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ASYMPTOTIC LIMITS OF ONE-DIMENSIONAL HYDRODYNAMIC MODELS FOR PLASMAS AND SEMICONDUCTORS

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Abstract

This paper studies the zero-electron-mass limit, the quasi-neutral limit and the zero-relaxation-time limit in one-dimensional hydrodynamic models of Euler-Poisson system for plasmas and semiconductors. For each limit in the steady-state models, the author proves the strong convergence of the sequence of solutions and gives the corresponding convergence rate. In the time-dependent models, the author shows some useful estimates for the quasi-neutral limit and the zero-electron-mass limit. This study completes the analysis made in [11,12,13,14,19].

Keywords Zero-electron-mass limit, Quasi-neutral limit, Zero-relaxation-time limit, Hydrodynamic models, Plasmas, Semiconductors
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§1. Introduction

In mathematical modeling and numerical simulation for plasmas and semiconductors devices, the hydrodynamic model like the Euler-Poisson system is wildly used. Due to the hyperbolic feature of the Euler equations, the study of weak solutions to the Euler-Poisson system is limited in one space dimension. In such situation, the existence of global weak solutions can be proved under natural assumptions (see [22, 20, 17, 5, 18]). In a series of papers[11,12,13,14], we are interested in some asymptotic limits in the hydrodynamic models for plasmas. In particular, the zero-relaxation-time limit, the zero-electron-mass limit and the zero-Debye-length (or quasi-neutral) limit have been studied. In one-dimensional transient Euler-Poisson system, the first limit has been rigorously justified[11,12], whereas the second and the third limits have not been proved yet, although the analysis has been made for the Drift-Diffusion equations[13,14].

From physical point of view for plasmas, the zero-Debye-length limit implies the quasi-neutrality of the plasmas, and the zero-electron-mass limit leads to the well-known Boltzmann-Maxwell relation. The zero-relaxation-time limit for the Euler-Poisson system gives the Drift-Diffusion equations, in which the zero-electron-mass limit and the zero-Debye-length limit were performed[13,14]. Since the results on these limits are wildly used in practice (see for instance [2, 21]), it is important to give their mathematical justifications. In this paper, we first show that these three limits can be rigorously made for one-dimensional steady-state Euler-Poisson system. For the one-dimensional transient Euler-Poisson system

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we present some energy type estimates which provide useful information to these limits. This study completes the analysis made in [11, 12, 13, 14, 19].

Let n, j and ϕ be the electron density, the current density and the electrostatistic potential respectively. The one-dimensional transient Euler-Poisson system in the case of unipolar model is written as

$$\partial_t n + \partial_x j = 0, \tag{1.1}$$

$$\varepsilon \partial_t j + \partial_x \Big(\frac{\varepsilon j^2}{n} + p(n) \Big) = n \partial_x \phi - \frac{\varepsilon j}{\tau(n,j)}, \tag{1.2}$$

$$-\lambda^2 \partial_{xx} \phi = b(x) - n \tag{1.3}$$

for $x \in (0,1)$. Here $b = b(x)$ is the doping profile for semiconductors or the ions density for plasmas, $p = p(n)$ is the pressure function, $\tau = \tau(n,j)$ is the momentum relaxation-time. The small physical parameters are the scaled electron mass $\varepsilon > 0$, the Debye-length $\lambda > 0$ and the relaxation time $\tau = \tau(n,j)$. They are independent of each other. As in [6] we suppose that

(H1) $b \in L^\infty(0,1)$ and $b(x) \geq b_0 > 0$, $\forall x \in (0,1)$;

(H2) $n \to n^2 p'(n)$ is strictly increasing from $[0,+\infty)$ to $[0,+\infty)$;

(H3) $\tau \in C^1([0,+\infty) \times I\!R)$ and $\tau(n,j) \geq \tau_0 > 0$, $\forall (n,j) \in [0,+\infty) \times I\!R$

for some constants b_0 and τ_0. Assumption (H2) includes the usual γ-law $p(n) = an^\gamma$, where $a > 0$ and $\gamma \geq 1$ are constants. In the zero-relaxation-time limit, τ is also assumed to be a constant.

In the steady-state case $n = n(x)$, $j = j(x)$ and $\phi = \phi(x)$, system (1.1)–(1.3) reduces to

$$j(x) = j = \text{constant}, \tag{1.4}$$

$$\frac{d}{dx}\Big(\frac{\varepsilon j^2}{n} + p(n) \Big) = n \frac{d\phi}{dx} - \frac{\varepsilon j}{\tau(n,j)}, \tag{1.5}$$

$$-\lambda^2 \frac{d^2 \phi}{dx^2} = b(x) - n, \quad x \in (0,1). \tag{1.6}$$

According to [6,8], system (1.4)–(1.6) is supplemented by the following boundary conditions:

$$n(0) = n_0, \quad n(1) = n_1, \tag{1.7}$$

$$\phi(0) = 0, \tag{1.8}$$

where $n_0 > 0$ and $n_1 > 0$. In the study of the quasi-neutral limit $\lambda \to 0$ for (1.4)–(1.6) when $\tau = +\infty$, another boundary condition can also be used which leads to quite different situation. We refer to [1] on this subject. For smooth solutions, after eliminating ϕ in (1.5)–(1.6), the density satisfies the following equation, parameterized by $j \in I\!R$:

$$\frac{d^2 F(n)}{dx^2} + \frac{d}{dx}\Big(\frac{\varepsilon j}{\tau n} \Big) - \frac{n - b(x)}{\lambda^2} = 0, \tag{1.9}$$

where F is defined by

$$F(n) = \frac{\varepsilon j^2}{2n^2} + h(n) \text{ with } h'(n) = \frac{1}{n} p'(n), \quad h(1) = 0. \tag{1.10}$$

Equation (1.9) is uniformly elliptic if $F'(n) > 0$, i.e., $n^2 p'(n) > \varepsilon j^2$. It is equivalent to the subsonic condition (see [6]). In this situation, we may first solve problem (1.7) and (1.9), then determine the electrostatistic potential by (1.6) and (1.8) together with the boundary condidition of ϕ at $x = 1$ given by

$$\phi(1) = F(n_1) - F(n_0) + \int_0^1 \frac{\varepsilon j}{\tau(n(x), j) n(x)} dx,$$

which is obtained by integrating equation (1.5) over interval $(0,1)$.

The existence of smooth solution in $H^1(0,1)$ has been proved in [6] under assumptions (H1)–(H3) by Schauder's fixed point theorem. The uniqueness of solution is also obtained under a supplementary condition. See also [8] for the weak solutions constructed by the artificial viscosity method. Let

$$\underline{n} = \min\Big(n_0, n_1, \inf_{x\in(0,1)} b(x)\Big), \quad \overline{n} = \max\Big(n_0, n_1, \sup_{x\in(0,1)} b(x)\Big).$$

The results on the existence and uniqueness of solution can be stated as follows.

Theorem 1.1 *Under assumptions* (H1)–(H3)*, suppose that* $\varepsilon, j \in \mathbb{R}$ *such that* $\underline{n} > \rho_\varepsilon$*, where* $\rho_\varepsilon > 0$ *is determined by equation* $\rho_\varepsilon^2 p'(\rho_\varepsilon) = \varepsilon j^2$*. Then problem* (1.5)–(1.8) *admits a solution* (n,ϕ) *satisfying* $n \in H^2(0,1)$*,* $\phi \in W^{2,\infty}(0,1)$ *and* $\underline{n} \le n(x) \le \overline{n}$ *for all* $x \in [0,1]$*. If in addition,* $|\frac{\varepsilon j}{\tau}|$ *is sufficiently small, then the solution of* (1.5)–(1.8) *is unique.*

The regularity of solution $n \in H^2(0,1)$ and $\phi \in W^{2,\infty}(0,1)$ is a direct consequence of equations (1.5)–(1.6). Noting that $\underline{n}$ and $\overline{n}$ are independent of ε, λ and τ, the inequality $\underline{n} \le n(x) \le \overline{n}$ obtained by the maximum principle provides an a priori L^∞ estimate for the sequence of solutions in the asymptotic analysis.

The zero-electron-mass limit $\varepsilon \to 0$ has been investigated in [13] for the Drift-Diffusion equations. The limit is proved if the Dirichlet boundary data are in equilibrium. The zero-Debye-length limit $\lambda \to 0$ was first considered in [2] for the nonlinear Poisson equation and in [4] for locally smooth solutions to the time-dependent Euler-Poisson system (1.1)–(1.3). This limit has been rigorously justified in [14] for the Drift-Diffusion equations under the initial quasi-neutrality assumption. A boundary layer analysis in this limit is given in [19]. The zero-relaxation-time limit $\tau \to 0$ has been performed for (1.1)–(1.3) with γ-law in [11, 12] for $\gamma > 1$ and in [10] for $\gamma = 1$. Both proofs are based on the entropy inequality of weak solutions. The zero-electron-mass limit is an open problem for the transient Euler-Poisson system. We refer to [9] for this limit.

In this paper we will give the justification of these three limits for the steady-state Euler-Poisson system (1.5)–(1.8) and the energy estimates for the transient Euler-Poisson system (1.1)–(1.3) with γ-law. It is organized as follows. In the next section, we prove the zero-electron-mass limit by establishing some uniform estimates for the sequence of solutions. The uniform convergence with rate $O(\varepsilon)$ is given. Section 3 is devoted to the zero-Debye-length limit together with a boundary layer analysis. We first show the existence and uniqueness of such boundary layer with exponential decay which allows to obtain the convergence results with rate $O(\lambda^{\frac{1}{2}})$. In a particular case which excludes the boundary layer, we obtain a better convergence rate $O(\lambda)$. In Section 4, we give a simple description of convergence results for the zero-relaxation-time limit by a method similar to that used in Section 2. The convergence rate $O(\tau^2)$ is obtained if $|\varepsilon j|$ is small enough. Finally, we prove in the last section some useful estimates for the quasi-neutral limit and the zero-electron-mass limit. In particular, an a priori estimate for weak quasi-neutrality in H^{-1} and a necessary condition on the given data for the zero-electron-mass limit are given.

Remark that the results obtained for the unipolar model in this paper can be easily extended to the bipolar models in a similar way. We refer to [3, 15, 16] for the models.

§2. Zero-Electron-Mass Limit

In the study of the zero-electron-mass limit $\varepsilon \to 0$ in (1.5)–(1.8), the Debye-length λ and the relaxation-time τ are supposed to be independent of ε. Since $\varepsilon \to 0$, the condition $\underline{n} > \rho_\varepsilon$ is satisfied for all $j \in \mathbb{R}$. Therefore, the existence and uniqueness of solution given by Theorem 1.1 is valid for all $j \in \mathbb{R}$.

Now we rewrite problem (1.5)–(1.8) under the form

$$\frac{d}{dx}\Big(\frac{\varepsilon j^2}{n_\varepsilon}+p(n_\varepsilon)\Big)=n_\varepsilon\frac{d\phi_\varepsilon}{dx}-\frac{\varepsilon j}{\tau(n_\varepsilon,j)}, \tag{2.1}$$

$$-\lambda^2\frac{d^2\phi_\varepsilon}{dx^2}=b(x)-n_\varepsilon \tag{2.2}$$

for $x\in(0,1)$ and

$$n_\varepsilon(0)=n_0,\quad n_\varepsilon(1)=n_1, \tag{2.3}$$

$$\phi_\varepsilon(0)=0. \tag{2.4}$$

Setting $\varepsilon=0$, we obtain the formal zero-electron-mass limit (n,ϕ) satisfying

$$-\lambda^2\frac{d^2\phi}{dx^2}=b(x)-n,\quad h(n)=h(n_0)+\phi,\quad x\in(0,1), \tag{2.5}$$

with boundary conditions

$$n(1)=n_1,\quad \phi(0)=0. \tag{2.6}$$

In particular, if $h(n)=\log n$, i.e., $p(n)=n$, we obtain the Boltzmann-Maxwell relation

$$n=n_0\exp(\phi).$$

Now we prove rigorously this limit and give its convergence rate. The strong convergence of the sequence $(n_\varepsilon,\phi_\varepsilon)_{\varepsilon>0}$ in $H^1(0,1)$ allows to pass to the limit in system (2.1)–(2.4) to obtain (2.5)–(2.6).

Theorem 2.1. *Under assumptions* (H1)–(H3)*, let* $(n_\varepsilon,\phi_\varepsilon)_{\varepsilon>0}$ *be the sequence of solutions to* (2.1)–(2.4) *and* (n,ϕ) *be the unique solution to problem* (2.5)–(2.6)*. Then as* $\varepsilon\to 0$*, we have*

$$\|n_\varepsilon-n\|_{H^1(0,1)}\le A_1\varepsilon,\quad \|\phi_\varepsilon-\phi\|_{H^1(0,1)}\le A_1\varepsilon, \tag{2.7}$$

where $A_1>0$ *is a constant independent of* ε*.*

Proof. From the $L^\infty(0,1)$ boundedness of $(n_\varepsilon)_{\varepsilon>0}$ given in Theorem 1.1 and noting (H1), we get that the sequence $(\phi_\varepsilon)_{\varepsilon>0}$ is bounded in $W^{2,\infty}(0,1)$. It follows from (H2)–(H3), $\underline{n}>\rho_\varepsilon$ and equation

$$\Big(p'(n_\varepsilon)-\frac{\varepsilon j^2}{n_\varepsilon^2}\Big)\frac{dn_\varepsilon}{dx}=n_\varepsilon\frac{d\phi_\varepsilon}{dx}-\frac{\varepsilon j}{\tau(n_\varepsilon,j)}$$

that the sequence $(n_\varepsilon)_{\varepsilon>0}$ is bounded in $W^{1,\infty}(0,1)$. By compact imbedding $W^{1,\infty}(0,1)\hookrightarrow C^0([0,1])$, $W^{2,\infty}([0,1])\hookrightarrow C^1([0,1])$ and Ascoli's theorem, we obtain the uniform convergence of a subsequence of $(n_\varepsilon,\phi_\varepsilon)_{\varepsilon>0}$, which allows to pass to the limit in the system and the boundary conditions (2.1)–(2.4) to obtain (2.5)–(2.6). The uniqueness of solution to the limiting problem implies the convergence of the whole sequence.

To prove the convergence rates (2.7), eliminating ϕ_ε in (2.1)–(2.2) and ϕ in (2.5) yields

$$\frac{d^2}{dx^2}\Big(\frac{\varepsilon j^2}{2n_\varepsilon^2}+h(n_\varepsilon)-h(n)\Big)-\frac{n_\varepsilon-n}{\lambda^2}+\frac{d}{dx}\Big(\frac{\varepsilon j}{\tau(n_\varepsilon,j)n_\varepsilon}\Big)=0.$$

Multiplying this equation by $h(n_\varepsilon)-h(n)$ and then integrating it over $(0,1)$, by means of $n_\varepsilon(0)=n(0)=n_0$ and $n_\varepsilon(1)=n(1)=n_1$, we obtain

$$\begin{aligned}&\int_0^1\Big[\Big(\frac{d}{dx}(h(n_\varepsilon)-h(n))\Big)^2+\frac{(n_\varepsilon-n)(h(n_\varepsilon)-h(n))}{\lambda^2}\Big]dx\\ =&\int_0^1\Big[(h(n_\varepsilon)-h(n))\frac{d}{dx}\Big(\frac{\varepsilon j}{\tau(n_\varepsilon,j)n_\varepsilon}\Big)-\frac{d}{dx}\Big(\frac{\varepsilon j^2}{2n_\varepsilon^2}\Big)\frac{d}{dx}(h(n_\varepsilon)-h(n))\Big]dx.\end{aligned}$$

From the $W^{1,\infty}(0,1)$ boundedness of $(n_\varepsilon)_{\varepsilon>0}$, (H3) and Poincaré's inequality, there is a constant $A_2 > 0$ independent of ε, such that

$$\begin{aligned}&\Big\|\frac{d}{dx}(h(n_\varepsilon)-h(n))\Big\|^2_{L^2(0,1)}+\frac{1}{\lambda^2}\int_0^1(n_\varepsilon-n)(h(n_\varepsilon)-h(n))dx\\ &\le A_2\varepsilon\Big\|\frac{d}{dx}(h(n_\varepsilon)-h(n))\Big\|_{L^2(0,1)},\end{aligned}$$

then

$$\Big\|\frac{d}{dx}(h(n_\varepsilon)-h(n))\Big\|_{L^2(0,1)}\le A_2\varepsilon,$$

where we have used the fact that h is smooth and strictly increasing. Let g be defined by $g = h^{-1}$. From Poincaré's inequality and the relation

$$\frac{d(n_\varepsilon-n)}{dx}=[g'(h(n_\varepsilon))-g'(h(n))]\frac{h(n_\varepsilon)}{dx}+g'(h(n))\frac{d(h(n_\varepsilon)-h(n))}{dx}, \tag{2.8}$$

we obtain the first estimate in (2.7). The estimate for $\phi_\varepsilon - \phi$ is derived directly from

$$\lambda^2\frac{d^2(\phi_\varepsilon-\phi)}{dx^2}=n_\varepsilon-n,\quad (\phi_\varepsilon-\phi)(0)=0,\quad (\phi_\varepsilon-\phi)(1)=\alpha_\varepsilon$$

with

$$\alpha_\varepsilon=\frac{\varepsilon j^2}{2n_1^2}-\frac{\varepsilon j^2}{2n_0^2}+\int_0^1\frac{\varepsilon j}{\tau(n_\varepsilon(x),j)n_\varepsilon(x)}dx.$$

This ends the proof.

§3. Quasi-Neutral Limit

In the study of the quasi-neutral limit $\lambda \to 0$ in (1.5)–(1.8), the electron mass ε and the relaxation-time τ are supposed to be independent of λ. The uniformly elliptic condition $\underline{n} > \rho_\varepsilon$ is needed in this case not only for the existence of solution to (1.5)–(1.8) but also for establishing some uniform estimates of solutions. We assume furthermore that

(H4) $b \in H^1(0,1)$.

This makes the sense of $b(0)$ and $b(1)$ because of Sobolev's imbedding $H^1(0,1) \hookrightarrow C^0([0,1])$.

For the quasi-neutral limit we rewrite the problem (1.5)–(1.8) under the form

$$\frac{d}{dx}\Big(\frac{\varepsilon j^2}{2n_\lambda^2}+h(n_\lambda)-\phi_\lambda\Big)=-\frac{\varepsilon j}{\tau(n_\lambda,j)n_\lambda}, \tag{3.1}$$

$$-\lambda^2\frac{d^2\phi_\lambda}{dx^2}=b(x)-n_\lambda \tag{3.2}$$

for $x \in (0,1)$ and

$$n_\lambda(0)=n_0,\quad n_\lambda(1)=n_1, \tag{3.3}$$

$$\phi_\lambda(0)=0. \tag{3.4}$$

Let $(n_\lambda, \phi_\lambda)_{\lambda>0}$ be a sequence of solutions to (3.1)–(3.4) and (n, ϕ) be its limit. It is clear that (n, ϕ) formally satisfies

$$\frac{d}{dx}\Big(\frac{\varepsilon j^2}{2n^2}+h(n)-\phi\Big)=-\frac{\varepsilon j}{\tau(n,j)n},\quad n=b(x). \tag{3.5}$$

Obviously, boundary layers will occur near $x = 0$ if $n_0 \ne b(0)$ or near $x = 1$ if $n_1 \ne b(1)$. For simplicity, in the sequel we assume that

(H5) $n_1 = b(1)$.

Therefore, only boundary layer near $x = 0$ should be considered.

We first give an a priori estimate for defining boundary conditions for the limit (n, ϕ). To this end, let

$$G_\lambda = \frac{\varepsilon j^2}{2n_\lambda^2} + h(n_\lambda) - \phi_\lambda, \quad G = \frac{\varepsilon j^2}{2n^2} + h(n) - \phi. \tag{3.6}$$

From (3.1), we have

$$\frac{dG_\lambda}{dx} = -\frac{\varepsilon j}{\tau(n_\lambda, j)n_\lambda}. \tag{3.7}$$

Lemma 3.1. *Under assumptions* (H1)–(H3)*, the sequence* $(G_\lambda)_{\lambda>0}$ *is bounded in* $H^1(0,1)$ *and the sequence* $(\phi_\lambda)_{\lambda>0}$ *is bounded in* $L^\infty(0,1)$.

Proof. Since $n_\lambda \geq \underline{n}$ and $\tau(n_\lambda, j) \geq \tau_0$, it follows from (3.7) and Poincaré's inequality that the sequence $(G_\lambda)_{\lambda>0}$ is bounded in $H^1(0,1)$. Therefore, $(G_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,1)$ by the imbedding $H^1(0,1) \hookrightarrow L^\infty(0,1)$. From (3.6) and the $L^\infty(0,1)$ boundedness of $(n_\lambda)_{\lambda>0}$, this implies that $(\phi_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,1)$.

Since $n = b \in H^1(0,1)$ and by Lemma 3.1, $G \in H^1(0,1)$, we have $\phi \in H^1(0,1)$ although the sequences $(n_\lambda)_{\lambda>0}$ and $(\phi_\lambda)_{\lambda>0}$ may not be bounded in $H^1(0,1)$ because of the boundary layer phenomenon. By means of the compact imbedding $H^1(0,1) \hookrightarrow C^0([0,1])$ and Ascoli's theorem, this lemma implies the uniform convergence of the sequence $(G_\lambda)_{\lambda>0}$ which allows to define the boundary condition $\phi(0)$ by the relation

$$\phi(0) = \frac{\varepsilon j^2}{2n^2(0)} + h(n(0)) - \frac{\varepsilon j^2}{2n_0^2} - h(n_0), \quad n(0) = b(0). \tag{3.8}$$

It is clear that equation (3.5) with boundary condition (3.8) admits a unique solution (n, ϕ).

Now we consider the boundary layer near $x = 0$, which will give useful information to prove the convergence of the sequence of solutions. In a neighborhood of $x = 0$, the solution $(n_\lambda, \phi_\lambda)_{\lambda>0}$ of (3.1)–(3.4) may be approximated by $(n(0)+u(y), \phi(0)+\psi(y))$, where $y = x/\lambda$ is the fast variable. We expect that $(u(y), \psi(y))$ describes the boundary layer near $x = 0$. Putting the approximate solution $(n(0) + u(y), \phi(0) + \psi(y))$ into (3.1)–(3.2) and neglecting the error term $O(\lambda)$, we obtain

$$\frac{d}{dy}\Big(\frac{\varepsilon j^2}{v(y)} + p(v(y))\Big) = v(y)\frac{d\psi}{dy}, \quad y \in (0, +\infty), \tag{3.9}$$

$$\frac{d^2\psi}{dy^2} = v(y) - n(0), \quad y \in (0, +\infty), \tag{3.10}$$

where $v(y) = n(0) + u(y)$. The boundary conditions for (v, ψ) are given by

$$v(0) = n_0, \quad \psi(0) = -\phi(0), \quad v(\infty) = n(0), \quad \psi(\infty) = 0. \tag{3.11}$$

It follows from (3.9) and (3.11) that $F(v) = \psi + F(n(0))$, where F is defined by (1.10). Under the uniformly elliptic condition $\underline{n} > \rho_\varepsilon$, function F is strictly increasing. Therefore, there is a smooth and strictly increasing function f, given by $f = F^{-1}$. Hence $v(y) = f(\psi + F(n(0)))$, and then

$$\frac{d^2\psi}{dy^2} = f(\psi + F(n(0))) - n(0). \tag{3.12}$$

The function $H(\psi) = f(\psi + F(n(0))) - n(0)$ is also smooth, strictly increasing and $H(0) = 0$. Thus we may apply the results in [7, 19] to equation (3.12) and obtain the following result.

Lemma 3.2. *Under assumptions* (H1)–(H2) *and* (H4)*, the boundary layer equations* (3.9)–(3.11) *have a unique smooth solution* (u, ψ) *satisfying*

$$|u(y)|, |\psi(y)|, |u'(y)|, |\psi'(y)| \leq B_1|\phi(0)|\exp(-B_2 y), \quad \forall y \in (0, +\infty),$$

where $B_1 > 0$ *and* $B_2 > 0$ *are constants.*

Now we are ready to prove the quasi-neutral limits $\lambda \to 0$ in (3.1)–(3.4) and give an appropriate convergence rate for the sequence $(n_\lambda, \phi_\lambda)_{\lambda>0}$.

Theorem 3.1. *Let $(n_\lambda, \phi_\lambda)_{\lambda>0}$ be a sequence of solutions to problem* (3.1)–(3.4) *and (n, ϕ) be the unique solution to* (3.5) *and* (3.8). *Under assumptions* (H1)–(H5), *the sequence $(\lambda^{\frac{1}{2}}\phi_\lambda)_{\lambda>0}$ is bounded in $H^1(0,1)$ and as $\lambda \to 0$, we have*

$$\|n_\lambda - n\|_{L^2(0,1)} \le B_3\lambda^{\frac{1}{2}}, \quad \|G_\lambda - G\|_{H^1(0,1)} \le B_4\lambda^{\frac{1}{2}}, \quad \|\phi_\lambda - \phi\|_{L^2(0,1)} \le B_5\lambda^{\frac{1}{2}}. \tag{3.13}$$

In particular, if $n_0 = b(0)$, then both sequences $(n_\lambda)_{\lambda>0}$ and $(\phi_\lambda)_{\lambda>0}$ are bounded in $H^1(0,1)$ and we have the following better estimates

$$\|n_\lambda - n\|_{L^2(0,1)} \le B_3\lambda, \quad \|G_\lambda - G\|_{H^1(0,1)} \le B_4\lambda, \quad \|\phi_\lambda - \phi\|_{L^2(0,1)} \le B_5\lambda, \tag{3.14}$$

where $B_3 > 0$, $B_4 > 0$ and $B_5 > 0$ are constants independent of λ.

Proof. In the proof of this theorem, B_i $(i \ge 6)$ denote various positive constants independent of λ. Let $u_\lambda(x) = u\Big(\frac{x}{\lambda}\Big)$, $\quad R_\lambda(x) = n_\lambda(x) - b(x) - u_\lambda(x)$. Obviously, $R_\lambda(0) = 0$ and $R_\lambda(1) = -u(\frac{1}{\lambda})$. Using the Poisson equation (3.2), we have

$$\begin{aligned}
\|R_\lambda\|^2_{L^2(0,1)} &= \int_0^1 (n_\lambda(x) - b(x) - u_\lambda(x))^2 dx \\
&= \int_0^1 (n_\lambda(x) - b(x))(n_\lambda(x) - b(x) - u_\lambda(x))dx - \int_0^1 u_\lambda(x)R_\lambda(x)dx \\
&= \lambda^2 \int_0^1 \frac{d^2\phi_\lambda}{dx^2}(n_\lambda(x) - b(x) - u_\lambda(x))dx - \int_0^1 u_\lambda(x)R_\lambda(x)dx \\
&= -\lambda^2 \int_0^1 \frac{d\phi_\lambda}{dx}\frac{d}{dx}(n_\lambda(x) - b(x) - u_\lambda(x))dx - \lambda^2\frac{d\phi_\lambda(1)}{dx}u\Big(\frac{1}{\lambda}\Big) \\
&\quad - \int_0^1 u_\lambda(x)R_\lambda(x)dx.
\end{aligned}$$

In view of the relation $F(n_\lambda) = \phi_\lambda + G_\lambda$, i.e., $n_\lambda = f(\phi_\lambda + G_\lambda)$, we obtain

$$\begin{aligned}
\|R_\lambda\|^2_{L^2(0,1)} &= -\lambda^2 \int_0^1 \frac{d\phi_\lambda}{dx}\frac{df(\phi_\lambda + G_\lambda)}{dx}dx + \lambda^2 \int_0^1 \frac{d\phi_\lambda}{dx}\frac{d}{dx}(b(x) + u_\lambda(x))dx \\
&\quad - \lambda^2\frac{d\phi_\lambda(1)}{dx}u\Big(\frac{1}{\lambda}\Big) - \int_0^1 u_\lambda(x)R_\lambda(x)dx \\
&= -\lambda^2 \int_0^1 f'(\phi_\lambda + G_\lambda)\Big(\frac{d\phi_\lambda}{dx}\Big)^2 dx - \lambda^2 \int_0^1 f'(\phi_\lambda + G_\lambda)\frac{d\phi_\lambda}{dx}\frac{dG_\lambda}{dx}dx \\
&\quad + \lambda^2 \int_0^1 \frac{d\phi_\lambda}{dx}\frac{d}{dx}(b(x) + u_\lambda(x))dx - \lambda^2\frac{d\phi_\lambda(1)}{dx}u\Big(\frac{1}{\lambda}\Big) - \int_0^1 u_\lambda(x)R_\lambda(x)dx.
\end{aligned}$$

From Lemma 3.2, it is easy to see that $\|u_\lambda\|_{L^1(0,1)} \le B_6\lambda$ and for sufficiently small λ, $\Big|u\Big(\frac{1}{\lambda}\Big)\Big| \le B_7\lambda$. Noting that $(n_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,1)$ and $b \in H^1(0,1)$, the sequence $(R_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,1)$, and it is easy to see that sequence of numbers $(\lambda^2\frac{d\phi_\lambda(1)}{dx})_{\lambda>0}$ is bounded. Again from Lemma 3.2, the sequence $(\lambda\frac{du_\lambda}{dx})_{\lambda>0}$ is bounded in $L^2(0,1)$. On the other hand, since $f = F^{-1}$, we have

$$f'(\phi_\lambda + G_\lambda) = \frac{1}{F'(n_\lambda)} = \frac{1}{h'(n_\lambda) - \frac{\varepsilon j^2}{n_\lambda^3}} = \frac{n_\lambda^3}{n_\lambda^2 p'(n_\lambda) - \varepsilon j^2}.$$

Therefore, the uniformly elliptic condition $\underline{n} > \rho_\varepsilon$ and the $L^\infty(0,1)$ boundedness of $(n_\lambda)_{\lambda>0}$ imply the existence of a constant $B_8 > 0$ such that $f'(\phi_\lambda + G_\lambda) \ge B_8$. Finally, by the

$L^\infty(0,1)$ boundedness of $(n_\lambda)_{\lambda>0}$, the $H^1(0,1)$ boundness of $(G_\lambda)_{\lambda>0}$ and Young's inequality, we obtain

$$\|R_\lambda\|^2_{L^2(0,1)} + \frac{B_8\lambda^2}{2}\Big\|\frac{d\phi_\lambda}{dx}\Big\|^2_{L^2(0,1)} \le B_9\lambda.$$

This shows the boundedness of the sequence $(\lambda^{\frac12}\phi_\lambda)_{\lambda>0}$ in $H^1(0,1)$. From Lemma 3.2, we have $\|u_\lambda\|_{L^2(0,1)} \le B_{10}\lambda^{\frac12}$. These two inequalities show that $\|n_\lambda - b\|_{L^2(0,1)} \le B_3\lambda^{\frac12}$.

Next, subtracting (3.1) from (3.5), and using the above estimate and Poincaré's inequality, we obtain

$$\|G_\lambda - G\|_{H^1(0,1)} \le B_{11}\|n_\lambda - n\|_{L^2(0,1)} \le B_4\lambda^{\frac12},$$

and then, by the definition of G_λ and G, we get

$$\|\phi_\lambda - \phi\|_{L^2(0,1)} \le B_{12}\Big(\|n_\lambda - n\|_{L^2(0,1)} + \|G_\lambda - G\|_{L^2(0,1)}\Big) \le B_5\lambda^{\frac12}.$$

This shows (3.13).

If $n_0 = b(0)$, then $u = 0$. Hence

$$\|n_\lambda - b\|^2_{L^2(0,1)} = -\lambda^2\int_0^1 f'(\phi_\lambda + G_\lambda)\Big(\frac{d\phi_\lambda}{dx}\Big)^2 dx - \lambda^2\int_0^1 f'(\phi_\lambda + G_\lambda)\frac{d\phi_\lambda}{dx}\frac{dG_\lambda}{dx}dx + \lambda^2\int_0^1 \frac{d\phi_\lambda}{dx}b'(x)dx,$$

then

$$\|n_\lambda - n\|^2_{L^2(0,1)} + \frac{B_8\lambda^2}{2}\Big\|\frac{d\phi_\lambda}{dx}\Big\|^2_{L^2(0,1)} \le B_{13}\lambda^2.$$

As above this shows (3.14) and, by Poincaré's inequality, the boundedness of $(\phi_\lambda)_{\lambda>0}$ in $H^1(0,1)$. Noting that $(G_\lambda)_{\lambda>0}$ is bounded in $H^1(0,1)$, $(n_\lambda)_{\lambda>0}$ is also bounded in $H^1(0,1)$. This finishes the proof.

The strong convergence of the sequences $(G_\lambda)_{\lambda>0}$ in $H^1(0,1)$ and $(n_\lambda)_{\lambda>0}$ in $L^2(0,1)$ allows to pass to the limit in the system (3.1)–(3.2) to obtain (3.5). Since $(n,\phi) \in (H^1(0,1))^2$, we may rewrite the first equation in (3.5) as

$$\frac{d}{dx}\Big(\frac{\varepsilon j^2}{n} + p(n)\Big) = n\frac{d\phi}{dx} - \frac{\varepsilon j}{\tau(n,j)},$$

which justifies the limit $\lambda \to 0$ in the original system (1.5)–(1.6).

§4. Zero-Relaxation-Time Limit

In the study of the zero-relaxation-time limit $\tau \to 0$, we assume that $\tau > 0$ is a constant and ε and λ are independent of τ. The limit $\tau \to 0$ has been rigorously justified for the transient Euler-Poisson system with γ-law ($\gamma \ge 1$) as pressure equation [11, 12, 10]. For the sake of completeness, we give an analysis of this limit in the steady-state problem for general pressure equation. In particular, the convergence rate is obtained if $|\varepsilon j|$ is small enough.

We first observe that if j is independent of τ, the limit $\tau \to 0$ in (1.5)–(1.6) is not meaningful. In the transient Euler-Poisson system, a new scaling is introduced to study the zero-relaxation-time limit. This scaling can be found by a formal asymptotic expansion in powers of τ. This enables us to consider the same scaling $j_\tau = \tau j$ as in the time-dependent problem, where $j \in I\!R$. In this situation, the problem (1.5)–(1.8) reads

$$\frac{d}{dx}\Big(\frac{\varepsilon\tau^2 j^2}{n_\tau} + p(n_\tau)\Big) = n_\tau\frac{d\phi_\tau}{dx} - \varepsilon j, \tag{4.1}$$

$$-\lambda^2\frac{d^2\phi_\tau}{dx^2} = b(x) - n_\tau \tag{4.2}$$

for $x \in (0,1)$ and

$$n_\tau(0) = n_0, \quad n_\tau(1) = n_1, \tag{4.3}$$

$$\phi_\tau(0) = 0. \tag{4.4}$$

Since $\tau \to 0$, the uniformly elliptic condition is satisfied and there is a smooth solution $(n_\tau, \phi_\tau) \in H^2(0,1) \times W^{2,\infty}(0,1)$ to (4.1)–(4.4). The limit (n, ϕ) of $(n_\tau, \phi_\tau)_{\tau>0}$ is formally governed by

$$\frac{dp(n)}{dx} = n\frac{d\phi}{dx} - \varepsilon j, \quad -\lambda^2 \frac{d^2\phi}{dx^2} = b(x) - n, \quad x \in (0,1) \tag{4.5}$$

and

$$n(0) = n_0, \quad n(1) = n_1, \quad \phi(0) = 0. \tag{4.6}$$

By Theorem 1.1, problem (4.5)–(4.6) has a unique solution if $|\varepsilon j|$ is sufficiently small. Now we prove the convergence of (4.1)–(4.4) to (4.5)–(4.6).

Theorem 4.1. *Under assumptions* (H1)–(H2)*, let* $(n_\tau, \phi_\tau)_{\tau>0}$ *be a sequence of solutions to* (4.1)–(4.4)*. Then there is a subsequence, still denoted by* $(n_\tau, \phi_\tau)_{\tau>0}$*, converging to* (n, ϕ) *in the following sense:*

$$n_\tau \to n \text{ uniformly in } C^0([0,1]), \quad n_\tau \to n \text{ uniformly in } C^1([0,1]), \tag{4.7}$$

where (n, ϕ) *solves problem* (4.5)–(4.6)*. Moreover, there exists two constants* $C_1 > 0$ *and* $C_2 > 0$ *independent of* τ*, such that if* $|\varepsilon j| < C_1$*, then the whole sequence* $(n_\tau, \phi_\tau)_{\tau>0}$ *converges to* (n, ϕ) *with*

$$\|n_\tau - n\|_{H^1(0,1)} \le C_2 \tau^2, \quad \|\phi_\tau - \phi\|_{H^1(0,1)} \le C_2 \tau^2. \tag{4.8}$$

Proof. The proof of (4.7) follows from the boundedness of the sequences $(n_\tau)_{\tau>0}$ and $(\phi_\tau)_{\tau>0}$ in $W^{1,\infty}(0,1)$ and $W^{2,\infty}(0,1)$ respectively, just like the proof of Theorem 2.1. Now we show (4.8) for small $|\varepsilon j|$. Using (4.1)–(4.6), we have

$$\frac{d^2}{dx^2}\Big(\frac{\varepsilon \tau^2 j^2}{2n_\tau^2} + h(n_\tau) - h(n)\Big) - \frac{n_\tau - n}{\lambda^2} + \frac{d}{dx}\Big(\frac{\varepsilon j}{n_\tau} - \frac{\varepsilon j}{n}\Big) = 0$$

with boundary conditions $n_\tau(0) - n(0) = n_\tau(1) - n(1) = 0$. Then

$$\int_0^1 \Big[\Big(\frac{d}{dx}(h(n_\tau) - h(n))\Big)^2 + \frac{(n_\tau - n)(h(n_\tau) - h(n))}{\lambda^2}\Big] dx$$

$$= -\frac{\varepsilon \tau^2 j^2}{2} \int_0^1 \frac{d}{dx}\Big(\frac{1}{n_\tau^2}\Big) \frac{d}{dx}(h(n_\tau) - h(n)) dx + \varepsilon j \int_0^1 (h(n_\tau) - h(n)) \frac{d}{dx}\Big(\frac{1}{n_\tau} - \frac{1}{n}\Big) dx.$$

Therefore, if $|\varepsilon j|$ is small enough, by Poincaré's inequality and a relation similar to (2.8), we obtain (4.8). The convergence of the whole sequence $(n_\tau, \phi_\tau)_{r>0}$ follows from the uniqueness of solution to problem (4.5)–(4.6).

§5. Energy Estimates for the Transient System

In this section we deal with the asymptotic limits for the transient Euler-Poisson system (1.1)–(1.3). Our goal is to present some energy estimates which provide useful information on the limits. To this end, we introduce the electric field $E = -\partial_x \phi$ and take the pressure function of γ-law:

$$p(n) = an^\gamma, \quad a > 0 \text{ and } \gamma \ge 1. \tag{5.1}$$

For convenience, we consider only the case $\gamma > 1$. The case $\gamma = 1$ can be treated in a similar way. See for instance [10] for the construction of entropies. Since the zero-relaxation-time limit has been justified in this situation, we will focus our attention on the zero-electron-mass limit and the quasi-neutral limit, of which the formal derivation can be found in [11,13]. In the sequel we assume that $\tau = 1$.

We consider system (1.1)–(1.3) in the whole space with initial conditions for (n, j) and boundary condition for E :

$$\partial_t n + \partial_x j = 0, \tag{5.2}$$

$$\varepsilon\partial_t j + \partial_x\Big(\frac{\varepsilon j^2}{n} + p(n)\Big) = -nE - \varepsilon j, \tag{5.3}$$

$$\lambda^2\partial_x E = b(x) - n \tag{5.4}$$

for $(t, x) \in (0, +\infty) \times I\!R$ and

$$t = 0 : n = n_0(x), \quad j = j_0(x), \quad x \in I\!R, \tag{5.5}$$

$$\lim_{x\to-\infty} E(t, x) = E_1(t), \text{ a.e.} \quad t \in (0, +\infty). \tag{5.6}$$

The global existence of weak solution has been proved in [22, 17] under the following assumptions:

(H6) $E_1 \in L^\infty(0, +\infty)$ and $b \in L^\infty(I\!R) \cap L^1(I\!R)$,

(H7) $n_0, j_0/n_0 \in L^\infty(I\!R)$ and $n_0 \geq 0$ has a compact support.

For any fixed $T > 0$, the solution (n, j, E) satisfies $E \in L^\infty(0, T; W^{1,\infty}(I\!R))$, $n, j/n \in L^\infty(Q_T)$, $n \geq 0$ and j have a compact support in x, where $Q_T = (0, T) \times I\!R$. Moreover, for any pair of smooth entropy-flux (η, q) with η convex, the following entropy condition holds in the sense of distributions:

$$\partial_t\eta(n, j) + \partial_x q(n, j) \leq -(nE/\varepsilon + j)\partial_j\eta(n, j). \tag{5.7}$$

To establish uniform estimates in the quasi-neutral limit, we assume that

(H8) $E_1 \in W^{1,\infty}(0, T)$,

(H9) $b = n_0$.

In the zero-electron-mass limit, we assume furthermore that

(H10) $E_1 = 0$,

(H11) $B, N_0 \in L^2(I\!R)$, where $B(x) = \int_{-\infty}^x b(y)dy$, $\ N_0(y) = \int_{-\infty}^x n_0(y)dy$.

From the Poisson equation (5.4), we obtain $\lambda^2 E = \lambda^2 E_1 + \int_{-\infty}^x (b - n)dx$. It follows from (5.2) that

$$\lambda^2\partial_t E = \lambda^2 E_1'(t) + j. \tag{5.8}$$

Therefore, (H8) implies that $\partial_t E \in L^\infty(Q_T)$ and $\partial_t(E - E_1)$ has a compact support in x. (H9) means the quasi-neutrality at the initial time. It implies that $E(0, x) - E_1(0) = 0$ for all $x \in I\!R$ and then the boundedness of the initial energy. Condition (H10) is necessary in the zero-electron-mass limit. If it is not satisfied, we will show by a counter example that the sequence of solutions can not converge to its equilibrium state.

Lemma 5.1. (Energy estimate) *Let $T > 0$ and* (H6)–(H8) *hold. Then for almost all $t \in (0, T)$, the solution (n, j, E) of* (5.1)–(5.6) *satisfies*

$$\mathcal{E}(t) + \int_0^t\int_{I\!R}\frac{\varepsilon j^2}{n}dxds \leq -\int_0^t\int_{I\!R} E_1 j dxds + \mathcal{E}(0),$$

where the total energy $\mathcal{E}$ is defined by

$$\mathcal{E}(t) = \int_{I\!R}\Big(\frac{\varepsilon j^2}{2n} + \frac{p(n)}{\gamma - 1} + \frac{\lambda^2(E - E_1)^2}{2}\Big)(t, x)dx.$$

Proof. Equation (5.3) is equivalent to $\partial_t j + \partial_x\Big(\frac{j^2}{n} + \frac{p(n)}{\varepsilon}\Big) = -\Big(\frac{nE}{\varepsilon} + j\Big)$, which allows to write down explicitly the convex entropy $\eta_1(n, j) = \frac{j^2}{2n} + \frac{p(n)}{\varepsilon(\gamma-1)}$. Let q_1 be its associated entropy-flux. Then (5.7) leads to

$$\partial_t\Big(\frac{j^2}{2n} + \frac{p(n)}{\varepsilon(\gamma - 1)}\Big) + \partial_x q_1(n, j) \leq -\Big(\frac{jE}{\varepsilon} + \frac{j^2}{n}\Big)$$

in the sense of distributions. Since n and j have compact support in x, the integration of the above equation over $I\!R$ gives

$$\frac{d}{dt}\int_{I\!R}\Big(\frac{\varepsilon j^2}{2n}+\frac{p(n)}{\gamma-1}\Big)dx+\int_{I\!R}\frac{\varepsilon j^2}{n}dx\le-\int_{I\!R}jEdx.$$

On the other hand, from (5.8) we have

$$\int_{I\!R}jEdx=\frac{d}{2dt}\int_{I\!R}\lambda^2(E-E_1)^2dx+E_1\int_{I\!R}jdx.$$

Integrating the last two equations with respect to t, we get the energy estimate.

Now we give the main estimates in the quasi-neutral limit and the zero-electron-mass limit.

Proposition 5.1. (Estimates in the quasi-neutral limit $\lambda\to 0$)

Under assumptions (H7)–(H9), *let $\varepsilon>0$ be a fixed constant independent of λ and $(n_\lambda, j_\lambda, E_\lambda)_{\lambda>0}$ be a sequence of weak solutions to* (5.1)–(5.6). *Then for any fixed $T>0$,*

(i) *the sequence $(n_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,T;L^s(I\!R))$, $\forall 1\le s\le\gamma$;*

(ii) *the sequence $(j_\lambda^2/n_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,T;L^1(I\!R))$;*

(iii) *the sequence $(j_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,T;L^1(I\!R))$;*

(iv) *the sequence $(\lambda(E_\lambda-E_1))_{\lambda>0}$ is bounded in $L^\infty(0,T;L^2(I\!R))$.*

Moreover, we have the weak quasi-neutrality in the following sense:

$$\|n_\lambda-b\|_{L^\infty(0,T;H^{-1}(I\!R))}\le B_T\lambda, \tag{5.9}$$

where $B_T>0$ is a constant independent of λ.

Proof. Integrating (5.2) over $I\!R$ gives $\frac{d}{dt}\int_{I\!R}n_\lambda(t,x)dx=0$, then $(n_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,T;L^1(I\!R))$ since $n_\lambda\ge 0$. By Young's inequality, for almost all $t\in(0,T)$,

$$\|j_\lambda(t,.)\|_{L^1(I\!R)}\le\frac12\Big(\|j_\lambda^2(t,.)/n_\lambda(t,.)\|_{L^1(I\!R)}+\|n_\lambda(t,.)\|_{L^1(I\!R)}\Big).$$

Therefore, there is a constant $D_T>0$ independent of λ, such that

$$\Big|\int_0^t\int_{I\!R}E_1jdxds\Big|\le D_T\Big(\int_0^t\mathcal{E}(s)ds+\|n_\lambda\|_{L^\infty(0,T;L^1(I\!R))}\Big),$$

then it follows from the energy estimate that

$$\mathcal{E}(t)+\int_0^t\int_{I\!R}\frac{\varepsilon j^2}{n}dxds\le D_T\Big(\int_0^t\mathcal{E}(s)ds+\|n_\lambda\|_{L^\infty(0,T;L^1(I\!R))}\Big)+\mathcal{E}(0).$$

Noting (H7)–(H9), $\mathcal{E}(0)$ is bounded, then we obtain (ii)–(iv) from the definition of $\mathcal{E}$ and Gronwall's inequality. Moreover, $(n_\lambda)_{\lambda>0}$ is bounded in $L^\infty(0,T;L^\gamma(I\!R))$. Therefore, (i) follows from an interpolation argument. Finally, (5.9) is a consequence of (iv) and the Poisson equation (5.4).

Proposition 5.2. (Estimates in the zero-electron-mass limit $\varepsilon\to 0$)

Assume (H7) *and* (H10)–(H11) *hold. Let $\lambda>0$ be a fixed constant independent of ε and $(n_\varepsilon, j_\varepsilon, E_\varepsilon)_{\varepsilon>0}$ be a sequence of weak solutions to* (5.1)–(5.6). *Then for any fixed $T>0$,*

(i) *the sequence $(n_\varepsilon)_{\varepsilon>0}$ is bounded in $L^\infty(0,T;L^s(I\!R))$, $\forall 1\le s\le\gamma$;*

(ii) *the sequence $(\varepsilon j_\varepsilon^2/n_\varepsilon)_{\varepsilon>0}$ is bounded in $L^\infty(0,T;L^1(I\!R))$;*

(iii) *the sequence $(\sqrt{\varepsilon}j_\varepsilon)_{\varepsilon>0}$ is bounded in $L^\infty(0,T;L^1(I\!R))$;*

(iv) *the sequence $(E_\varepsilon)_{\varepsilon>0}$ is bounded in $L^\infty(Q_T)\cap L^\infty(0,T;W^{1,r}_{\rm loc}(I\!R))$.*

The proof of Proposition 5.2 is similar to that of Proposition 5.1 and will be omitted here. Now we give an example to show that condition (H10) can not be removed for obtaining the limit equation from (5.1)–(5.6) as $\varepsilon\to 0$. Indeed,

$$n_\varepsilon = b \in I\!R, \quad j_\varepsilon = j_0 \exp(-t) - \frac{1}{\varepsilon}\int_0^t E_1(s)\exp(s-t)ds, \quad E_\varepsilon = E_1(t)$$

is a particular solution of (5.1)–(5.6). If $n > 0$ and $E_1 \neq 0$, we can not obtain the relation $\partial_x p(n) = -nE$ which is the formal limit of (5.3) as $\varepsilon \to 0$.

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