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SPECTRUM-PRESERVING ELEMENTARY OPERATORS ON $\mathcal{B}(\mathcal{X})$**

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Abstract

Characterizations for elementary operators of length 2 to be invertibility-preserving, spectrum-preserving or spectrum-compressing are obtained.

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§1. Introduction

Let $\mathcal{X}$ be an infinite dimensional complex Banach space and $\mathcal{B}(\mathcal{X})$ the Banach algebra of all bounded linear operators on $\mathcal{X}$. For $T \in \mathcal{B}(\mathcal{X})$, $\sigma(T)$, as usual, will denote the spectrum of T. Let Φ be a linear map from $\mathcal{B}(\mathcal{X})$ into itself. Φ is spectrum-preserving if $\sigma(\Phi(T)) = \sigma(T)$ for all $T \in \mathcal{B}(\mathcal{X})$; Φ is spectrum-compressing if $\sigma(\Phi(T)) \subseteq \sigma(T)$ for all $T \in \mathcal{B}(\mathcal{X})$. It is clear that if Φ is unital (i.e., $\Phi(I) = I$), then Φ is spectrum-preserving (spectrum-compressing) if and only if Φ preserves invertibility in both directions (preserves invertibility), i.e., $\Phi(T)$ is invertible if and only if T is ($\Phi(T)$ is invertible if T is). Spectrum-preserving linear maps have been studied by some authors, e.g., see [1, 2, 4-6] and the references therein. In fact, this is one of the so-called linear preserver problems.

Jafarian and Sourour[4] proved that a spectrum-preserving linear map Φ from $\mathcal{B}(\mathcal{X})$ onto itself (i.e., Φ is surjective) is an automorphism or anti-automorphism, that is, there exists an invertible operator A in $\mathcal{B}(\mathcal{X})$ or $\mathcal{B}(\mathcal{X}, \mathcal{X}^*)$ such that $\Phi(T) = A^{-1}TA$ for all T or $\Phi(T) = A^{-1}T^*A$ for all T.

Note that Φ is assumed to be surjective is crucial for the results in [4]. So it is interesting to ask if one can give a characterization for the structure of spectrum-preserving linear maps which are not surjective. But this question seems very difficult to answer. An important class of linear maps on $\mathcal{B}(\mathcal{X})$ which contains many non-surjective maps is the class of elementary operators. Recall that Φ is called an elementary operator if there exist operators $A_1, \cdots, A_n$, $B_1, \cdots, B_n$ in $\mathcal{B}(\mathcal{X})$ such that

$$\Phi(T) = \sum_{i=1}^{n} A_i T B_i \quad \text{for all} \quad T \in \mathcal{B}(\mathcal{X}).$$

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The number $l(\Phi) = \inf\{\ n\ ;\ \Phi(\cdot) = \sum_{i=1}^{n} A_i(\cdot)B_i\}$ is called the length of Φ. For a Banach space $\mathcal{X}$ and an operator $T \in \mathcal{B}(\mathcal{X})$, $\mathcal{R}(T)$ and $\ker T$ will denote the range and the null space of T, respectively. Let $\mathcal{X}^{(n)} = \mathcal{X} \oplus \mathcal{X} \oplus \ldots \oplus \mathcal{X}$, the direct sum of n copies of $\mathcal{X}$ and

$$T^{(n)} = T \oplus T \oplus \cdots \oplus T \in \mathcal{B}(\mathcal{X}^{(n)}).$$

M. Gao[1] considered the spectrum-preserving problem for elementary operators $\Phi(\cdot) = A_1(\cdot)B_1 + A_2(\cdot)B_2$ of length 2. He proved that if $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$ and if $I \in \mathcal{R}(\Phi)$, then Φ is spectrum-preserving if and only if $\mathbf{A} = (\,A_1 \quad A_2\,) \in \mathcal{B}(\mathcal{X}^{(2)}, \mathcal{X})$ is invertible with $\mathbf{A}^{-1} = \mathbf{B}^t = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$. In other words, Φ has the form of

$$\Phi(T) = \mathbf{A}T^{(2)}\mathbf{A}^{-1}, \tag{1.1}$$

which is clearly an injective endomorphism of $\mathcal{B}(\mathcal{X})$.

In the present paper, we show that the assumption $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$ in the Gao's result above can be omitted, and more generally, we discuss the invertibility-preserving and spectrum-compressing elementary operators of length 2 and obtain the characterizations for them respectively.

§2. Results and Proofs

The main result of this paper is the following theorem:

Theorem 2.1. *Let $\Phi(\cdot) = A_1(\cdot)B_1 + A_2(\cdot)B_2$ be an elementary operator of length 2 with $I \in \mathcal{R}(\Phi)$. Then the following statements are equivalent.*

(i) Φ *is spectrum-compressing;*

(ii) Φ *is spectrum-preserving;*

(iii) $\mathbf{A} = (\,A_1 \quad A_2\,)$ *is invertible with* $\mathbf{A}^{-1} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$*;*

(iv) *There exists an invertible operator $\mathbf{A} \in \mathcal{B}(\mathcal{X}^{(2)}, \mathcal{X})$ such that $\Phi(T) = \mathbf{A}T^{(2)}\mathbf{A}^{-1}$ for all T.*

Remark 2.1. This theorem particularly improves the result in [1] mentioned in Introduction by ommitting the assumption $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$.

To prove our main result we begin with a discussion of elementary operators of length 2 which preserve invertibility. Note that if operators T and S with rank great than 1 are linearly independent, then there is a vector x such that Tx and Sx are linearly independent. In fact, Tx and Sx are linearly dependent for all x if and only if T and S either are linearly dependent or both are operators of rank one. This fact will be used frequently in the sequel and a proof of it may be found in [3].

Lemma 2.1. *Let $\Phi(\cdot) = A_1(\cdot)B_1 + A_2(\cdot)B_2$ be an elementary operator of length 2 on $\mathcal{B}(\mathcal{X})$. If Φ is invertibility-preserving, then the following statements are true:*

(a) $\ker B_1 \cap \ker B_2 = \{0\}$*,*

(b) $\mathcal{R}(A_1) + \mathcal{R}(A_2) = \mathcal{X}$*,*

(c) *Both A_1 and A_2 are injective,*

(d) *A_1x is linearly independent to A_2x for every nonzero vector $x \in \mathcal{X}$,*

(e) $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$*.*

Proof. (a) and (b) are obvious.

(c) First note that $\{A_1, A_2\}$ and $\{B_1, B_2\}$ are linear independent sets of operators. So there is a vector x_0 such that B_1x_0 and B_2x_0 are linearly independent. Fix such an x_0.

Claim 1. $\ker A_1 \cap \ker A_2 = \{0\}$.

Assume, on the contrary, that $\mathcal{N} = \ker A_1 \cap \ker A_2 \neq \{0\}$; we shall induce a contradiction.

If $\dim \mathcal{N} \geq 2$, take two vectors $y_1,\ y_2 \in \mathcal{N}$ such that they are linearly independent. For any invertible operator T which sends B_ix_0 to y_i, $i = 1,\ 2$, $\Phi(T)$ can not be invertible since $\Phi(T)x_0 = 0$. Hence, we must have $\dim \mathcal{N} = 1$, i.e., $\mathcal{N} = \{\lambda y_0;\ \lambda \in \mathbf{C}\}$, where $\mathbf{C}$ is the field of complex numbers. We assert that

$$\ker A_1 = \ker A_2 = \mathcal{N}.$$

In fact, if $\ker A_1 \neq \mathcal{N}$, and take $y \in \ker A_1 \setminus \mathcal{N}$, then there is an invertible operator T satisfying $TB_1x_0 = y$ and $TB_2x_0 = y_0$. However, $\Phi(T)x_0 = 0$, contrary to the invertibiliy-preserveness of Φ.

In the case that $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) \neq \{0\}$, there exist vectors y_1 and y_2 which are linearly independent such that $A_1y_1 = -A_2y_2 \neq 0$. Thus for any invertible operator T satisfying $TB_ix_0 = y_i$ we have $\Phi(T)x_0 = 0$, which is impossible.

In the case that $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$, both B_1 and B_2 are invertible. In fact, it is obvious that B_1 and B_2 are surjective since, by (b), $\mathcal{X} = \mathcal{R}(A_1) + \mathcal{R}(A_2)$ is a direct sum. If there is a nonzero vector x for which $B_2x = 0$, then $B_1x \neq 0$ by (a). Let T be an invertible operator such that $TB_1x = y_0$; we get $\Phi(T)x = A_1TX_1x + 0 = A_1y_0 = 0$, a contradiction. Therefore, B_2, as well as B_1, is injective. Because B_1^* and B_2^* are invertible and linearly independent to each other, we can find $f,\ g \in \mathcal{X}^*$ such that f is linearly independent to g and $B_1^*f = -B_2^*g$. Take $h \in \mathcal{X}^*$ such that A_1^*h and A_2^*h are linearly independent. Then there exists an invertible operator T such that $T^*A_1^*h = f$ and $T^*A_2^*h = g$. Now $\Phi(T)$ is invertible but

$$\Phi(T)^*h = B_1^*T^*A_1^*h + B_2^*T^*A_2^*h = B_1^*f + B_2^*g = 0,$$

again a contradiction.

Claim 2. Either A_1 or A_2 is injective.

If, on the contrary, neither of A_1 and A_2 is injective, then there are nonzero vectors $y_1 \in \ker A_1$ and $y_2 \in \ker A_2$. It follows from Claim 1 that y_1 and y_2 are linearly independent. Then for any invertible operator T satisfying $TB_ix_0 = y_i$, $i = 1,\ 2$, we would have $\Phi(T)x_0 = 0$; this completes the proof of Claim 2.

Without loss of generality, we may assume that A_2 is injective. We have to show that A_1 is injective, too.

If $\ker A_1 \neq \{0\}$ and if $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) \neq \{0\}$, then there exist vectors y_1 and y_2 so that they are linearly independent and $A_1y_1 = -A_2y_2$. Now for any invertible operator T such that $TB_ix_0 = y_i$, $i = 1, 2$, we would have $\Phi(T)x_0 = 0$, which is impossible.

If $\ker A_1 \neq \{0\}$ and if $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$, then B_2 must be invertible. In fact, by (a) and (b), B_i are obviously surjective. If $B_2x = 0$ for some x, then $B_1x \neq 0$ and for invertible T with $TB_1x \in \ker A_1$, $\Phi(T)$ would not be invertible since $\Phi(T)x = 0$. Now, let T be an invertible operator. For any $x \in \mathcal{X}$, there exists a vector y such that

$$A_1x + A_2x = \Phi(T)y = A_1TB_1y + A_2TB_2y.$$

Since $\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}$, we get

$$A_ix = A_iTB_iy, \ i = 1, \ 2.$$

Note that A_2 is injective, so we have $x = TB_2y$ and $A_1x = A_1TB_1B_2^{-1}T^{-1}x$ since B_2 is invertible. By the arbitrariness of x, we must have $A_1 = A_1TB_1B_2^{-1}T^{-1}$. Again, since T is arbitrary, we get

$$A_1T(B_2 - B_1) = 0$$

for all invertible operators T. But, this leads to $B_1 = B_2$ which is not the case. Hence we must have $\ker A_1 = \{0\}$, completing the proof of (c).

(d) Let $C_1, \ C_2 \in [A_1, \ A_2]$, the linear span of A_1 and A_2. If C_1 and C_2 are linearly independent, then it is clear that there exist D_1 and $D_2 \in [B_1, \ B_2]$ such that $\Phi(\cdot) = C_1(\cdot)D_1 + C_2(\cdot)D_2$. By (c), both C_1 and C_2 are injective. This means that, for any nonzero linear combination $\lambda_1A_1+\lambda_2A_2$ of A_1 and A_2, $\ker(\lambda_1A_1+\lambda_2A_2) = \{0\}$, i.e., for each nonzero vector x, A_1x and A_2x are linearly independent.

(e) Suppose, on the contrary, that $\mathcal{R}(A_1)\cap\mathcal{R}(A_2) \neq \{0\}$. Then there exist vectors y_1 and y_2 such that $A_1y_1 = -A_2y_2$. By (d), y_1 and y_2 must be linearly independent. Now, for any invertible operator T satisfying $TB_ix_0 = y_i$, $i = 1, \ 2$, $\Phi(T)$ can not be invertible because $\Phi(T)x_0 = 0$. This shows that (e) is true.

The theorem below is a characterization for the elementary operators of length 2 which preserve invertibility.

Theorem 2.2. *Let Φ be an elementary operator on $\mathcal{B}(\mathcal{X})$ of length 2. Then Φ is invertibility-preserving if and only if there exist invertible operators $\mathbf{A} = (\, A_1 \quad A_2 \,)$ and $\mathbf{B} = (\, B_1 \quad B_2 \,)$ in $\mathcal{B}(\mathcal{X}^{(2)}, \mathcal{X})$ such that*

$$\Phi(T) = \mathbf{A}T^{(2)}\mathbf{B}^t \text{ for all } T \in \mathcal{B}(\mathcal{X}). \tag{2.1}$$

Proof. There are operators $A_i, \ B_i \in \mathcal{B}(\mathcal{X})$, $i = 1, \ 2$, such that

$$\Phi(T) = A_1TB_1 + A_2TB_2 = \mathbf{A}T^{(2)}\mathbf{B}^t \quad \text{for all} \quad T \in \mathcal{B}(\mathcal{X}).$$

Since Φ preserves invertibility, by Lemma 2.1, we have

$$\ker A_1 = \ker A_2 = \{0\}, \quad \mathcal{R}(A_1) + \mathcal{R}(A_2) = \mathcal{X}$$

and

$$\mathcal{R}(A_1) \cap \mathcal{R}(A_2) = \{0\}.$$

These together will imply that $\mathbf{A} = (\, A_1 \quad A_2 \,) \in \mathcal{B}(\mathcal{X}^{(2)}, \mathcal{X})$ is invertible. Since $\mathbf{AB}^t = A_1B_1 + A_2B_2 = \Phi(I)$ is invertible, $\mathbf{B}^t$, as well as $\mathbf{B}$, is invertible, too.

The converse is obvious.

Proof of Theorem 2.1. It is obvious that (iii) $\Rightarrow$ (iv) $\Rightarrow$ (ii) $\Rightarrow$ (i). So we need only to prove (i) $\Rightarrow$ (iii).

Assume that Φ is spectrum-compressing. Then Φ preserves invertibility and by Theorem 2.2, $\mathbf{A} = (\, A_1 \quad A_2 \,)$ and $\mathbf{B} = (\, B_1 \quad B_2 \,)$ are invertible. Since $I \in \mathcal{R}(\Phi)$, $I = \Phi(E)$ for some

E. Let Ψ be the elementary operator defined by

$$\Psi(T) = \Phi(TE) = A_1TEB_1 + A_2TEB_2.$$

It is clear that $\Psi(I) = I$ and $\Psi(T) = \mathbf{A}T^{(2)}\mathbf{A}^{-1}$ with

$$\mathbf{A}^{-1} = \begin{pmatrix} EB_1 \\ EB_2 \end{pmatrix} = E^{(2)}\mathbf{B}^t.$$

Therefore, E is invertible and Ψ is spectrum-preserving. Now

$$\sigma(TE^{-1}) = \sigma(\Psi(TE^{-1})) = \sigma(\Phi(TE^{-1}E)) = \sigma(\Phi(T)),$$

which implies that $\sigma(TE^{-1}) \subseteq \sigma(T)$ holds for every $T \in \mathcal{B}(\mathcal{X})$. If $E \neq I$, then there exists $f \in \mathcal{X}^*$ such that $E^{*-1}f \neq f$. Take $x \in \mathcal{X}$ such that

$$\langle x, f\rangle = 1 \neq \langle x, E^{*-1}f\rangle \neq 0.$$

Let $T = x \otimes f$ be the rank one operator defined by $Ty = \langle y, f\rangle x$. Then $TE^{-1} = x \otimes E^{*-1}f$. It is easily seen that

$$\sigma(TE^{-1}) = \{\langle x, E^{*-1}f\rangle,\ 0\} \not\subseteq \{1,\ 0\} = \sigma(T).$$

So, we have $E = I$ and $\Phi(I) = I$. It follows that $\mathbf{A}^{-1} = \mathbf{B}^t$ and $\Phi(T) = \mathbf{A}T^{(2)}\mathbf{A}^{-1}$ for all T, i.e., (iii) is true.

It is worth to note that Theorem 2.1 says particularly that if Φ is a spectrum-compressing elementary operator of length 2 with $I \in \mathcal{R}(\Phi)$, then $\mathcal{R}(\Phi)$ is a subalgebra of $\mathcal{B}(\mathcal{X})$ and Φ is in fact an endomorphism of $\mathcal{B}(\mathcal{X})$. The next theorem explains what will happen for an invertibility-preserving elementary operator of length 2 with the range a subalgebra.

Theorem 2.3. *Let Φ be an elementary operator of length 2 on $\mathcal{B}(\mathcal{X})$. Then Φ is invertibility-preserving with $\mathcal{R}(\Phi)$ a subalgebra of $\mathcal{B}(\mathcal{X})$ if and only if there are invertible operators $\mathbf{A}$, $\mathbf{B} \in \mathcal{B}(\mathcal{X}^{(2)}, \mathcal{X})$ and invertible operator $E \in \mathcal{B}(\mathcal{X})$ such that*

$$\mathbf{A}^{-1} = {E^{(2)}}^{-1}\mathbf{B}^t \text{ and } \Phi(T) = \mathbf{A}T^{(2)}\mathbf{B}^t$$

for all T.

Proof. Suppose that $\mathbf{A}^{-1} = {E^{(2)}}^{-1}\mathbf{B}^t$ and $\Phi(T) = \mathbf{A}T^{(2)}\mathbf{B}^t$. Then Φ preserves invertibility and $B_iA_j = \delta_{ij}E$. Now for any operators T and S, we have

$$\Phi(S)\Phi(T) = \mathbf{A}S^{(2)}\mathbf{B}^t\mathbf{A}T^{(2)}\mathbf{B}^t = \mathbf{A}S^{(2)}E^{(2)}T^{(2)}\mathbf{B}^t = \Phi(SET) \in \mathcal{R}(\Phi).$$

So $\mathcal{R}(\Phi)$ is closed under the product computation and hence is a subalgebra. Note that $\Phi(E^{-1}) = I$, i.e., $\mathcal{R}(\Phi)$ is also unital.

Conversely, assume that Φ is invertibility-preserving with range a subalgebra. By Theorem 2.2, there are invertible operators $\mathbf{A} = (\,A_1 \quad A_2\,)$ and $\mathbf{B} = (\,B_1 \quad B_2\,)$ from $\mathcal{X}^{(2)}$ onto $\mathcal{X}$ such that $\Phi(T) = \mathbf{A}T^{(2)}\mathbf{B}^t$. Since $\mathcal{R}(\Phi)$ is a subalgebra, for any S and T, there is an operator W such that $\Phi(S)\Phi(T) = \Phi(W)$, that is,

$$\mathbf{A}W^{(2)}\mathbf{B}^t = \mathbf{A}\begin{pmatrix} SB_1A_1T & SB_1A_2T \\ SB_2A_1T & SB_2A_2T \end{pmatrix}\mathbf{B}^t.$$

Thus we have

$$\begin{pmatrix} W & 0 \\ 0 & W \end{pmatrix} = \begin{pmatrix} SB_1A_1T & SB_1A_2T \\ SB_2A_1T & SB_2A_2T \end{pmatrix}.$$

Since S and T are arbitrary, we must have

$$B_1A_2 = B_2A_1 = 0 \text{ and } B_1A_1 = B_2A_2 = E.$$

It is clear that E is invertible and $\Phi(S)\Phi(T) = \Phi(SET)$ for any S and T. Notice that

$$\Phi(E^{-1})\Phi(T) = \Phi(E^{-1}ET) = \Phi(T) = \Phi(T)\Phi(E^{-1}).$$

Hence $I = \Phi(E^{-1})$ (i.e., $\mathcal{R}(\Phi)$ is unital) and $\mathbf{A}^{-1} = {E^{(2)}}^{-1}\mathbf{B}^t$.

The following corollary is immediate from Theorem 2.3.

Corollary 2.1. *Let Φ be an elementary operator of length 2 on $\mathcal{B}(\mathcal{X})$ with $\mathcal{R}(\Phi)$ a subalgebra. Then Φ is spectrum-compressing if and only if there exists an invertible operator $\mathbf{A} \in \mathcal{B}(\mathcal{X}^{(2)}, \mathcal{X})$ such that*

$$\Phi(T) = \mathbf{A}T^{(2)}\mathbf{A}^{-1} \quad \textit{for all } T.$$

Let $\mathcal{H}$ be a Hilbert space. Recall that an operator $A \in \mathcal{B}(\mathcal{H})$ is said to be positive if $A = A^*$ and $\sigma(A) \subset [0, +\infty)$; an map Ψ: $\mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is said to be positive if $\Psi(A)$ is positive whenever A is positive. The next corollary is also a generalization of a main result (i.e., (ii) $\Leftrightarrow$ (iii)) due to Gao[1, Theorem 3.2], but the proof here is much simpler by virtue of Theorem 2.1.

Corollary 2.2. *Let Φ be an elementary operator of length 2 on $\mathcal{B}(\mathcal{H})$, where $\mathcal{H}$ is an infinite dimensional complex Hilbert space. The following statements are equivalent.*

(i) Φ *is positive and spectrum-compressing;*

(ii) Φ *is positive and spectrum-preserving;*

(iii) *There exists a unitary operator $\mathbf{U} \in \mathcal{B}(\mathcal{H}^{(2)}, \mathcal{H})$ such that $\Phi(T) = \mathbf{U}T^{(2)}\mathbf{U}^*$ for all $T \in \mathcal{B}(\mathcal{H})$.*

Proof. Since Φ is positive elementary operator of length 2, by a result in [2], Φ is completely positive and there exist operators A_1 and $A_2 \in \mathcal{B}(\mathcal{H})$ such that

$$\Phi(T) = A_1TA_1^* + A_2TA_2^* \quad \text{for all} \quad T \in \mathcal{B}(\mathcal{X}).$$

Let $\mathbf{U} = (\,A_1 \quad A_2\,)$. If Φ is spectrum-compressing, then $\Phi(I) = I$ since $\Phi(I)$ is positive and $\sigma(\Phi(I)) \subseteq \{1\}$. Now it is clear from Theorem 2.1 that $\mathbf{U} \in \mathcal{B}(\mathcal{H}^{(2)}, \mathcal{H})$ is invertible and $\mathbf{U}^{-1} = \mathbf{U}^*$, i.e., $\mathbf{U}$ is unitary.

Remark 2.2. We prove in [7] that Corollary 2.2 is true for elementary operator of any length which answers affirmatively a problem in [1]. So a positive elementary operator which compresses spectrum (preserves spectrum) on $\mathcal{B}(\mathcal{H})$ must be an injective $*$-endomorphism of $\mathcal{B}(\mathcal{H})$. Similar questions for point spectrum preserving elementary operator is considered in [8].

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