-28 in [4]), (3.ii) can be shown.

By statements (3.i) and (3.ii) , we obtain the main theorem:

Theorem 3.1. *With the notations as above, the map η defined by $(3.\eta)$ is an algebra isomorphism from $H' = H(G'//J', \psi'_x)$ onto $H = H(G//J, \sigma)$, and the map η is an isometry of L^2-space.*

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