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# FUZZY PRETOPOLOGICAL SPACES, AN EXTENSIONAL TOPOLOGICAL EXTENSION OF FTS**

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**Abstract**

The category of fuzzy pretopological spaces is introduced, and it is proved that this category is a well-fibred extensional topological construct, and it is a finally dense extension of the category of fuzzy topological spaces. Moreover this category contains both the category of pretopological spaces and the category of probabilistic neighbourhood spaces as simultaneously bireflective and bicoreflective full subcategories.

**Keywords** Fuzzy topological spaces, Fuzzy pretopological spaces, Extensional topological category

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## §0. Introduction

By a fuzzy topology on a set $X$ we mean a subset of $I^X$ which is closed under finite intersections, arbitrary unions and contains all the constant fuzzy sets. It is well known that the category **FTS** of fuzzy topological spaces is a well-fibred topological construct, and since it contains the category **Top** as a both reflective and coreflective subcategory, like **Top**, this category lacks many convenient properties such as extensionality (for definition see [6] or 4.1 in this paper), cartesian closedness and being a topological universe (see [1,6] for definitons). So Herrlich[9] raised the natural question: Determine the extensional topological hull, the cartesian closed topological hull and the topological universe hull of **FTS**.

Recall that the category **PrTop** of pretopological spaces is the extensional topological hull of **Top**[7]; the category **PsTop** of pseudotopologcal spaces is the topological universe hull of **Top**[10] and the cartesian closed topological hull of **Top** is given by the Antoine spaces[2,3].

In [14, 15] the authors introduced the concept of fuzzy convergence spaces by weakening the axioms of the fuzzy topological convergence (in the spirit of defining pseudotopological spaces), they proved that the well-fibred topological construct **FCS** is a topological universe extension of **FTS**.

In this paper dropping the axiom of idempotency in the axioms of fuzzy topological interior operators we introduce the concept of fuzzy pretopological spaces. It is proved that

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the category **FPrTop** of fuzzy pretopological spaces is a well-fibred extensional topological construct and it is a finally dense extension of **FTS**, hence it is a likely candidate for the extensional topological hull of **FTS**. Moreover this category contains both **Top** and the category **P-Neigh** of probabilistic neighbourhood spaces[12] as simultaneously bireflective and bicoreflective subcategories.

The relationship between fuzzy pretopological spaces and the fuzzy convergence spaces in [14,15] will be discussed in future work.

In this paper we adhere to [1] for categorical terminologies, and $I$ denotes the unit interval [0, 1], $I_0 = I \backslash \{0\}$. We also use $a$ to denote the constant fuzzy set with value $a$ for all $a \in I$. For all $a \in I, U \in I^X$, the $a$-level set of $U$ is defined by $U_a = \{x \in X | U(x) \geq a\}$, and for each $x \in X, \dot{x}$ denotes the filter generated by $x$.

## §1. Fuzzy Pretopological Spaces

**Definition 1.1.** *A fuzzy pretopological structure on a set $X$ is a family of functions $P = \{p_x : I^X \longrightarrow I | x \in X\}$ with the following properties: for each $x \in X, U, V \in I^X$,*

(FP1) $p_x(a) = a$ *for all* $a \in I$*;*

(FP2) $p_x(U) \leq U(x)$*;*

(FP3) $p_x(U \wedge V) = p_x(U) \wedge p_x(V)$*.*

*The pair $(X, P)$ is called a fuzzy pretopological space, and a fuzzy pretopological structure $P$ will be called topological if it satisfies moreover*

(FP4) $p_x(y \mapsto p_y(U)) = p_x(U)$*, this means $P$ is idempotent.*

*A function $f : (X, P) \longrightarrow (Y, Q)$ between fuzzy pretopological spaces is called continuous if for each $x \in X, U \in I^Y, p_x(f^{-1}(U)) \geq q_{f(x)}(U)$.*

**Note.** A function $p_x : I^X \longrightarrow I$ satisfying (FP1)–(FP3) is a special case of a fuzzy filter on $X$ defined in [4,13]. The interested reader is also refered to [18] for the definition of a fuzzy ideal on a distributive lattice.

**Proposition 1.1.** *The category* **FPrTop** *of fuzzy pretopological spaces is a well-fibred topological construct, this is to say, a concrete category over* **Set** *with the following properties*:

(1) (*Existence of initial structures*)*: Given any family $\{(X_t, P_t)\}_{t \in T}$ of fuzzy pretopological spaces indexed by a class $T$ and a family of functions $f_t : X \longrightarrow X_t, t \in T$, there is a unique fuzzy pretopological structure $P$ on $X$ such that the source $\{(X, P) \xrightarrow{f_t} (X_t, P_t)\}_{t \in T}$ is initial.*

(2) (*Fiber-smallness*)*: For each set $X$, the collection of the fuzzy pretopological structures on $X$ is a set.*

(3) (*Terminal separator property*)*: On any singelton set $X$ there is exactly one fuzzy pretopological structure on it.*

**Proof.** (2), (3) are trivial.

(1) For this it suffices to oberve that a source $\{(X, P) \xrightarrow{f_t} (X_t, P_t)\}_{t \in T}$ in **FPrTop** is initial if and only if for each $x \in X, U \in I^X$,

$$p_x(U) = \bigvee_{A \in T^{<\omega}} \Big\{ \bigwedge_{t \in A} p_{t, f_t(x)}(U_t) | U_t \in I^{X_t}, \bigwedge_{t \in A} f_t^{-1}(U_t) \leq U \Big\}.$$

Given a fuzzy topological space $X$, define a family of functions $P = \{p_x : I^X \longrightarrow I | x \in X\}$ as follows: for each $x \in X, U \in I^X, p_x(U) = U^\circ(x)$, where $^\circ$ denotes the interior operator corresponding to the fuzzy topology on $X$, then trivially $P$ satisfies (FP1)–(FP4). And easily a fuzzy pretopological structure is topological if and only if it is induced from a fuzzy topology.

Now it is easy to see the category **FTS** of fuzzy topological spaces is a full subcategory of **FPrTop**, moreover

**Proposition 1.2.** **FTS** *is initially closed in* **FPrTop**, *hence reflective in it.*

**Proof.** Suppose $\{(X,P) \xrightarrow{f_t} (X_t, P_t)\}_{t\in T}$ is an initial source in **FPrTop** with $(X_t, P_t)$ in **FTS** for each $t \in T$. We prove $(X,P)$ is in **FTS**, or equivalently $(X,P)$ satisfies (FP4).

Given $U \in I^X$, define $U^\circ \in I^X$ by $U^\circ(x) = p_x(U), x \in X$. It suffices to prove $p_x(U^\circ) = p_x(U)$.

Since the source is initial,

$$\begin{aligned} p_x(U^\circ) &= \bigvee_{A\in T^{<\omega}} \Big\{ \bigwedge_{t\in A} p_{t,f_t(x)}(U_t) | U_t \in I^{X_t}, \bigwedge_{t\in A} f_t^{-1}(U_t) \le U^\circ \Big\} \\ &= \bigvee_{A\in T^{<\omega}} \Big\{ \bigwedge_{t\in A} p_{t,f_t(x)}(U^\circ) | U_t \in I^{X_t}, \bigwedge_{t\in A} f_t^{-1}(U_t) \le U^\circ \Big\} \\ &= \bigvee_{A\in T^{<\omega}} \Big\{ \bigwedge_{t\in A} p_{t,f_t(x)}(U^\circ) | U_t \in I^{X_t}, \bigwedge_{t\in A} f_t^{-1}(U_t) \le U \Big\}, \end{aligned}$$

the last equality holds since if $\bigwedge\limits_{t\in A} f_t^{-1}(U_t) \le U$, we have

$$\bigwedge_{t\in A} f_t^{-1}(U_t^\circ) \le \bigwedge_{t\in A} (f_t^{-1}(U_t))^\circ = \Big( \bigwedge_{t\in A} f_t^{-1}(U_t) \Big)^\circ \le U^\circ$$

and

$$\bigwedge_{t\in A} p_{t,f_t(x)}(U_t^\circ) = \bigwedge_{t\in A} p_{t,f_t(x)}(U_t),$$

whence $p_x(U^\circ) = p_x(U)$.

## §2. Embedding PrTop in FPrTop

**PrTop** stands for the category of pretopological spaces. By a pretopological structure on a set $X$ we mean a collection of functions $P = \{p_x : 2^X \longrightarrow 2 | x \in X\}$ with the following properties:

(Pr1) $p_x(X) = 1$;

(Pr2) $p_x(U) \neq 0$ implies $x \in U$;

(Pr3) $p_x(U \wedge V) = p_x(U) \wedge p_x(V)$.

The pair $(X, P)$ is called a pretopological space, and continuous maps can be defined in an obvious way. It is easy to see pretopological spaces can also be equivalently (and traditionally as in the theory of convergence spaces) defined by assigning each point a filter which is the smallest among those converging to it[10,12].

In the following we will show that **PrTop** can be embedded into **FPrTop** in an extremly nice way, namely it is a simultaneously bireflective and bicoreflective subcategory of **FPrTop**.

**Proposition 2.1.** *Given a pretopological space* $(X,P)$, *define*

$$\omega(P) = \{p_x^\omega : I^X \longrightarrow I | x \in X\}$$

*as follows: for each $x \in X, U \in I^X$,*

$$p_x^\omega(U) = \bigvee_{a \in I} a \wedge p_x(U_a),$$

*where $U_a = \{x \in X | U(x) \geq a\}$. Then $(X, \omega(P))$ is a fuzzy pretopological space, and the correspondence $\omega : \mathbf{P}rTop \longrightarrow$ **FPrTop** is functorial and is an embedding.*

**Proof.** Straightforward verifications.

**Theorem 2.1.** **PrTop** *is both bireflective and bicoreflective in* **FPrTop**:

(1) *Given a fuzzy pretopological space $(X, P)$, its* **PrTop** *bireflection is given by*

$$(X, P) \xrightarrow{id_X} (X, \omega\gamma(P))$$

*where $\gamma(P) = \{p_x^\gamma : 2^X \longrightarrow 2 | x \in X\}$ and for each $x \in X, U \in 2^X$,*

$$p_x^\gamma(U) = \begin{cases} 1, & p_x(U) = 1, \\ 0, & p_x(U) \neq 1. \end{cases}$$

(2) *Given a fuzzy pretopological space $(X, P)$, its* **PrTop** *bicoreflection is given by*

$$(X, \omega\iota(P)) \xrightarrow{id_X} (X, P)$$

*where $\iota(P) = \{p_x^\iota : 2^X \longrightarrow 2 | x \in X\}$ and for each $x \in X, U \in 2^X$,*

$$p_x^\iota(U) = \begin{cases} 1, & p_x(U) \neq 0, \\ 0, & p_x(U) = 0. \end{cases}$$

**Proof.** Routine verifications, similar to the proof of Theorem 3.2, left to the reader.

**Corollary 2.1.** **FPrTop** *is not cartesian closed.*

**Proof.** If **FPrTop** is cartesian closed, **PrTop** would be cartesian closed since it is a coreflective subcategory of **FPrTop** and it is closed under finite products[16], but this is not the case[10].

## §3. Embedding P-Neigh in FPrTop

**Definition 3.1.**[12] *A probabilistic neighbourhood space is a pair $(X, P)$, where $P = \{p_x : 2^X \longrightarrow I | x \in X\}$ with the following properties:*

(PN1) $p_x(X) = 1$*;*

(PN2) $p_x(U) \neq 0$ *implies $x \in U$;*

(PN3) $p_x(U \cap V) = p_x(U) \wedge p_x(V)$ *for all $U, V \in 2^X$.*

*$(X, P)$ will be called a probabilistic topological space if it satisfies moreover*

(PN4) $p_x(U) = \bigvee_{V \in \dot{x}|U} \bigwedge_{y \in V} p_y(V)$, *where $\dot{x}|U = \{V \subseteq X | x \in V \subseteq U\}$.*

Continuous maps between probabilistic neighbourhood (topological) spaces are defined in an obvious manner.

**Theorem 3.1.**[12] (1) *Both the category* **P-Neigh** *of probabilistic neighbourhood spaces and the category* **P-Top** *of probabilistic topological spaces are well-fibred topological construct and* **P-Top** *is bireflective in* **P-Neigh**.

(2) **Top** *is both bireflective and bicoreflective in* **P-Top***;* **PrTop** *is both bireflective and bicoreflective in* **P-Neigh**.

(3) **P-Neigh** *is the extensional topological hull of* **P-Top**.

Now we will show that like **PrTop**, **P-Neigh** can be embedded into **FPrTop** as a simultaneously bireflective and bicoreflective subcategory of **FPrTop**.

Suppose $(X, P)$ is a probabilistic neighbourhood space. It is easy to verify the family of functions

$$i(P) = \{p_x^i : I^X \longrightarrow I | x \in X\}$$

is a fuzzy pretopological structure on $X$, where for each $x \in X, U \in I^X$,

$$p_x^i(U) = \bigvee_{a \in I} (a \wedge p_x(U_a)).$$

If $f : (X, P) \longrightarrow (Y, Q)$ is a continuous map between probabilistic neighbourhood spaces, then $f : (X, i(P)) \longrightarrow (Y, i(Q))$ is continuous, hence

**Proposition 3.1.** $i$: **P-Neigh** $\longrightarrow$ **FPrTop** *is functorial and moreover* $i$ *is an embedding.*

**Theorem 3.2. P-Neigh** *is bireflective in* **FPrTop**.

**Proof.** Given a fuzzy pretopological space $(X, P)$, define

$$\delta(P) = \{p_x^\delta : 2^X \longrightarrow I | x \in X\}$$

by $p_x^\delta(U) = p_x(U)$ for all $x \in X, U \in 2^X$. It is easy to see $\delta(P)$ is a probabilistic neighbourhood structure on $X$. We say $(X, \delta(P))$ is the **P-Neigh**-reflection of $(X, P)$, for this it suffices to verify the following claims:

(1) $id_X : (X, P) \longrightarrow (X, i\delta(P))$ is continuous.

(2) Given any probabilistic neighbourhood space $(Y, Q)$ and any function $f : X \longrightarrow Y$, the continuity of $f : (X, P) \longrightarrow (Y, i(Q))$ implies the continuity of $f : (X, \delta(P)) \longrightarrow (Y, Q)$.

**Proof of (1).** That is to say, for each $x \in X, U \in I^X$,

$$p_x(U) \geq p_x^{\delta i}(U).$$

Indeed,

$$p_x^{\delta i}(U) = \bigvee_{a \in I} (a \wedge p_x^\delta(U_a)) = \bigvee_{a \in I} (a \wedge p_x(U_a)) \leq p_x(U).$$

The last inequality holds since for each $a \in I, a \wedge U_a \leq U$, thus

$$a \wedge p_x(U_a) = p_x(a \wedge U_a) \leq p_x(U).$$

**Proof of (2).** If $f : (X, P) \longrightarrow (Y, i(Q))$ is contiunous, for all $x \in X, U \in 2^Y$, we have

$$p_x^\delta(f^{-1}(U)) = p_x(f^{-1}(U)) \geq q_{f(x)}(U).$$

Thus $f : (X, \delta(P)) \longrightarrow (Y, Q)$ is continuous.

**Lemma 3.1.** *A fuzzy pretopological structure $P$ on $X$ is induced by a probabilistic neighbourhood structure if and only if $(X, P)$ satisfies moreover*

(FN) $p_x(a \vee U) = a \vee p_x(U)$ *for all* $x \in X, a \in I, U \in I^X$.

**Proof.** Necessity. Suppose $(X, P) = (X, i(S))$ for some probabilistic neighbourhood structure $S$ on $X$. For all $x \in X, a \in I, U \in I^X$, we have

$$\begin{aligned} p_x(a \vee U) &= \bigvee_{b \in I} (b \wedge s_x((a \vee U)_b)) \\ &= a \vee \bigvee_{b > a} (b \wedge s_x(U_b)) = a \vee p_x(U). \end{aligned}$$

Sufficiency. We prove in this case $(X,P)=(X,i\delta(P))$. This means for all $x\in X, U\in I^X$,

$$p_x(U)=\bigvee_{a\in I}(a\wedge p^\delta_x(U_a))=\bigvee_{a\in I}(a\wedge p_x(U_a)).$$

At first $p_x(U)\geq \bigvee\limits_{a\in I}(a\wedge p_x(U_a))$ is trivial; conversely suppose

$$p_x(U)>\bigvee_{a\in I}(a\wedge p_x(U_a)),$$

there is some $\epsilon>0$,

$$p_x(U)-\epsilon \ > a\wedge p_x(U_a) \quad \text{for all} \quad a\in I.$$

Let $a=p_x(U)-\frac{\epsilon}{2}$. Then

$$p_x(U_a)<p_x(U)-\epsilon.$$

Since $a\vee U_a\geq U$, we get

$$\begin{aligned} p_x(U)&\leq p_x(a\vee U_a)=a\vee p_x(U_a)\\ &=p_x(U)-\frac{\epsilon}{2}, \end{aligned}$$

a contradiction.

**Theorem 3.3. P-Neigh** *is bicoreflective in* **FPrTop**.

**Proof.** It suffices to prove that **P-Neigh** is finally closed in **FPrTop**, that is to say, if $\{(X_t,P_t)\xrightarrow{f_t}(X,P)\}_{t\in T}$ is a final sink in **FPrTop** with $(X_t,P_t)$ in **P-Neigh** for all $t\in T$, then $(X,P)$ is an object in **P-Neigh**, or equivalently $(X,P)$ satisfies (FN) by the above lemma.

Since the sink is final, for all $x\in X, U\in I^X$, we have

$$p_x(U)=\bigwedge_{t\in T}\bigwedge_{f_t(x_t)=x}p_{t,x_t}(f_t^{-1}(U)).$$

Therefore for all $x\in X, a\in I, U\in I^X$, we have

$$\begin{aligned} p_x(a\vee U)&=\bigwedge_{t\in T}\bigwedge_{f_t(x_t)=x}p_{t,x_t}(f_t^{-1}(a\vee U))\\ &=\bigwedge_{t\in T}\bigwedge_{f_t(x_t)=x}p_{t,x_t}(a\vee f_t^{-1}(U))\\ &=a\vee\bigwedge_{t\in T}\bigwedge_{f_t(x_t)=x}p_{t,x_t}(f_t^{-1}(U))\\ &=a\vee p_x(U). \end{aligned}$$

## §4. Extensionality of FPrTop

**Definition 4.1.**[7,8] *Let* **A** *be a well-fibred topological construct,*

(1) *A partial morphism from $X$ to $Y$ is a morphism $f:Z\longrightarrow Y$ whose domain $Z$ is a subspace of $X$.*

(2) *Partial morphisms into $Y$ are representable provided $Y$ can be embedded via the addition of a single point $\infty$ into an object $Y^\#$ with the property that for every partial morphism $f:Z\longrightarrow Y$, the map $f^X:X\longrightarrow Y^\#$ defined by $f^X(x)=f(x)$ if $x\in Z$; $f^X(x)=\infty$ if $x\notin Z$, is a morphism. The object $Y^\#$ is called the one point extension of $Y$.*

(3) **A** *is called extensional if all partial morphisms into all* **A***-objects are representable.*

**Example.** Both **PrTop** and **P-Neigh** are extensional[10,12], while neither **Top** nor **P-Top** is extensional.

**Proposition 4.1. FPrTop** *is extensional.*

**Proof.** Given a fuzzy pretopological space $(X,P)$, let $X^{\#}=X\cup\{\infty\}$, for each $x\in X^{\#}, U:X^{\#}\longrightarrow I$, define

$$p_x^{\#}(U)=\begin{cases}\bigwedge\limits_{y\in X^{\#}} U(y), & x=\infty;\\ p_x(U|X)\wedge U(\infty), & x\in X.\end{cases}$$

Then it is routine to verify that (1) $(X^{\#},P^{\#})$ is a fuzzy pretopological space; (2) The inclusion $(X,P)\longrightarrow(X^{\#},P^{\#})$ is initial; (3) For each partial morphism $X\longleftarrow A\hookrightarrow Y$, the function $f^Y:Y\longrightarrow X^{\#}$ defined by $f^Y(y)=f(y)$ if $y\in A, f^Y(y)=\infty$ if $y\notin A$, is continuous. Therefore **FPrTop** is extensional.

**Proposition 4.2. FTS** *is finally dense in* **FPrTop***, this means for every fuzzy pretopological space $(X,P)$ there is a final sink $\{(X_t,P_t)\xrightarrow{f_t}(X,P)\}_{t\in T}$ with $(X_t,P_t)$ in* **FTS**.

**Proof.** Given a fuzzy pretopological space $(X,P)$, for each $x\in X$, we define a family of functions $P^x=\{p_y^x:I^X\longrightarrow I|y\in X\}$ as follows: for all $y\in X, U\in I^X$,

$$p_y^x(U)=\begin{cases}p_x(U), & y=x,\\ U(y), & y\neq x.\end{cases}$$

Now we have

(1) For each $x\in X,(X,P^x)$ is a fuzzy pretopological space satisfying (FP4), hence an object in **FTS**.

At first the fact that $(X,P^x)$ is a fuzzy pretopological space is trival, what remains is to prove that $(X,P^x)$ satisfies (FP4). Indeed, for each $U\in I^X, z\in X$, let

$$U^\circ(z)=p_z^x(U)=\begin{cases}p_x(U), & z=x,\\ U(z), & z\neq x.\end{cases}$$

We need only prove $p_y^x(U)=p_y^x(U^\circ)$ for all $y\in Y$.

**Case 1.** $y=x$. Since $U^\circ\geq U\wedge p_x(U)$,

$$\begin{aligned}p_y^x(U^\circ)&=p_x^x(U^\circ)=p_x(U^\circ)\\ &\geq p_x(U\wedge p_x(U))=p_x(U).\end{aligned}$$

**Case 2.** $y\neq x$. In this case

$$p_y^x(U^\circ)=U^\circ(y)=U(y)=p_y^x(U).$$

(2) $id_X:(X,P^x)\longrightarrow(X,P)$ is continuous, this is trivial.

(3) The sink $\{(X,P^x)\xrightarrow{id_X}(X,P)\}_{x\in X}$ is final. Indeed, for each $y\in X, U\in I^X$, since $p_y^y(U)=p_y(U)$ by definition, we get

$$p_y(U)=\bigwedge_{x\in X}p_y^x(U),$$

hence the sink is final.

By Propositions 4.1, 4.2 and the characterization theorem of extensional topological hull in [10] whether **FPrTop** is the extensional topological hull of **FTS** depends on the solution of the following:

**Question**: Is the class $\{X^{\#}|X\in\textbf{FTS}\}$ initially dense in **FPrTop**?

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