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ON THE CLASSIFICATION OF AF-ALGEBRAS AND THEIR DIMENSION GROUPS (II)**

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Abstract

This paper is a continuation of [1]. It gives some applications of the results in [1], containing some examples of pure-infinite AF-algebras and the invariant properties of the types of the C^*-extensions by two AF-algebras of the same type.

Keywords AF-algebra, Dimension group, C^*-extension.
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§1. Introduction

This is a continuation of paper [1] by the author. In [1], we gave the characters for the classification of AF-algebras defined by J. Cuntz and G. K. Pedersen[2]. In this paper, we shall give some applications of these characters. First we give some examples of III-type C^*-algebras (i.e., pure-infinite C^*-algebras) which are defined in [2], and then we study the quotients and extensions of AF-algebras. We will prove that the types of the C^*-extensions by two AF-algebras of the same types are invariant.

In this paper, we will quote all the notations in [1]. The main results of this paper have been announced in [10].

§2. Examples of III-Type C^*-Algebras

In order to give examples of pure-infinite AF-algebras, by [1, Theorem 3.20], it suffices to give dimension groups of the same type.

Example 2.1. Let

$$\begin{aligned} G &= \bigoplus_{i=1}^{\infty} \mathbb{Z}_i \\ &= \{(a_1,\ a_2,\ \dots a_n,\ \dots):\ a_n \in \mathbb{Z} \text{ and only a finite munber of } a_n \text{ are nonzero}\}, \\ G_+ &= \{a \in G:\ \text{the first nonzero coordinate of } a \text{ is bigger than } 0\} \\ &\quad \bigcup \{(0,\ 0,\ 0, \dots)\}. \end{aligned}$$

Then (G, G_+) is a dimension group, and it does not contain any archimedean elements. Thus (G, G_+) is pure-infinite.

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In the following, we give another interesting example of pure-infinite AF-algebras with unit by using the Bratteli diagram. For the Bratteli diagrams, one can consult the references [4, 5].

Example 2.2. Let A be the AF-algebra whose Bratteli diagram is shown in Figure (1).

Figure 1. The Bratteli Diagram for a Pure-Infinite AF-algebra

The construction of this Bratteli diagram can be described as follows.

First, we let the dyadic number 1 stand for the unique point in the first generation D_1, and let the dyadic numbers $10, 11$ stand for the two points in the second generation $D_2, \cdots$; in general, we let the 2^{n-1}-many dyadic numbers from $1\ \underbrace{00\cdots 0}_{\text{n-1 many 0}}$ to $\underbrace{11\cdots 1}_{\text{n many 1}}$ stand for the 2^{n-1}-many points in the n-th generation D_n. Let $x = x_1x_2\cdots x_n(x_1 = 1,\ x_i = 1,\ \text{or},\ 0 \text{ if } i > 1)$ denote one point in D_n. Then the changing rules are as follows:

1) there exist lines connecting x with $x1,\ x0$ in D_{n+1};

2) if $x_i = 0(i > 1)$ in x for some one or many i $(i > 1)$, then there exist lines moving from the point x to the points in D_{n+1} which are obtained by changing such x_i all or in part to 1 in the corresponding positions.

In another way, one may think of putting a switch on each x_i of $x \in D_n$, and let the number 1 stand for the locking state of the switch, 0 stand for the unlocking state, then the point x in D_n may be thought of representing a state for the Bratteli diagram. The changing rules are as follows:

1) the points in D_n can change to the points in D_{n+1} which are obtained by adding a locking or unlocking switch to the end of the state $x \in D_n$;

2) the unlocking states in x can change into the locking states all or in part, but the switch positions in x are not changed in the next generation.

From this diagram, it is easy to see that if we change all the numbers zero in the second positions into one in the right half of the figure, we obtain exactly the left half of the figure. Thus we may consider that the figure is symmetric about the center axis in some senses. If we let the number $d(x)$ be equal to the sum of all possible moving ways from 1 to x, for example, $d(1) = 1,\ d(11) = d(10) = 1,\ d(111) = d(110) = 2,\ d(101) = d(100) = 1,\ \cdots$, then the values of $d(x)$ in the left of D_n are just $(n-1)$-times of the correspondence x (by the above symmetry) in the right half (if $n \geq 2$). From these facts, it is easy to verify that A is a unital, pure-infinite AF-algebra.

Example 2.3. In [2], J. Cuntz and G. K. Pedersen gave an example of non-semifinite AF-algebras which was first studied by Dixmier in [3]. Here, we shall give another proof of their result in [2] with the characters given in [1].

Let H be an infinitely dimensional, separable Hilbert space. Consider the C^*-subalgebras A_n on the infinite tensor product space $\bigotimes_1^\infty H$, where A_n are generated by the operators of the following forms:

$$x = x_1 \otimes x_2 \otimes \cdots \otimes x_n \otimes \cdots \tag{2.1}$$

where $x_k \in K(H)$ for $k \leq n$, and $x_k = 1$ for $k > n$. It is well-known that A_n are isomorphic to $K(\bigotimes_1^n H)$ and they are closed ideals of $B(\bigotimes_1^n H)$. Moreover for every $n \in N$, $B_n = \sum_{k=1}^n A_k$ is a C^*-subalgebra of $B(\bigotimes_1^n H)$. Let $A = \varinjlim_n (B_n,\ i_n)$ be the direct limit of B_n where $i_n:\ B_n \longrightarrow B_{n+1}$ are the natural embedding maps. Then we have

$$B_1 \xrightarrow{i_1} B_2 \xrightarrow{i_2} B_3 \xrightarrow{i_3} \ldots \to B_n \xrightarrow{i_n} \ldots \tag{2.2}$$

and

$$K_0(B_1) \xrightarrow{i_{1*}} K_0(B_2) \xrightarrow{i_{2*}} K_0(B_3) \xrightarrow{i_{3*}} \cdots \to K_0(B_n) \xrightarrow{i_{n*}} \cdots . \tag{2.3}$$

$K_0(A) = \varinjlim_n (K_0(B_n),\ i_{n*})$ and $i_n,\ i_{n*}$ are injective maps. Denote the maps from $K_0(B_n)$ to $K_0(A)$ by

$$\varphi_n^\infty : K_0(B_n) \longrightarrow K_0(A).$$

By [1, Theorem 3.17], to prove that the C^*-algebra A is non-semifinite, it suffices to find a pure-infinite element in $K_0(A)$. This is very easy. For example, take rank one projections p_0, p_1 in $K(H)$, then the operator $p_2 = p_0 \otimes p_1 \otimes I \otimes \cdots$ is in B_2, if we let the projection p_1 vary in its equivalent class, we can get infinitely many projections in B_2 such that they are equivalent to p_2 and pairwise perpendicular, moreover, any finite sum of them are smaller than $p_0 \otimes I \otimes \cdots$. Therefore, we have got a pure-infinite element in $K_0(A)$.

§3. Quotients and Extensions of AF-Algebras

Theorem 3.1. *Let (G, G_+, Γ) be a scaled dimension group, J be an ideal of G, $J_s = \Gamma_s - \Gamma_s$ where Γ_s is the stable cone of G. Then*

(1) *$(G/J, G_+/J, \Gamma/J)$ is a scaled dimension group;*

(2) *$(G/J_s, G_+/J_s, \Gamma/J_s)$ is a finite scaled dimension group;*

(3) *if $(G/J, G_+/J, \Gamma/J)$ is a finite scaled dimension group, then $J_s \subseteq J$.*

Proof. (1) We have already known that the quotient group $(G/J, G_+/J)$ is a dimension group[6]. Thus it suffices to prove that Γ/J is a scale of $(G/J, G_+/J)$.

Since (G, G_+, Γ) is a scaled dimension group, there exists an AF-algebra A such that its dimension group $(G(A), G(A)_+, \Gamma(A)) = (G, G_+, \Gamma)$. Since J is an ideal of G, by the correspondent relation between the ideals of AF-algebras and their dimension groups[7], there exists an ideal I of A such that $G(I) = J, G(I)_+ = J_+$, and $\Gamma(I) = J_+ \bigcap \Gamma$. For the AF-algebra A and its ideal I, we have the following short exact sequence

$$0 \longrightarrow I \xrightarrow{i} A \xrightarrow{\pi} A/I \to 0. \tag{3.1}$$

By the six-term exact sequence in K-theory[8], and $K_1(A) = 0$ for any AF-algebras, we have

$$0 \to K_0(I) \xrightarrow{i_*} K_0(A) \xrightarrow{\pi_*} K_0(A/I) \to 0 \tag{3.2}$$

i.e.,

$$0 \to J \xrightarrow{i_*} G \xrightarrow{\pi_*} K_0(A/I) \to 0. \tag{3.3}$$

By L. G. Brown's lemma[9, Lemma 9.7], π_* maps the scale of A onto that of A/I, i.e., $\Gamma(A/I) = \pi_*(\Gamma(A))$. Then we get an order preserved isomorphism by the above short exact sequence (3.3)

$$\omega: \quad K_0(A/I) \longrightarrow G/J \tag{3.4}$$

such that $\omega(\Gamma(A/I)) = \Gamma/J$. From this fact, it is easy to verify that Γ/J is a scale of $(G/J, G_+/J)$.

(2) Take an arbitrary element $\tilde{a}_0 \in (\Gamma/J_s) \setminus \{0\}$. Similar to the above proof, we have the short exact sequence

$$0 \to J_s \xrightarrow{i} G \xrightarrow{\pi} G/J_s \to 0 \tag{3.5}$$

and $\pi(\Gamma) = \Gamma/J_s$. Therefore there exists an a_0 in Γ such that $\pi(a_0) = \tilde{a}_0$, and $a_0 \notin \Gamma_s$, i. e., a_0 is a finite element of Γ. By [1, Proposition 3.7], there exists a bounded functional φ on G such that $\varphi(a_0) > 0$ and $\varphi|_{\Gamma_s} = 0$ for φ is bounded. Therefore, we can induce a bounded functional $\tilde{\varphi}$ on G/J_s by

$$\tilde{\varphi}(\tilde{a}) = \varphi(a); \qquad \forall a \in G,\ \pi(a) = \tilde{a} \tag{3.6}$$

and $\tilde{\varphi}(\tilde{a_0}) = \varphi(a_0) > 0$. Since $\tilde{a}_0$ is arbitrary, we have proved that Γ/J_s is a finite scale of $(G/J_s,\ G_+/J_s)$ by [1, Theorem 3.8].

(3) If J is an ideal of G such that Γ/J is a finite scale of $(G/J, G_+/J)$, there exists a faithful bounded functional φ on G/J by [1, Theorem 3.8]. Set

$$f(a) = \varphi(\pi(a)), \quad \forall a \in G. \tag{3.7}$$

Then f is a bounded functional on G with $f|_{\Gamma_s} = 0$, i.e., $\varphi(\pi(a)) = 0,\ \forall a \in \Gamma_s$. Since φ is faithful, it follows that $\pi(a) = 0$, i.e., $a \in J,\ \forall a \in \Gamma_s$. Therefore, $\Gamma_s \subseteq J$.

Remark. The converse of the statement (3) in the above theorem is false, since there exists a finite AF-algebra A (thus $\Gamma_s(A) = \{0\}$), but it has an ideal I such that the quotient A/I is not finite again (see [2, Example 3.9]). However, we have the following proposition (cf. [2, Theorem 3.8]).

Proposition 3.1. *Let (G, G_+, Γ) be a scaled dimension group, J be an ideal of G. Then the following two statements are equivalent*

(1) *$(G/J, G_+/J, \Gamma/J)$ is a finite scaled dimension group;*

(2) *Given $a \in G_+$, if for any bounded functional φ on G with $\varphi(J) = 0$ one always has $\varphi(a) = 0$, then $a \in J_+$.*

Proof. 1) $\Longrightarrow$ 2): Since $(G/J, G_+/J, \Gamma/J)$ is a finite scaled dimension group, there is a faithful bounded functional $\tilde{\varphi_0}$ on G/J. Set

$$\varphi_0(a) = \tilde{\varphi_0}(\pi(a)), \qquad \forall a \in G. \tag{3.8}$$

Then φ_0 is a bounded functional on G and $\varphi_0|_J = 0$. If $a_0 \in G_+$ satisfies the condition in (2), then $\varphi_0(a_0) = 0$, i.e., $\tilde{\varphi_0}(\pi(a_0)) = 0$. Since $\tilde{\varphi_0}$ is faithful, $\pi(a_0) = 0$, i.e., $a \in J_+$.

2) $\Longrightarrow$ 1) : if G/J is not finite, then it contains a nonzero stable element $\tilde{a}_0$ in Γ/J . Take an $a_0 \in \Gamma$ such that $\tilde{a_0} = \pi(a_0)$. If φ is an arbitrary bounded functional on G which satisfies $\varphi(J) = 0$, then by the following equality

$$\tilde{\varphi}(\tilde{a}) = \varphi(a), \qquad \forall \tilde{a} \in G/J \text{ with } \pi(a) = \tilde{a}, \tag{3.9}$$

it can induce a bounded functional $\tilde{\varphi}$ on G/J. Because $\tilde{a_0}$ is a stable element in Γ/J, one has $\varphi(a_0) = \tilde{\varphi}(\tilde{a_0}) = 0$. Thus by the condition (2), we have $a_0 \in J_+$, i.e., $\tilde{a_0} = \pi(a_0) = 0$, which is a contradiction.

Corollary 3.1. *If A is an AF-algebra, then there exists a stable ideal I_s of A such that A/I_s is a finite AF-algebra; if I is an ideal of A such that A/I is a finite AF-algebra, then $I_s \subseteq I$.*

Theorem 3.2. *If A, B are finite AF-algebras, then the C^*-extensions of A by B are also finite AF-algebras.*

Proof. Let C be a C^*-extension of A by B, i.e., the following short sequence is exact:

$$0 \to A \xrightarrow{i} C \xrightarrow{\pi} B \to 0. \tag{3.10}$$

By L. G. Brown's Lemma[9], C is also an AF-algebra. Therefore we have the following commutative diagram

$$\begin{array}{ccccccccc} & & \mathrm{Proj}(A) & \xrightarrow{i} & \mathrm{Proj}(C) & \xrightarrow{\pi} & \mathrm{Proj}(B) & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & K_0(A) & \xrightarrow{i_*} & K_0(C) & \xrightarrow{\pi_*} & K_0(B) & \longrightarrow & 0 \end{array} \tag{3.11}$$

and $\pi_*(\Gamma(C)) = \Gamma(B)$. To prove the theorem, it suffices to prove that $\Gamma(C)$ is a finite scale of $(G(C),\ G(C)_+)$.

If $\Gamma(C)$ were not a finite scale of $(G(C),\ G(C)_+)$, we could get an $a \in \Gamma(C)$ such that $na \in \Gamma(C),\ \forall n \in N$, and then

$$\pi_*(na) = n\pi_*(a) \in \Gamma(B).$$

It follows that $\pi_*(a) = 0$ for the finiteness of B. Then by the exact sequence in (3. 11), there exists a unique sequence of elements $\{b_n\}_{n=1}^{\infty} \in \Gamma(A)$ such that

$$i_*(b_n) = na \in \Gamma(C). \tag{3.12}$$

Since $na \in \Gamma(C)$, there exist $p_n \in \mathrm{Proj}(C)$ such that $[p_n] = na,\ \forall n \in N$. Noticing that

$$[\pi(p_n)] = \pi_*([p_n]) = \pi_*(na) = 0,$$

we have $\pi(p_n) = 0$, i.e., $p_n \in \mathrm{Ker}\pi = \mathrm{Im}i$. Thus, there exist $q_n \in \mathrm{Proj}(A)$ with $i(q_n) = p_n$ and

$$i_*([q_n]) = [i(q_n)] = [p_n] = na \in \Gamma(C). \tag{3.13}$$

By (3.12) and (3.13), we have $i_*(b_n) = i_*([q_n])$, so $b_n = [q_n] \in \Gamma(A)$. Since $i_*([q_n]) = na = ni_*([q_1]) = i_*(n[q_1])$, we have

$$[q_n] = n[q_1], \qquad \text{i.e.,} \quad b_n = nb_1 \in \Gamma(A), \quad \forall n \in N. \tag{3.14}$$

Therefore $b_1 = 0$ for $\Gamma(A)$ is finite, and $a = i_*(b_1) = 0$. Consequently, $\Gamma(C)$ has no nonzero stable element, which completes the proof.

For the semi-finite and pure-infinite AF-algebras, we have similar results.

Theorem 3.3. *If A, B are semi-finite AF-algebras, then all the C^*-extensions of A by B are semi-finite.*

Proof. Let C be a C^*-extension of A by B. Then we have the short exact sequence (3.10) and the commutative diagram (3.11) as in the proof of Theorem 3.2.

If $K_0(C)$ were not semi-finite, then there would be a pure-infinite element c_0 in $\Gamma(C)$, i.e., for any $c_1 > 0$ with $c_1 \le c_0$, there would always exist $c_2 > 0$ such that $nc_2 \le c_1$, $\forall n \in N$.

(1) If $\pi_*(c_0) = 0$, then there exists $a_0 \in \Gamma(A)$ such that $i_*(a_0) = c_0$.

Since $K_0(A)$ is semi-finite, there exists an archimedean element a_1 in $\Gamma(A)$ such that $0 < a_1 \le a_0$. Set $c_1 = i_*(a_1)$. Then $c_1 \le c_0 = i_*(a_0)$, and there is $c_2 > 0$ such that $nc_2 \le c_1$, $\forall n \in N$, for c_0 is pure-infinite. Noticing that

$$\pi_*(c_2) = \pi_*(c_1) = \pi_*(c_0) = 0,$$

we get an $a_2 \in \Gamma(A)$ such that

$$i_*(a_2) = c_2,\ i_*(na_2) = nc_2 \le i_*(a_1) = c_1,\ \forall n \in N.$$

By the injectiveness of i_*, we have $na_2 \le a_1$, $\forall n \in N$; this implies that $a_2 = 0$ for a_1 is archimedean. Consequently, $c_2 = i_*(a_2) = 0$, which is a contradiction. Therefore it is impossible for $\pi_*(c_0)$ to be equal to 0.

(2) If $b_0 = \pi_*(c_0) > 0$, there is an archimedean element $b_1 \in \Gamma(B) \setminus \{0\}$ such that $b_1 \le b_0$ for $K_0(B)$ is semi-finite. Take a projection $p_0 \in \mathrm{Proj}(C)$ with $[p_0] = c_0$. Then

$$[\pi(p_0)] = \pi_*([p_0]) = \pi_*(c_0) = b_0.$$

Since $b_1 \le b_0$, we can find a projection $q_1 \in \mathrm{Proj}(B)$ such that

$$q_1 \le q_0 = \pi(p_0) \ \text{ and } \ [q_1] = b_1.$$

Considering the hereditary C^*-subalgebra p_0Cp_0 of C generated by the projection p_0 and the hereditary C^*-subalgebra q_0Bq_0 of B generated by q_0, we see that the map π restricted to p_0Cp_0 is surjective onto q_0Bq_0. It is well-known that p_0Cp_0, q_0Bq_0 are AF-algebras.

Since $q_1 \le q_0$, $q_1 \in q_0Bq_0$. By the L. G. Brown's Theorem[9], there exists a projection $p_1 \in p_0Cp_0$ such that $\pi(p_1) = q_1$. Let $c_1 = [p_1](> 0, \le c_0)$. Then, by the assumption that c_0 is pure-infinite, there exists $c_2 > 0$ such that $nc_2 \le c_1$ $(\forall n \in N)$. Therefore

$$n\pi_*(c_2) = \pi_*(nc_2) \le \pi_*(c_1) = \pi_*([p_1]) = [\pi(p_1)] = [q_1] = b_1. \tag{3.15}$$

Since b_1 is archimedean, it follows that $\pi_*(c_2) = 0$. However $c_2 \le c_0$, so c_2 is also a pure-infinite element in $\Gamma(C)$. By (1), this is impossible.

Consequently, C is semi-finite.

Theorem 3.4. *If A, B are pure-infinite AF-algebras, then all the C^*-extensions of A by B are pure-infinite.*

Proof. Let C be a C^*-extension of A by B. We have also the exact sequences (3.10) and (3.11).

If C were not pure-infinite, then $\Gamma(C)$ would contain at least one nonzero archimedean element c_0.

(1) If $J = i_*(K_0(A))$ is an essential ideal of $K_0(C)$, then there exists a $c_1 \in J$ with $0 < c_1 \leq c_0$, and c_1 is archimedean by the assumption that c_0 is archimedean. Take an $a_1 \in K_0(A)_+$ such that $i_*(a_1) = c_1$ and an $a_2 \in K_0(A)_+ \setminus \{0\}$ such that $na_2 \leq a_1$ $(\forall n \in N)$; it is possible for $K_0(A)$ is pure-infinite. Then we have $ni_*(a_2) \leq i_*(a_1) = c_1$, which implies that $i_*(a_2) = 0$. Since the map i_* is injective, we have $a_2 = 0$, which is a contradiction. Consequently, it is impossible for J to be an essential ideal of $K_0(A)$.

(2) If J is not an essential ideal of $K_0(A)$, we set

$$J^\perp = \{c \in K_0(C) : \ c \perp J\}.$$

Then $J^\perp \neq 0$ is an ideal of $K_0(C)$ and $J + J^\perp$ is an essential ideal of $K_0(C)$. We first show that $\pi_*(J^\perp)$ is an ideal of $K_0(B)$. To prove this statement, it suffices to prove the equality

$$\pi_*(J^\perp_+) = \pi_*(J^\perp) \bigcap K_0(B)_+. \tag{3.16}$$

Obviously, $\pi_*(J^\perp_+) \subseteq \pi_*(J^\perp) \bigcap K_0(B)_+$.

Conversely, take $b_0 \in \pi_*(J^\perp) \bigcap K_0(B)_+$. Then there is $d_0 \in J^\perp$ such that $\pi_*(d_0) = b_0$ and $d_0' \in K_0(C)_+$ such that $\pi_*(d_0') = b_0$. So

$$\pi_*(d_0 - d_0') = 0, \quad \text{i. e,} \ \ d_0' - d_0 \in J. \tag{3.17}$$

$$d_0' = (d_0' - d_0) + d_0 \in J + J^\perp. \tag{3.18}$$

By [1, Lemma 2.6], we know that every element in $J + J^\perp$ has only a unique decomposition and

$$(J + J^\perp)_+ = J_+ + J^\perp_+,$$

so we have $d_0 \in J^\perp_+$. Thus $\pi_*(J^\perp)$ is an ideal of $K_0(B)$. Moreover it is easy to verify that $\pi_*(J^\perp)$ is an essential ideal of $K_0(B)$.

(i) If $\pi_*(c_0) = 0$, then $c_0 \in i_*(K_0(A))$. Similar to the case (1), we can prove that c_0 is not an archimedean element, which contradicts the assumption. Therefore, we must have $\pi_*(c_0) > 0$.

(ii) If $\pi_*(c_0) > 0$, there exists $c_1 \in (J + J^\perp)_+$ such that $c_1 \leq c_0$ for $J + J^\perp$ is an essential ideal of $K_0(C)$. Decompose c_1 as

$$c_1 = c_1^1 + c_1^2, \ c_1^1 \in J_+, \ c_1^2 \in J^\perp_+.$$

If $c_1^1 \neq 0$, then by (i), c_1^1 is not archimedean. It follows immediately that c_0 is not archimedean, which contradicts the assumption. Therefore, $c_1^1 = 0$, i. e, $c_1 \in J^\perp_+$.

Let $b_1 = \pi_*(c_1) \in K_0(B)_+ \setminus \{0\}$. Then there is $b_2 \in K_0(B)_+$ such that $nb_2 \leq b_1$, $\forall n \in N$, for $K_0(B)$ is pure-infinite. Noticing that $\pi_*(J^\perp)$ is an essential ideal of $K_0(B)$, one can get a $b_3 \in \pi_*(J^\perp_+) \setminus \{0\}$ such that $b_3 \leq b_2$. Take a $c_3 \in J^\perp_+$ such that $\pi_*(c_3) = b_3$, so

$$n\pi_*(c_3) = nb_3 \leq nb_2 \leq b_1 = \pi_*(c_1). \tag{3.19}$$

It follows that $\pi_*(c_1 - nc_3) \geq 0,\ \forall n \in N$. But $\pi_*|_{J^\perp}$ is injective, so $c_1 - nc_3 \geq 0,\ \forall n \in N$. Thus $c_3 = 0$, which is a contradiction.

Consequently, C is pure-infinite.

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