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ON INTERACTION OF SHOCK AND SOUND WAVE (I)

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Abstract

This paper studies the interaction of shock and gradient wave (sound wave) of solutions to the system of inviscid isentropic gas dynamics as a model for the corresponding problems for nonlinear hyperbolic systems. The problem can be reduced to a boundary value problem in a wedged domain. By using the method of constructing asymptotic solutions and Newton's iteration process it is proved that if a weak shock hits a gradient wave, then the grandient wave will split into two gradient waves, while the shock continuses propagating. In this paper the author reduces the problem to a standard form and constructs asymptotic solution of the problem. The existence of the genuine solution will be given in the following paper.

Keywords Shock wave, Sound wave, Nonliner hyperbolic system, Asymptotic solution
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§1. Introduction

Recently the study of discontinuous solution for the system of conservation laws in higher dimensional space has been considerably developed. In [1,9–12] the local existence of solution for such system with discontinuity involving single shock, rarefaction wave or sound wave (gradient wave) has been established. In [2] and [14] the problems on interaction of two shocks or interaction of weak singularities are also considered. It is natural to ask what about the result when a shock is interacted by a wave with weaker singularities, particularly, for the n by n system with $n > 2$. The purpose of this paper and [3] is to solve this problem. For definiteness we restrict ourselves to the system of inviscid isentropic flow in gas dynamics under the assumption that both shock and sound wave are extreme waves. It is proved that when such two waves meet together, the sound waves will split into two gradient waves while the shock continues propagating. Since the head and the tail of rarefaction wave is a gradient wave, the conclusion in this paper has also intepreted the character of interaction between shock and rarefaction wave. Besides, the method in our paper can also be applied to treating the interaction of shock and singularities weaker than gradient waves.

Since the shock and sound wave before interaction are extreme waves, by using the conclusions in [9,12] and the fact that the hyperbolic system has the property of finite propagation velocity one can determine the intersection of shock and sound wave. Hence the remaining problem is to determine the solution of the system of isentropic flow in a wedged domain with an unknown shock front as its free boundary and a given characteristic surface as its

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another fixed boundary. Meanwhile, inside the domain there could be a surface, on which the gradient of the solution is discontinuous. Certainly, the surface is also characteristic and is to be determined.

Our plan is as follows. In §2 we give a precise formulation of the problem . In §3 the problem is reduced to a nonlinear Goursat problem in a wedged domain, and then in §4 an asymptotic solution is constructed. The existence of exact solution to the problem will be given in [3]. There the asymptotic solution will be chosen as the first term in the iterative process. After some technical preparation in §5, the iterative process and corresponding estimates are established in §6. Finally, we complete our proof of the main theorem in §7 and thus establish the main conclusion of this paper.

§2. Formulation

Let us consider the system of unsteady inviscid isentropic flow in two dimensional space with the form of conservation laws

$$\frac{\partial F_0}{\partial t}+\frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}=0, \tag{2.1}$$

where

$$F_0= {}^t(\rho\ \rho u\ \rho v),\quad F_1= {}^t(\rho u\ p{+}\rho u^2\ \rho uv),\quad F_2= {}^t(\rho v\ \rho uv\ p{+}\rho v^2),$$

u, v, ρ, p represent the components of velocity, density and pressure respectively. $p = p(\rho)$ is a given function with $p'(\rho) > 0$. Denote the sound speed by $a = (p'(\rho))^{\frac{1}{2}}$. (2.1) can be written in the form of symmetric hyperbolic system

$$B_0\frac{\partial U}{\partial t}+B_1\frac{\partial U}{\partial x}+B_2\frac{\partial U}{\partial y}=0, \tag{2.2}$$

where $U = {}^t(u, v, p)$,

$$B_0=\begin{pmatrix}\rho & & \\ & \rho & \\ & & a^{-2}\rho^{-1}\end{pmatrix},\quad B_1=\begin{pmatrix}\rho u & & 1\\ & \rho u & \\ 1 & & a^{-2}\rho^{-1}u\end{pmatrix},\quad B_2=\begin{pmatrix}\rho v & & \\ & \rho v & 1\\ & 1 & a^{-2}\rho^{-1}v\end{pmatrix}.$$

We consider the weak solution of (2.1). If the solution U is discontinuous on $S : x = \psi(t, y)$, then the Rankine-Hugoniot condition

$$[F_0]\psi_t-[F_1]+[F_2]\psi_y=0 \tag{2.3}$$

should be satisfied. In this paper we always assume that the jump of U on shock is small.

If the surface $\Lambda(t, x, y)$ =const. is characteristic, then one of the following three relations will hold on it:

$$\Lambda_t+u\Lambda_x+v\Lambda_y+a(\Lambda_x^2\pm\Lambda_y^2)^{\frac{1}{2}}=0, \tag{2.4}$$

$$\Lambda_t+u\Lambda_x+v\Lambda_y=0. \tag{2.5}$$

Obviously, if the characteristic surface is discribed by $x = \phi(t, y)$, then $\phi(t, y)$ satisfies

$$\phi_t-u+v\phi_y+a(1\pm\phi_y^2)^{\frac{1}{2}}=0 \tag{2.6}$$

or

$$\phi_t-u+\phi_y=0. \tag{2.7}$$

Suppose that the system (2.1) has a weak solution for $t < 0$, which contains a weak shock of first class $S : x = \psi(t, y)$ and a sound wave $R_1 : x = \phi_1(t, y)$ propagating along the first characteristics,

$$\psi(0,0) = \phi_1(0,0) = \frac{\partial \psi}{\partial y}(0,0) = \frac{\partial \phi_1}{\partial y}(0,0) = 0. \tag{2.8}$$

Then the solution $U(t, x, y)$ can be expressed as

$$U(t,x,y) = \begin{cases} U^-(t,x,y), & x < \psi(t,y), \\ U_1^+(t,x,y), & \psi(t,y) < x < \phi_1(t,y), \\ U_2^+(t,x,y), & x > \phi_1(x,y), \end{cases}$$

where U^- and U_1^+ satisfy (2.3) on $x = \psi(t, y)$, (2.7) and the equality $U_1^+ = U_2^+$ on $x = \phi_1(t, y)$. The problem is to determine the solution of (2.1) at $t > 0$.

The wave graph of the solution of our problem can be looked as a perturbation of the similar problem in one space-dimensional case. That is, S and R will intersect at a curve Γ, and then the shock S continues propagating, while weak singularities will split into two waves propagating along characteristics R_2 and R_3 , issuing from Γ. Our main result can be precisely expressed as follows.

Theorem 2.1. *For given $s > 0$, there exists an integer $\mu > 0$ (to be determined), such that for $U^-, U_1^+, U_2^+ \in H^\mu, \phi_1(t, y) \in H^\mu, \psi(t, y) \in H^{\mu+1}$ in $t < 0$ we have the following conclusions:*

1) *We can find an integer λ satisfying $0 < \lambda < \mu$, a neighborhood Ω_0 of the origin, and a function $\tau(y)$ defined on $\Omega_0 \cap \{t = x = 0\}$, such that $\phi_1(t, y)$ (resp.$\psi(t, y)$) can be continuously extended to $0 < t < \tau(y)$ as $H^\lambda(H^{\lambda+1})$ function, and U^-, U_1^+, U_2^+ can be continuously extended to $\{0 < t < \tau(y), x < \psi\}, \{0 < t < \tau(y), \psi < x < \phi_1\}, \{0 < t < \tau(y), x > \psi\}$ as H^λ functions respectively. Besides,* (2.1)-(2.7) are still satisfied.

2) For $t > \tau(y)$, there are functions $\tilde\psi(t, y) \in H^{s+1}, \tilde\phi_1(t, y) \in H^s$ defined on $\Omega_0 \cap \{x = 0\}$, $\Lambda(t, x, y) \in H^s$ defined on Ω_0 and a piesewise H^s function

$$\widetilde U(t,x,y) = \begin{cases} \widetilde U^-(t,x,y), & x < \tilde\phi_1(t,y), \\ \widetilde U_2^-(t,x,y), & x > \tilde\phi_1(t,y), \Lambda(t,x,y) < 0, \\ \widetilde U_1^-(t,x,y), & \Lambda(t,x,y) > 0, x < \tilde\psi(t,y), \\ \widetilde U^+(t,x,y), & x > \tilde\psi(t,y), \end{cases}$$

such that

$$\tilde\psi(\tau(y), y) = \psi(\tau(y), y) = \tilde\phi_1(\tau(y), y), \quad \Lambda(\tau(y), \tilde\psi(\tau(y), y), y) = 0,$$

$$\widetilde U^+(\tau(y), x, y) = U_2^+(\tau(y), x, y), \quad \widetilde U^-(\tau(y), x, y) = U^-(\tau(y), x, y).$$

Besides, (2.3), (2.5) and (2.7) are satisfied on $x = \tilde\psi, \Lambda = 0$ and $x = \tilde\phi_1$ respectively, while the system (2.1) is satisfied in the domain where U is in H^s.

§3. Nonlinear Gousart Problem

Before the proof of Theorem 2.1 we remark that in this theorem we pay main attention to the existence of the smooth solution to the problem of wave interaction; meanwhile, we allow the smoothness loss of the solution comparing the smoothness of the initial data. The similar treatment was accepted in [1,12], when the authors proved the existence of sound wave or rarefaction wave, even without wave interaction. Under such an understanding the first part of Theorem 2.1 can be easily proved by applying the result in [8] and [10,11]. Since $\frac{\partial \phi_1}{\partial t}(0,0) > \frac{\partial \psi}{\partial t}(0,0)$, by implicit function theorem we can determine a function $t = \tau(y)$ for small $|y|$ from the equality $\psi(t, y) = \phi_1(t, y)$, and then the intersection $\Gamma : t = \tau(y), x = \psi(\tau(y), y)$ is obtained. The uniqueness of the solution in between the extension of S and R is garanteed by the property of finite propagation velocity for hyperbolic systems.

The proof of the second part is our main task. The functions $\widetilde{U}^-(t, x, y)$, $\widetilde{U}^+(t, x, y)$ and $\tilde{\phi}_1(t, y)$ can also be determined by using the property of finite propagation velocity. However, in order to find other unknown functions for $t > \tau(y)$, we need to take a long procedure. Next in order to alleviate the notational burden we simply omit the notation above U, ψ and ϕ_1, if no confusion arises.

We first reduce the problem to a Goursat problem on a wedged domain by several coordinate transformations. The first transformation is

$$\chi_1 : t_1 = t - \tau(y), \quad x_1 = x - \xi(y), \quad y_1 = y, \tag{3.2}$$

where $\xi(y) = \phi_1(\tau(y), y)$. It is easy to see that χ_1 transforms Γ to $t_1 = x_1 = 0$ and transforms the system (2.2) to

$$B_0^{(1)} \frac{\partial U}{\partial t_1} + B_1^{(1)} \frac{\partial U}{\partial x_1} + B_2^{(1)} \frac{\partial U}{\partial y_1} = 0, \tag{3.3}$$

where $B_0^{(1)} = B_0 - \tau'(y) B_2, B_1^{(1)} = B_1 - \xi'(y) B_2, B_2^{(1)} = B_2$.

As the second step we introduce

$$\chi_2 : t_2 = t_1,\ y_2 = y_1,\ x_2 = \frac{2x_1 - \psi(t_1, y_1) - \phi_1(t_1, y_1)}{\psi(t_1, y_1) - \phi_1(t_1, y_1)} t_1 \tag{3.4}$$

which transforms the shock S and the characteristic surface of first class R_1 to $x_2 = t_2$ and $x_2 = -t_2$ respectively. Denote the image of the family of characteristic surfaces of second class by $\Lambda(t_2, x_2, y_2)$ =const. For definitness we determine these constants in the expression by adding boundary conditions as

$$\Lambda(x_2, x_2, y_2) = x_2 \text{ for } x_2 > 0, \quad \Lambda(-x_2, x_2, y_2) = x_2 \text{ for } x_2 < 0. \tag{3.5}$$

The next transformation is

$$\chi_3 : t_3 = t_2, y_3 = y_2, x_3 = \Lambda(t_2, x_2, y_2) \tag{3.6}$$

which maps all characteristic surfaces of second class to x_3 =const. Particularly, the characteristic surface starting from the intersection Γ is $x_3 = 0$ now. Meanwhile, (3.6) holds the form of the boundary $x_2 = \pm t_2$. Under the coordinate system (t_3, x_3, y_3) the system (3.3) has the form

$$B_0^{(3)} \frac{\partial U}{\partial t_1} + B_1^{(3)} \frac{\partial U}{\partial x_1} + B_2^{(3)} \frac{\partial U}{\partial y_1} = 0, \tag{3.7}$$

where $B_0^{(3)} = B_0^{(2)} = B_0^{(1)},\ B_2^{(3)} = B_2^{(2)} = B_2^{(1)},\ B_1^{(3)} = B_0^{(2)} \frac{\partial \Lambda}{\partial t_2} + B_1^{(2)} \frac{\partial \Lambda}{\partial x_2} + B_2^{(2)} \frac{\partial \Lambda}{\partial y_2}$,

$$\begin{aligned} B_1^{(2)} = {} & B_0^{(1)} \Big(\frac{x_2}{t_2} - \frac{\psi_t - \phi_{1t}}{\psi - \phi_1} x_2 - \frac{\psi_t + \phi_{1t}}{\psi - \phi_1} t_2 \Big) + B_1^{(1)} \frac{2t_2}{\psi - \phi_1} \\ & + B_2^{(1)} \Big(-\frac{\psi_y - \phi_{1y}}{\psi - \phi_1} x_2 - \frac{\psi_y + \phi_{1y}}{\psi - \phi_1} t_2 \Big). \end{aligned} \tag{3.8}$$

Here we emphasize that the transforms χ_2, χ_3 depend on the unknown function ψ, so the coefficients of (3.7) depend on the gradient of ψ.

Finally we introduce the transformation

$$\chi_4 : \begin{cases} t_4 = t_3, x_4 = x_3, y_4 = y_3 & \text{for} \quad x_3 > 0, \\ t_4 = t_3, x_4 = -x_3, y_4 = y_3 & \text{for} \quad x_3 < 0, \end{cases} \tag{3.9}$$

and set

$$A_0 = \operatorname{diag}(B_0^{(3)}, B_0^{(3)}), \quad A_1 = \operatorname{diag}(B_1^{(3)}, -B_1^{(3)}), \quad A_2 = \operatorname{diag}(B_2^{(3)}, B_2^{(3)}),$$

$$\underline{U}(t_4, x_4, y_4) = {}^t(\underline{U}_{(1)}(t_4, x_4, y_4), \underline{U}_{(2)}(t_4, x_4, y_4)) = {}^t(U(t_4, x_4, y_4), U(t_4, -x_4, y_4)).$$

We have

$$\underline{LU} \equiv A_0 \frac{\partial \underline{U}}{\partial t_4} + A_1 \frac{\partial \underline{U}}{\partial x_4} + A_2 \frac{\partial \underline{U}}{\partial y_4} = 0 \quad \text{in } \Omega : 0 < x_4 < t_4. \tag{3.10}$$

Obviously, the transform (3.9) is nothing but folding the domain $t_3 > 0, -t_3 < x_3 < t_3$ in half. Corresponding to the above transforms χ_1 to χ_4, the boundary conditions are

$$\underline{U}_{(2)} = U^-, \quad \Sigma_1 : x_4 = t_4, \tag{3.11}$$

$$F(\underline{U}, \psi) \equiv [F_0^*]\psi_{t_4} - [F_1^*] + [F_2^*]\psi_{y_4} = 0, \quad \Sigma_1 : x_4 = t_4, \tag{3.12}$$

$$\psi = 0, \quad \Gamma : x_4 = t_4 = 0, \tag{3.13}$$

$$\underline{U}_{(1)} = \underline{U}_{(2)}, \quad \Sigma_2 : x_4 = 0, \tag{3.14}$$

where F_j^* can be easily obtained by direct calculation.

Besides, the inverse function $x_2 = \Phi(t_2, x_3, y_2)$ of $x_3 = \Lambda(t_2, x_2, y_2)$ satisfies $a\Phi_{t_3} + b\Phi_{y_3} + d = 0$, where $a = 1 - \tau'(y)v, b = v$,

$$d = -a\Big(\frac{x_2}{t_2} - \frac{\psi_t - \phi_{1t}}{\psi - \phi_1}x_2 - \frac{\psi_t + \phi_{1t}}{\psi - \phi_1}t_2\Big) - 2t_2\frac{u - \xi'(y)v}{\psi - \phi_1} - \Big(\frac{\phi_{1y} - \psi_y}{\psi - \phi_1}x_2 - \frac{\psi_y + \phi_{1y}}{\psi - \phi_1}t_2\Big)v$$

with (t_2, x_2, y_2) replaced by (t_3, Φ, y_3). Let

$$\underline{\Phi}(t_4, x_4, y_4) = {}^t(\underline{\Phi}_{(1)} \ \underline{\Phi}_{(2)}) = {}^t(\Phi(t_4, x_4, y_4) \ \Phi(t_4, -x_4, y_4)).$$

Then $\underline{\Phi}$ satisfies the equation with the form as

$$a\underline{\Phi}_{t_4} + b\underline{\Phi}_{y_4} + d = 0. \tag{3.15}$$

Besides, the function $\underline{\Phi}$ satisfies the condition

$$\underline{\Phi}(t_4, x_4, y_4) = x_4 \qquad \text{on } \Sigma_1. \tag{3.16}$$

Therefore, our problem has been reduced to finding a solution $(\underline{U}, \underline{\Phi}, \psi)$ for the system (3.10),(3.15) in Ω, so that the boundary conditions (3.11)-(3.14) and (3.16) are satisfied. This problem is called Problem (G). Obviously, once the existence of solution for (G) with appropriate smoothness is proved, the second part of Theorem 2.1 is obtained.

§4. Asymptotic Solution

To prove the existence of the solution of (3.10)-(3.16) we will use Newton iterative scheme. As a first term of the iterative sequence we construct an asymptotic solution $(\underline{U}^{(a)}, \underline{\Phi}^{(a)}, \psi^{(a)})$, which satisfies the system with error $O(t^{\lambda_0})$ and the boundary condition with error $O(t^{\lambda_0+1})$.

In order to construct the asymptotic solution we have to show that the system and the boundary condition satisfy compatibility conditions. It means that by using different way to compute the derivatives of U, Φ, ψ at Γ, the results are identical. From the first part of Theorem 2.1, the initial data on $t = 0$ for the system (3.3) are known. These data have discontinuity at $x_1 = 0$ and satisfy the compatibility conditions of order zero. But in order to satisfy compatibility conditions of higher order we need the following lemma.

Lemma 4.1. *If the system of conservation laws*

$$\sum_{i=0}^{n} \frac{\partial F_i(u)}{\partial x_i} = 0 \tag{4.1}$$

is strictly hyperbolic, the initial data at $x_0 = 0$,

$$u = \begin{cases} u_0^+(x), & x_n > 0, \\ u_0^-(x), & x_n < 0 \end{cases} \tag{4.2}$$

have jump at $x_n = 0, u_0^\pm \in C^\ell$, and there is a function $\sigma_0(x')$, such that $\lambda_N(u_0^-) > \sigma_0 > \lambda_N(u_0^+)$ and

$$\sigma_0(F_0(u_0^+) - F_0(u_0^-)) = (F_n(u_0^+) - F_n(u_0^-)), \qquad x_0 = x_n = 0. \tag{4.3}$$

Then for any $k < \ell$ we can define $\sigma_k(x'), u_{j,k}(x') \in C^{\ell-k}$ for $1 \le j \le N-1$ and $\phi_{j,k}(x') \in C^{\ell-k}$ for $1 \le j \le N-1$, so that when we regard σ_k as the derivative $\frac{\partial^{k+1}\psi}{\partial x_0^{k+1}}$; $\phi_{j,k}$ as the derivative $\frac{\partial^k \phi_j}{\partial x_0^k}$; $u_{j-1,k}, u_{j,k}$ as the k-th derivative of u on both sides of characteristic surface R_j (for $j < N$) or shock S (for $j = N$), the compatibility conditions of order $\ell - 1$ at $x_0 = x_n = 0$ are satisfied.

Proof. Without loss of generality we assume that $F_0(u) \equiv u$. Denote the shock by $S : x_n = \psi(x')$, the characteristic surfaces by $R_j : x_n = \phi_j(x'), 2 \le j \le N$. The Rankine-Hugoniot condition on shock is

$$\sum_{i=0}^{n-i} \psi_{x_i}(f_i^+ - f_i^-) = f_n^+ - f_n^-. \tag{4.4}$$

Hence (4.3) is just the compatibility condition of order 0. Denoting x_0 by t and differentiating (4.4) along $x_n = \psi(x')$ with respect to t, we have

$$\sum_{i=0}^{n-1} \psi_{x_i t}(f_i^+ - f_i^-) + \sum_{i=0}^{n-1} \psi_{x_i}(\partial_t + \psi_t \partial_n)(f_i^+ - f_i^-) = f_n^{+\prime}(\partial_t + \psi_t \partial_n)u^+ - f_n^{-\prime}(\partial_t + \psi_t \partial_n)u^-. \tag{4.5}$$

Noticing $\psi(0, x') = 0$, we have

$$\sigma_1[u] - \psi_t(\psi_t - f_n^{-\prime})u_{N-1,1} = G_1(\sigma_0, \partial_{x'}\sigma_0, u_0^+, \nabla u_0^+, u_{N-1,0}) \tag{4.6}$$

at $t = 0$, where G_1 is a given function of its arguments. Similarly, differentiating along $x_n = \psi(t, x)$ successively implies

$$\sigma_k[u] - \psi_t^k(\psi_t - f_n^{-\prime})u_{N-1,k} = G_k(\partial_{x'}^{k_2}\sigma_k, \partial_{x_n}^{k_1}\partial_{x'}^{k_2}u_0^+, \partial_{x'}^{k_2}u_{N-1,k_1})_{k_1+k_2 \le k, k_1 < k} \ (k < \ell), \tag{4.7}$$

which is the compatibility condition of order k.

On the other hand, if $x_n = \phi(t, x)$ is a characteristic surface starting from $x_n = t = 0$, then $\phi_j(0, x') = 0$, and

$$\phi_{j,t} = \lambda_j(u, (\phi_j)_{x'}, -1), \tag{4.8}$$

where $\lambda_j(u,\xi,-1)$ is the j-th eigenvalue of $\Sigma\xi_i f_i'$. The compatibility condition of order 0 is $\phi_{j,1}=\lambda_j(u_0^-,0,-1)$. Differentiating (4.8) along $x_n=\phi_j(t,x')$, we have $(\phi_j)_{tt}=\dfrac{\partial\lambda_j}{\partial u}(\partial_t+\phi_{jt}\partial_n)u+\dfrac{\partial\lambda_j}{\partial\xi}(\phi_j)_{x't}$. Then the compatibility condition of order 1 is

$$\phi_{j2}=\frac{\partial\lambda_j}{\partial u}(\phi_{j,1}-f_n')u_{j,1}+\frac{\partial\lambda_j}{\partial\xi}\frac{\partial\phi_{j,1}}{\partial x'} \tag{4.9}$$

(or the equality with $u_{j,1}$ replaced by $u_{j-1,1}$). Subtracting the equiality for $u_{j-1,1}$ from (4.9) for $u_{j,1}$, we obtain $\dfrac{\partial\lambda_j}{\partial u}(\phi_{j,1}-f_n')(u_{j-1,1}-u_{j,1})=0$. Therefore, we require

$$(\phi_{j,1}-f_n')(u_{j-1,1}-u_{j,1})=0. \tag{4.10}$$

Similarly, by differentiating successively we can obtain

$$(\phi_{j,1}-f_n')(u_{j-1,k}-u_{j,k})=0. \tag{4.11}$$

In view of the fact that (3.4) is strictly hyperbolic, $u_{j-1,k}-u_{j,k}$ is the j-th eigenvector for f_n'. So we have

$$u_{j-1,k}-u_{j,k}=w_{j,k}e_j, \tag{4.12}$$

where e_j is the unit eigenvector of j-th class. Hence

$$u_{N-1,k}=\sum_{j=1}^{N-1}w_{j,k}e_j+u_{0,k}, \tag{4.13}$$

where $u_{0,k}$ represents the derivatives of $u_0^-(x)$ and then belongs to $C^{\ell-k}$. Substituting (4.13) to (4.7) we obtain

$$\sigma_k[u]-\psi_t^k(\psi_t-f_n')w_{j,k}e_j=\text{known}. \tag{4.14}$$

Now if the strength of the shock is weak, then $u_0^+(0)$ is near to $u_0^-(0)$, so $[u]$ is near to proportional to the first eigenvector e_1 for f_n', and $\psi_t^k(\psi_t-f_n')e_j$ is near to $\psi_t^k(\psi_t-\lambda_j)e_j$. In view of the strict hyperbolicity of (3.4), $e_1,\cdots,e_N$ are linearly independent. Thus regarding (4.14) as a linear system of σ_k and $w_{j,k}$, the determinant of the coefficient matrix is not zero, and then these unknowns are uniquely determined. After this $u_{j,k}$ and $\phi_{j,k}$ can also be determined. Certainly, these quantities satisfy compatibility conditions and smoothness conditions as required. Thus Lemma 3.1 is obtained.

Remark. If u on the both sides of $x_n=0$ belongs to H^ℓ with $l>s$, then similar conclusion is also satisfied, but the functions $\sigma_k,u_{j,k},\phi_{j,k}$ are only in $H^{\ell-k-\frac12}$.

Theorem 4.1. *For the problem* (G) *there is an asymptotic solution* $(\underline{U}^{(a)},\underline{\Phi}^{(a)},\psi^{(a)})$ *satisfying*

$$\begin{aligned}&\underline{LU}^{(a)}=O(t_4^{\lambda-2})\ \text{ in }\Omega,\quad a\underline{\Phi}_{t_4}^{(a)}+b\underline{\Phi}_{y_4}^{(a)}+d=0\ \text{ in }\Omega,\\&F(\underline{U}_{(1)}^{(a)},\psi^{(a)})=O(t_4^{\lambda-1}),\quad \underline{U}_{(2)}^{(a)}=U_3^-,\quad \underline{\Phi}^{(a)}=x_4\ \text{ on }\Sigma_1,\\&\underline{U}_{(1)}^{(a)}=\underline{U}_{(2)}^{(a)},\quad \underline{\Phi}_{(1)}^{(a)}=\underline{\Phi}_{(2)}^{(a)}\ \text{ on }\Sigma_2,\quad \psi^{(a)}=0\ \text{ on }\Gamma,\end{aligned} \tag{4.15}$$

where the arguments U,Φ,ψ in the coefficients of $\underline{L}$ and a,b,d should also be substituted by $\underline{U}^{(a)},\underline{\Phi}^{(a)},\psi^{(a)}$.

Proof. By using Lemma 4.1 we may determine the derivatives of higher order for ϕ_1,ϕ_2,ψ, as well as u in between R_1 and R_2 or in between R_2 and S. After transformations

χ_2, χ_3, χ_4, we know the derivatives of higher order for $\underline{U}$, $\underline{\Phi}$ and ψ on Γ.

Now we construct the asymptotic solution as follows. First, we determine all derivatives of $\partial^\alpha \underline{U}_{(2)}$ with $|\alpha| \le \lambda$. In fact, all tangential derivatives $D^\alpha \underline{U}_{(2)}$ are equal to corresponding $D^\alpha U_3^-$ on Σ_1. Besides, the normal coefficient matrix $\beta = n_0 A_0 + n_1 A_1 + n_2 A_2$ has constant rank on Σ_1, where (n_0, n_1, n_2) is the normal direction of Σ_1. According to the boundary condition on Σ_1 we may use a suitable transformation of unknown functions to reduce the vector $\underline{U}_{(2)}$ to ${}^t(\underline{U}'_{(2)}\ \underline{U}^*_{(2)})$, such that the corresponding normal coefficient matrix is $\begin{pmatrix} 0 & \\ & \beta^* \end{pmatrix}$ with β^* being 2×2 nonsingular. Therefore, if we know all derivatives $(\partial_n^{k'} D^\alpha \underline{U}_{(2)})_{|\alpha|+k' \le \lambda, k' < k}$, then $(\partial_n^k D^\alpha \underline{U}^*_{(2)})_{k+|\alpha| \le \lambda}$ can also be determined because of $\det\beta^* \ne 0$. On the other hand, $\partial_n^k D^\alpha \underline{U}'_{(2)}$ can be determined by solving a differential equation with initial data given on $x_4 = t_4 = 0$. Hence by induction we obtain all derivatives $\partial^\alpha \underline{U}_{(2)}$ with $|\alpha| \le \lambda$. Similarly, such a conclusion is also valid for $\underline{\Phi}_{(2)}$.

Regarding the value of $\partial^\alpha \underline{U}_{(2)}$ as trace we may determine the function $\underline{U}^{(a)}_{(2)} \in H^\lambda(\Omega)$ having the derivatives as given above. Combining with (3.14) we may construct an H^λ function $\underline{U}^{(a)}_{(1)}$ on Σ_2. Then in a similar way we obtain $\underline{U}^{(a)}_{(1)} \in H^\lambda(\Omega)$ with given trace on Σ_2. Afterwards, we determine a function $\underline{\Phi}^{(a)} \in H^\lambda(\Omega)$ so that it is the solution of (3.15) with its trace on Σ_1 satisfying (3.16). The function $\underline{\Phi}^{(a)}$ automatically satisfies $\underline{\Phi}^{(a)}_{(1)} = \underline{\Phi}^{(a)}_{(2)}$ on Σ_2, because Σ_2 is a characteristic for (3.15). Finally, the function $\psi^{(a)} \in H^{\lambda+1}(\Sigma_1)$ can be determined by finite Taylor series according to its derivatives on Γ. Since the derivatives of $(\underline{U}^{(a)}, \underline{\Phi}^{(a)}, \psi^{(a)})$ satisfies compatibility conditions of order $\lambda - 1$, which is just derived from (3.10)-(3.16), (4.15) is satisfied.

The above asymptotic solution will be chosen as the first term of the sequence, which converges to the genuine solution of the Problem (G). The details will be given in [3].

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