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6-CANONICAL MAPS OF NONSINGULAR MINIMAL 3-FOLDS**

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Abstract

The aim of this paper is to study 6-canonical system of a nonsingular minimal 3-fold X. If $|2K_X|$ is not composed of pencils, it is shown that $\Phi_{|6K_X|}$ is birational with possible exceptions for:

$$(K_X^3, \chi(\mathcal{O}_X), p_g(X)) = (2,-1,0) \text{ or } (2,-1,1) \text{ or } (4,-1,0) \text{ or } (4,-1,1).$$

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§1. Introduction and Main Results

In this paper, all the arguments proceed on base field **C**. Let X be a nonsingular projective 3-fold on which K_X is nef and big. Much progress has been achieved on pluricanonical map for X. As far as I know, X. Benveniste proved in [2] that $\Phi_{|nK_X|}$ is birational for $n \geq 8$, and then K.Matsuki proved in [7] that $\Phi_{|7K_X|}$ is birational. Following their ideas in both [2] and [7], we go on studing 6-canonical maps.

Main Results. *Let X be a nonsingular projective 3-fold on which K_X is nef and big. Suppose that $|2K_X|$ is not composed of pencils, i.e.,* $\dim\Phi_{|2K_X|}(X) \geq 2$. *Then $\Phi_{|6K_X|}$ is birational with the possible exceptions: $p_g \leq 1$ and $(K_X^3, \chi(\mathcal{O}_X)) = (2,-1)$, or $(4,-1)$.*

§2. Review on Surface

The pluricanonical maps for surfaces of general type have been studied in detail by E. Bombieri, Francia, Reider, G. Xiao, etc. We now list several results in the following which will be used or contrasted in next sections. They are well-known to specialists in surface.

Lemma 2.1. *Let S be a nonsingular projective surface of general type with the canonical divisor K_S. Then*

(1) $\Phi_{|nK_S|}$ *is birational for* $n \geq 5$;

(2) *if furthermore S is minimal and* $(K_S^2, p_g) \neq (1,2)$, *then* $\Phi_{|4K_S|}$ *is birational;*

(3) *if S is minimal and* $(K_S^2, p_g) \neq (1,2), (2,3)$, *then* $\Phi_{|3K_S|}$ *is birational;*

(4) *if S has no pencil of curves of genus 2,* $K_S^2 \geq 10$ *or* $p_g \geq 1$, *then* $\Phi_{|2K_S|}$ *is birational with the possible exception for* $p_g = q = 1, K_S^2 = 3, 4$.

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Lemma 2.2.[7,Theorem 5] *Let S be a nonsingular projective surface, $R \in$Pic S be a nef and big divisor on S, and m a positive integer which satisfies the following condition $(*)$:*

$(*)$ *Given arbitrary two distinct points $x_1, x_2 \in S$, letting $\pi : S'' \longrightarrow S$ be the blowing-up at x_1 and x_2, $L_1 = \pi^{-1}(x_1)$, $L_2 = \pi^{-1}(x_2)$, the linear system $|\pi^*(mR) - 2L_1 - 2L_2|$ is not empty. Then $\Phi_{|K_S+mR|}$ is birational in the following two cases:*

(1) *$R^2 \geq 2$ and $m \geq 3$;*

(2) *$R^2 \geq 1$ and $m \geq 4$.*

Remark 2.1. In Lemma 2.2, if condition $(*)$ is substituted by condition $h^0(S, O_S(mR)) \geq 7$, then Lemma 2.2 is also true (see [2], Proposition 3-0).

§3. Proof of Main Result

Let X be a nonsingular complete variety. $D(X) =$Div$(X) \otimes_{\mathbf{Z}} \mathbf{R}$. We denote the numerical equivalence and linear equivalence by $\approx$ and $\sim$, respectively. $N(X) = \{1-\text{cycles on } X\}/\approx \otimes_{\mathbf{Z}} \mathbf{R}$. $\overline{NE}(X)$ is the closure of the effective cone generated by effective 1-cycles. Let $D \in D(X)$. Then D is called nef if $D.C \geq 0$ for any element $C \in \overline{NE}(X)$. We say that D is big if $\kappa(D, X) = \dim X$(see [5]). For any $D \in$Div X with $h^0(X, D) \neq 0$, $\Phi_{|D|}$ denotes the rational map with respect to the complete linear system $|D|$.

At first we introduce Kawamata's theorems which will be used in our proofs.

Proposition 3.1.[6,Theorem 1.2] *Let X be a nonsingular complete variety, $D \in$ Div$(X) \otimes \mathbf{Q}$. Assume the following two conditions:*

(i) *D is nef and big;*

(ii) *the fractional part of D has the support with only normal crossings.*

Then

$$H^i(X, \mathcal{O}_X(\ulcorner D \urcorner + K_X)) = 0 \quad \text{for} \quad i > 0,$$

where $\ulcorner D \urcorner$ is the minimum integral divisor with $\ulcorner D \urcorner - D \geq 0$.

Proposition 3.2.[6,Theorem 2.6] *Let X be a nonsingular complete variety with the canonical divisor K_X. Then the following conditions are equivalent to each other:*

(1) *there exists a positive integer n such that the base locus $Bs|nK_X| = \emptyset$ and $\Phi_{|nK_X|}$ is birational;*

(2) *K_X is nef and big.*

Let X be a nonsingular projective 3-fold. The Riemman-Roch tells that

$$\chi(\mathcal{O}_X(D)) = D^3/6 - K_X.D^2/4 + D.(K_X^2 + C_2)/12 + \chi(\mathcal{O}_X),$$
$$\chi(\mathcal{O}_X) = -C_2.K_X/24.$$

Lemma 3.1. *Let X be a nonsingular projective 3-fold on which K_X is nef and big. Then*

(i) *$K_X.D^2$ is even, especially K_X^3 is even;*

(ii) *$p(n) = h^0(X, \mathcal{O}_X(nK_X)) = (2n-1)[n(n-1)K_X^3/12 - \chi(\mathcal{O}_X)]$, for $n \geq 2$;*

(iii) *$K_X^3 \leq -72\chi(\mathcal{O}_X)$.*

Proof. (i) $\chi(\mathcal{O}_X(D)) + \chi(\mathcal{O}_X(-D)) = -K_X.D^2/2 + 2\chi((\mathcal{O}_X) \in \mathbf{Z}$. Then $K_X.D^2 \in 2\mathbf{Z}$.

(ii) If $n \geq 2$, from Proposition 3.1, we get

$$h^i(X, nK_X) = 0 \ \forall i > 0.$$

Let $D = nK_X$. Using R-R, we obtain

$$\begin{aligned} p(n) &= H^0(X, \mathcal{O}_X(nK_X)) = \chi(nK_X) \\ &= n^3K_X^3/6 - n^2K_X^3/4 + nK_X^3/12 + nK_X.C_2/12 + \chi(\mathcal{O}_X) \\ &= (2n-1)[n(n-1)K_X^3/12 - \chi(\mathcal{O}_X)]. \end{aligned}$$

(iii) From Miyaoka's theorem,we know that $3C_2 - C_1^2$ is pseudo-effective. Thus $K_X.(3C_2 - K_X^2) \geq 0$, $K_X^3 \leq 3K_X.C_2 = -72\chi(\mathcal{O}_X)$. In particular we have $\chi(\mathcal{O}_X) < 0$. This completes the proof of Lemma 3.1.

Now we assume that $\Phi_{|nK_X|}$ is generically finite ($n \geq 2$). We have the following commutative diagram:

$$\begin{tikzcd} X' \arrow[rr, "h_n"] \arrow[d, "f_n"] \arrow[drr, "g_n"] & & W'_n \arrow[d, "s_n"] \\ X \arrow[rr, dashed, "\Phi_{|nK_X|}"] & & W_n \end{tikzcd}$$

where f_n is blowing-ups with nonsingular centers such that $g_n = \Phi_{|nK_X|} \circ f_n$ is a morphism, $g_n = s_n \circ h_n$ is the stein factorization. Let $b_n = \deg(s_n)$. Let H_n be a general hyperplane section of W_n in $\mathbf{P}^{p(n)-1}$. Let S_n be the general member of $|g_n^*(H_n)|$. Then S_n is a nonsingular projective surface. Let $R_n = f_n^*(K_X)|_{S_n}$. We have the following result.

Proposition 3.3. *Let X be a nonsingular projective minimal 3-fold. Assume that $\Phi_{|nK_X|}$ is generically finite. Then the generic degree b_n satisfies the following inequality:*

$$b_n \leq n^2R_n^2/[p(n) - 3].$$

Proof. We use the above diagram and set $nK_X \sim M_n + Z_n$, where Z_n is the fixed part of $|nK_X|$, and set $f_n^*(M_n) \sim S_n + E'_n$, $K'_X \sim f_n^*(K_X) + E_n$, where E_n is the ramification divisor for f_n, and E'_n is an exceptional divisor for f_n. Because $f_n^*(K_X)$ is nef and big, S_n is nef, $S_n \not\approx 0$, we get

$$R_n^2 = (f_n^*(K_X)|_{S_n})^2 = f_n^*(K_X)^2.S_n \geq 1.$$

Multiplying $nK_X \sim M_n + Z_n$ by $K_X.M_n$, we have

$$nK_X^2.M_n = K_X.M_n^2 + K_X.M_n.Z_n.$$

Since $M_n.Z_n \geq 0$ as a 1-cycle, we have $K_X.M_n.Z_n \geq 0$, and then

$$K_X.M_n^2 \leq nK_X^2.M_n = nf_n^*(K_X)^2.S_n = nR_n^2.$$

We have

$$nf_n^*(K_X) \sim S_n + f_n^*(Z_n) + E'_n,$$

considering the exact sequence

$$0 \longrightarrow H^0(X', f_n^*(Z_n) + E'_n) \longrightarrow H^0(X', nf_n^*(K_X)) \xrightarrow{r} H^0(S_n, \mathcal{O}_{S_n}(nR_n)).$$

Since $f_n^*(Z_n)+E_n'$ is the fixed part of $|nf_n^*(K_X)|$, we have $\dim_{\mathbf{C}}(\operatorname{Im} r)=p(n)-1$. Since we suppose $\dim\Phi_{|nK_X|}(X)=3$, we have $\dim g_n(S_n)=2$. Let $D=g_n^*(H_n)|_{S_n}$. Then

$$D^2\geq b_n(H_n|_{g_n(S_n)})^2\geq b_n(p(n)-3).$$

On the other hand, we know $R_n.D=f_n^*(K_X).S_n^2$ and

$$\begin{aligned}K_X.{M_n}^2&=f_n^*(K_X).f_n^*(M_n)^2\\&=f_n^*(K_X).f_n^*(M_n)(S_n+E_n')\\&=f_n^*(K_X).f_n^*(M_n).S_n\\&=f_n^*(K_X).S_n^2+f_n^*(K_X).S_n.E_n'\\&=R_n.D+f_n^*(K_X).S_n.E_n',\end{aligned}$$

where $f_n^*(K_X).S_n.E_n'\geq 0$, because $S_n.E_n'\geq 0$ as a 1-cycle. Therefore we get

$$R_n.D\leq K_X.M_n^2\leq nR_n^2.$$

Noting that $R_n.D\geq 1$, on surface S_n, we have $(R_n.D)^2\geq R_n^2.D^2$. Thus

$$D^2\leq (R_n.D)^2/R_n^2\leq n^2R_n^2$$

and then $b_n[p(n)-3]\leq D^2\leq n^2R_n^2$ and

$$b_n\leq n^2R_n^2/[p(n)-3].$$

The proof is completed.

Now we can give the proof of the main result.

Theorem 3.1. *Let X be a nonsingular projective 3-fold on which K_X is nef and big. Suppose that $|2K_X|$ is not composed of pencils, i.e, $\dim\Phi_{|2K_X|}(X)\geq 2$. Then $\Phi_{|6K_X|}$ is birational with the possible exceptions: $(K_X^3,\chi(\mathcal{O}_X))=(2,-1)$ or $(4,-1)$.*

Proof. From Lemma 3.1, we know that $p(2)=3[K_X^3/6-\chi(\mathcal{O}_X)]$. If $(K_X^3,\chi(\mathcal{O}_X))\neq$ (2,-1),(4,-1), then $p(2)\geq 6$.

If $\dim\Phi_{|2K_X|}(X)\geq 2$, we consider the following commutative diagram:

$$\begin{tikzcd} X' \arrow[r, "h_2"] \arrow[d, "f_2"'] \arrow[dr, "g_2"] & W_2' \arrow[d, "s_2"] \\ X \arrow[r, dashed, "\Phi_{|2K_X|}"'] & W_2 \end{tikzcd}$$

where f_2 is a succession of blowing-ups with nonsingular centers such that $g_2=\Phi_{|2K_X|}\circ f_2$ is a morphism, $g_2=s_2\circ h_2$ is the stein factorization. Let H_2 be a hyperplane section of $W_2=\Phi_{|2K_X|}(X)$ in $\mathbf{P}^{p(2)-1}$ and S_2 be a general member of $|g_2^*(H_2)|$. Since we suppose that $\dim W_2\geq 2$, i.e., $|g_2^*(H_2)|$ is not composed of pencils, S_2 is a nonsingular irreducible projective surface. We set $2K_X\sim N_2+Z_2$ where Z_2 is the fixed part of $|2K_X|$, and set

$$f_2^*(N_2)\sim S_2+E_2',\quad K_{X'}\sim f_2^*(K_X)+E_2,$$

where E_2 is the ramification divisor for f_2. E_2' is an exceptional divisor for f_2.

Assume that $\Phi_{|6K_X|}$ is not birational. Let

$$\psi = \Phi_{|K_{X'}+3f_2^*(K_X)+S_2|}.$$

We shall derive a contradiction. We have the relation

$$6K_{X'} \sim \{K_{X'} + 3f_2^*(K_X) + S_2\} + 5E_2 + f_2^*(Z_2) + E_2'.$$

Thus we infer that ψ is not birational.

Claim 3.1. $\psi|_{S_2}$ *is not birational.*

Proof. Fix an effective divisor $D_0 \in |4f_2^*(K_X) + E_2|$ and a section

$$t_0 \in H^0(X', \mathcal{O}_{X'}(4f_2^*(K_X) + E_2))$$

which determines D_0. Then there exists a nonempty Zariski open subset U of X' such that $U \cap D_0 = \phi$ and for an arbitrary point $x \in U$ there exists $y \in U$ distinct from x such that $\psi(x) = \psi(y)$. We may assume that $S_2 \cap U \neq \phi$, since S_2 is a general member.

Take $s \in H^0(X', g_2^*(H_2))$ so that s determines S_2. For any point $x \in S_2 \cap U$, there exists $y \in U$ distinct from x such that $\psi(x) = \psi(y)$. We shall show that y is in S_2. Since

$$t_0.s \in H^0(X', \mathcal{O}_{X'}(K_{X'} + 3f_2^*(K_X) + S_2)),$$

there exists $a \in \mathbf{C}^*$ such that $t_0(x).s(x) = at_0(y).s(y)$. We have $D_0 \cap U = \phi$, which implies $t_0(y) \neq 0$, whereas $s(x) = 0$, therefore $s(y) = 0$, i.e., $y \in S_2 \cap U$. Thus $\psi|_{S_2}$ is not birational.

We have an exact sequences

$$\begin{aligned} 0 &\longrightarrow \mathcal{O}_{X'}(K_{X'} + 3f_2^*(K_X)) \\ &\longrightarrow \mathcal{O}_{X'}(K_{X'} + 3f_2^*(K_X) + S_2) \\ &\longrightarrow O_{S_2}(K_{S_2} + 3R_2) \\ &\longrightarrow 0, \end{aligned}$$

where $R_2 = f_2^*(K_X)|_{S_2}$. From Proposition 3.1, we get

$$H^1(X', \mathcal{O}_{X'}(K_{X'} + 3f_2^*(K_X))) = 0.$$

Thus the homomorphism

$$H^0(X', K_{X'} + 3f_2^*(K_X) + S_2) \longrightarrow H^0(S_2, K_{S_2} + 3R_2) \longrightarrow 0$$

is surjective, i.e., $\psi|_{S_2} = \Phi_{|K_{S_2}+3R_2|}$.

Claim 3.2. $\Phi_{|K_{S_2}+3R_2|}$ *is birational.*

Proof. In order to use Lemma 2.2, we must verify the condition $(*)$ and that $R_2^2 \geq 2$.

We consider the blowing-up of X' at arbitrary two points x_1 and x_2 of S_2, denoted by $\theta : X'' \longrightarrow X'$. Let

$$M_1 = \theta^{-1}(x_1), \quad M_2 = \theta^{-1}(x_2),$$

S_2'' be the proper transform of S_2 and

$$\pi_2 = \theta|_{S_2''} : S_2'' \longrightarrow S_2,$$

the restriction of θ to S_2''. Then π_2 is the blowing-up of S_2 at x_1 and x_2 with the exceptional divisors

$$L_1 = \pi_2^{-1}(x_1) = M_1 \cap S_2'' \text{ and } L_2 = \pi_2^{-1}(x_2) = M_2 \cap S_2'.$$

We have

$$\begin{aligned} h^0(X'', \mathcal{O}_{X''}(3\theta^* f_2^*(K_X))) &= h^0(X', \mathcal{O}_{X'}(3f_2^*(K_X))) \\ &= h^0(X, \mathcal{O}_X(3K_X)) \\ &= 5[K_X^3/2 - \chi(\mathcal{O}_X)] \\ &\geq 15, \end{aligned}$$

where $(K_X^3, \chi(\mathcal{O}_X)) \neq (2,-1).(4,-1)$, and then

$$h^0(X^{''}, \mathcal{O}_{X''}(3\theta^* f_2^*(K_X) - 2M_1 - 2M_2)) \geq 15 - 4 - 4 = 7,$$

$$H^0(X'', \mathcal{O}_{X''}(3\theta^* f_2^*(K_X) - 2M1 - 2M_2)) \neq 0.$$

Since

$$\mathcal{O}_{X''}(3\theta^* f_2^*(K_X) - 2M_1 - 2M_2)|_{S_2''} = \mathcal{O}_{S_2''}(3\pi_2^*(R_2) - 2L_1 - 2L_2),$$

we have the natural restriction homomorphism

$$H^0(X'', \mathcal{O}_{X''}(3\theta^* f_2^*(K_X) - 2M_1 - 2M_2)) \xrightarrow{\delta} H^0(S_2'', \mathcal{O}_{S_2''}(3\pi_2^*(R_2) - 2L_1 - 2L_2)).$$

We claim that δ is not a zero homomorphism, otherwise we have

$$S_2'' \subset Bs|3\theta^* f_2^*(K_X) - 2M_1 - 2M_2|,$$

and then $h^0(X'', \mathcal{O}_{X''}(S_2'')) = 1$. On the other hand,

$$\begin{aligned} h^0(X'', \mathcal{O}_{X''}(S_2'')) &= h^0(X'', \mathcal{O}_{X''}(\theta^* g_2^*(H_2) - M_1 - M_2)) \\ &\geq h^0(W_2, H_2) - 1 - 1 = p(2) - 1 - 1 \\ &\geq 6 - 2 = 4. \end{aligned}$$

This is impossible. Thus δ is not a zero homomorphism. Then $|3\pi^*(R_2) - 2L_1 - 2L_2|$ is nonempty and condition $(*)$ is satisfied.

In the next we shall show that $R_2^2 \geq 2$. On the contrary, assuming that $R_2^2 = 1$, we shall derive a contradiction.

Multiplying $2K_X \sim N_2 + Z_2$ by $K_X.N_2$, we have

$$2K_X^2.N_2 = K_X.N_2^2 + K_X.N_2.Z_2.$$

Noting that $K_X^2.N_2 = f_2^*(K_X)^2.S_2 = R_2^2 = 1$, we have

$$K_X.N_2^2 + K_X.N_2.Z_2 = 2.$$

Since $|S_2|$ is not composed of pencils, $f_2^*(K_X)$ is nef and big and since S_2 is nef, $S_2 \not\approx 0$, we have

$$\begin{aligned} K_X.N_2^2 &= f_2^*(K_X).f_2^*(N_2)^2 \\ &= f_2^*(K_X).f_2^*(N_2).(S_2 + E_2') \\ &= f_2^*(K_X).f_2^*(N_2).S_2 \\ &= f_2^*(K_X).S_2^2 + f_2^*(K_X).S_2.E_2' \\ &\geq 1. \end{aligned}$$

Because $K_X.N_2^2$ is even by Lemma 3.1(i) and $K_X.N_2.Z_2 \geq 0$, we conclude that $K_X.N_2^2 = 2$,$K_X.N_2.Z_2 = 0$. Since

$$2 = K_X.N_2^2 = f_2^*(K_X).S_2^2 + f_2^*(K_X).S_2.E_2',$$

and $f_2^*(K_X).S_2^2 \geq 1$ and since

$$f_2^*(K_X).S_2.E_2' \geq 0,$$

we have the following two cases:

(I) $f_2^*(K_X).S_2^2 = 1$ and $f_2^*(K_X).S_2.E_2' = 1$;

(II) $f_2^*(K_X).S_2^2 = 2$ and $f_2^*(K_X).S_2.E_2' = 0$.

We consider an exact sequence

$$\begin{aligned} 0 &\longrightarrow H^0(X', \mathcal{O}_{X'}(f_2^*(Z_2) + E_2')) \\ &\longrightarrow H^0(X', \mathcal{O}_{X'}(2f_2^*(K_X)) \\ &\xrightarrow{r} H^0(S_2, \mathcal{O}_{S_2}(2R_2)). \end{aligned}$$

Since $f_2^*(Z_2) + E_2'$ is the fixed part of $|2f_2^*(K_X)|$, we have

$$\dim_{\mathbf{C}}(\operatorname{Im} r) = p(2) - 1 \geq 5.$$

Case 1. $\dim g_2(S_2) = 1$. Let

$$a_2 = g_2(S_2).H_2 \geq p(2) - 2 \geq 4, \quad D_2 = g_2^*(H_2)|_{S_2}.$$

Then $D \approx a_2 F$, where F is a general fiber of $g_2|_{S_2}$. Thus

$$R_2.D = a_2 R_2.F \geq a_2 \geq 4.$$

On the other hand, $R_2.D = f_2^*(K_X).S_2^2 = 1$ or 2 in (I) or (II). This is a contradiction.

Case 2. $\dim g_2(S_2) = 2$. In this case, $\Phi_{|2K_X|}$ is generically finite. From Proposition 3.3, we have

$$b_2 \leq \frac{4R_2^2}{[p(2) - 3]} \leq \frac{4}{3},$$

and then $b_2 = 1$, that is to say, $\Phi_{|2K_X|}$ is birational and then $\Phi_{|6K_X|}$ is birational too. This contradicts our assumption. Thus we have proved that $R_2^2 \geq 2$ and Claim 3.2 is proved.

Claim 3.1 and Claim 3.2 contradict each other. Thus $\Phi_{|6K_X|}$ is birational.

Theorem 3.2. *Let X be a nonsingular projective 3-fold on which K_X is nef and big. Suppose that $p_g \geq 2$ and that $|2K_X|$ is not composed of pencils. Then $\Phi_{|6K_X|}$ is birational.*

Proof. For the most part, the proof of Theorem 3.1 can be paralleled and we need to show that $R_2^2 \geq 2$.

Let $K_X \sim M_1 + Z_1$, where Z_1 is the fixed part of $|K_X|$, M_1 is the moving part. Because $p_g = h^0(X, K_X) \geq 2$, $2M_1$ is moving. Let $2K_X \sim M_2 + Z_2$, where Z_2 is the fixed part, M_2 is the moving one. We can write $M_2 \sim 2M_1 + D,\ D$ is effective.

$$R_2^2 = K_X^2.M_2 \geq 2K_X^2.M_1 \geq 2.$$

Thus we complete the proof.

Remark 3.1. $K_X^2.M_1 \geq 1$ can be obtained after studying the canonical map of X.

From the proof of Theorem 3.1, we know that the key point is to show that $R_2^2 \geq 2$. We keep the notations as in above and let $2K_X \sim M_2 + Z_2$, where M_2 is the moving part, Z_2 is the fixed part of $|2K_X|$. We can get the following corollary.

Corollary 3.1. *Let X be a nonsingular projective 3-fold on which K_X is nef and big. Assume that $|2K_X|$ is not composed of pencils and $\Phi_{|6K_X|}$ is not birational, then $K_X^2.M_2 = 1$.*

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