

JACQUES-LOUIS LIONS, 1928–2001**

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Jacques-Louis Lions was born in the heart of Provence in the charming city of Grasse, much renowned for its perfume industry and its historical center.

In spite of his young age, he had the courage and determination to join the French Résistance at the end of 1943, as a soldier in the FFI (French Forces of the Interior). There he met Andrée, his wife and life-long companion.

Their son Pierre-Louis, who was born in 1956, would also be distinguished by mathematical talent. This gift earned him the highest mathematical distinction, the Fields Medal, awarded to him during the 1994 International Congress of Mathematicians in Zurich. His parents had the great joy of being present for this unique occasion.

At the early age of nineteen, Jacques-Louis Lions passed the entrance exam to the highly coveted Ecole Normale Supérieure de la rue d'Ulm. There he met Bernard Malgrange among others and at the end of their studies, they both decided to opt for a university carrier in mathematics (a rather uncommon choice at a time when most "Normaliens" would rather teach the famed "classes de Mathématiques Spéciales" in the lycées). They were then awarded a grant by CNRS (National Center for Scientific Research) to prepare their doctoral dissertations. And so they both went to Nancy to work under the guidance of a prestigious thesis advisor, Laurent Schwartz, who had just received the Fields Medal for his theory of distributions in 1950.

After defending his thesis in 1954, Jacques-Louis Lions began his career "en province", which is to say outside of the Paris region as was then customary, at the University of Nancy, where he held a professorship from 1954 to 1962.

Far from keeping him fully occupied, his remarkable mathematical achievements during that period left him enough time to envision the immense opportunities offered by Scientific Computing, which was then coming of age, with the manifold industrial applications that would henceforth become amenable. This constant quest for applications, which was to guide him all his life and to become one of the most exceptional aspects of his career, materialized in 1958 when he became scientific consultant for the SEMA (Society for Economics and Applied Mathematics), a society headed by Robert Lattès, who had entered the Ecole Normale Supérieure one year after him.

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While this kind of inclination is common nowadays, it required lots of courage to follow it at the time. The applications of mathematics were then far from arousing the enthusiasm that they now generate!

After Nancy, Jacques-Louis Lions was named professor at the University of Paris, where he very quickly created a weekly seminar on “Numerical Analysis”, a discipline that was practically unheard of in France at that time. This seminar first met in the basement of the Institut Henri Poincaré, then in a dusty room of the Institut Blaise Pascal, which was situated rue du Maroc in the North of Paris.

When the University of Paris broke into thirteen distinct universities, he chose the sixth one, which was to be later named Université Pierre et Marie Curie. Two of his major initiatives there were to found the Laboratoire d’Analyse Numérique (after thirty years on the Jussieu campus, this department has been located rue du Chevaleret, near Place d’Italie, since 1999) and to create a DEA (Diplôme d’Etudes Approfondies, a set of advanced courses that a doctoral student has to pass before beginning a dissertation) specialized in Numerical Analysis. This DEA, from which a considerable number of applied mathematicians now holding positions in universities, at CNRS, or in industry graduated over the years, was always highly regarded. As a tribute to his memory and as an expression of gratitude, the Laboratoire d’Analyse Numérique, which is currently headed by Yvon Maday, has just been renamed Laboratoire Jacques-Louis Lions.

In 1973, at the early age of forty-five, Jacques-Louis Lions had the highly unusual honor of being simultaneously named professor at the celebrated Collège de France and elected to the French Academy of Sciences. At the Collège de France, he held the Chair entitled “Mathematical Analysis of Systems and of their Control” for twenty-five years. His series of lectures, which in the tradition of the Collège had to be renewed each year, were always followed by vast audiences, attracting not only his own students but also students of his students!

The “Seminar of Applied Mathematics” that he organized there until 1998, first with Jean Leray, then with Haïm Brezis for many years, soon became an “institution within an institution”. Indeed, countless applied mathematicians, either from Paris and its vicinity or French and foreign colleagues happening to be in Paris, gathered each Friday afternoon to hear prestigious lecturers, such as Stuart Antman, John Ball, Felix Browder, Ciprian Foias, Gu Chao-hao, Li Ta-tsien, Klaus Kirchgässner, Peter Lax, Andrew Majda, Louis Nirenberg, Olga Oleinik, Sergei Sobolev, Tosio Kato, Mark Vishik, and many others.

From 1966 to 1986, Jacques-Louis Lions was also part-time professor at the Ecole Polytechnique, where he created a course in numerical analysis from scratch that soon became a legend! Following the rule at the Ecole Polytechnique, he also wrote lecture notes, the contents of which were revolutionary for the time, at least in France. Indeed these notes constituted a kind of encyclopedia where, with his natural gift for teaching, Jacques-Louis Lions described and analyzed practically all that was then known about the numerical analysis of partial differential equations. Introductions to numerical optimization and numerical linear algebra were also presented in two separate chapters written by his first two doctoral students, Jean C  a and Pierre-Arnaud Raviart. A mystery remains about the first versions of these lecture notes. They were affectionately referred to as “the Diplodocus”, even though no one including their author ever seemed to understand the reason for this!

But all these essentially academic activities, that would normally occupy all of one’s time, did not take up all of his. Far from it!

From 1980 to 1984, he was also President of INRIA (National Institute for Research in Computer Science and Automatics), normally a full-time position! His leadership was of profound and lasting influence at INRIA: During his first weeks at the head of this institute, he used his incredible talents as an organizer to rejuvenate the organization and objectives, in particular by introducing the notion of project, gathering a clearly identified team around a well-defined objective on a specific theme.

During his four-year term as president, he strongly advocated the creation of start-up companies by researchers from the institute and he initialized its decentralization through the creation of similar institutes at Sophia-Antipolis and Rennes. Due to his personal prestige, the teams he was able to gather, and the numerous first-class international conferences that he organized there, he greatly contributed to the fame of INRIA.

From 1984 to 1992, he held another high-level, and also normally full-time, official position as President of CNES (National Center for Space Studies), where he continued and developed the action of his predecessor Hubert Curien (current President of the Academy of Sciences), who had just been named Minister of Research and Technology. There he used not only his eminent intellectual capacities but also his talents for intelligent persuasion to convince the French authorities that the orientations he advocated were well-founded. In this way, he played a major role in the conception of the French-American “Topex-Poseidon” space program for oceanography. Topex-Poseidon is also the name of the satellite that made it possible to at last understand “El Niño”, a major event in climatology. His influence was likewise a decisive factor in the French-Russian negotiations that ultimately allowed Jean-Loup Chrétien and Michel Tognini to participate in manned space missions.

His presence at the Monday afternoon seances at the French Academy of Sciences remained rare for many years. But eventually, he gave new life to this noble “Compagnie” (as it is traditionally known among its members) when he became President in January 1997 for the customary two years. Immediately after Lions took office, President Jacques Chirac gave him the mission of supervizing the drafting of a document concerning the state of the art worldwide in each of the following areas: “Access to knowledge for all and electronic processing of information, knowledge of our planet and ways of life, understanding life systems and improving health-care for all”. He immediately began to work on this ambitious undertaking by creating and heading a “Committee 2000”, whose task was to analyze the three questions and make proposals, under his ongoing close supervision. Remarkably, in spite of the scope of this project, he was able to meet the 2000 deadline he had set himself.

Indeed, he personally handed President Jacques Chirac the requested document during a ceremony held at the Elysée Palace on 25 January, 2000. He even succeeded in having all the Members and Corresponding Members of the Academy invited for the occasion, a first indeed!

However, Jacques-Louis Lions’s actions during his presidency were not limited to the Committee 2000. His efforts were decisive in promoting the need for a profound reform in the status of the Academy. The principles of this reform have now been accepted. He also played a major role in the creation of an Academy of Technology, always desired but never achieved before. This academy was eventually created on 12 December, 2000.

As exemplified by his presidencies at INRIA and CNES, Lions was an exceptionally successful promoter of ever closer ties between academic research, too often seen as disconnected from the real world, and the more pragmatic industrial research. In this spirit, he headed scientific committees in major public utility companies, such as Météorologie Nationale, Gaz

de France, France Telecom, or Electricité de France and he held high level advisory positions in major companies, such as Pechiney, Dassault Aviation, or Elf.

Jacques-Louis Lions's influence extended far beyond frontiers. Since the beginning of his career, he was an indefatigable traveler who, in addition to traditional venues in Europe or the Americas, very quickly added less canonical ones to his list of destinations. For instance, as early as 1957 he set out for a three-month visit to the Tata Institute of Fundamental Research in Bombay, at the time a genuinely adventurous trip! He enjoyed the splendor of the ancient Taj Mahal Hotel and the hospitality of Kollagunta Gopalaiyer Ramanathan, with whom he contributed to the creation of an applied branch of the Tata Institute on the campus of the Indian Institute of Sciences in Bangalore twenty years later.

In 1966 he began a long series of visits in the former Soviet Union. Often invited by the USSR Academy of Sciences or by the Novosibirsk Institute for Computation, he initiated manifold scientific exchanges with eminent soviet mathematicians such as Guri Marchuk, Olga Oleinik, Lev Semenovitch Pontryagin, Ilia Vekua, Mark Visik, or Nicolay Nicolayevich Yanenko. One of his merits during this period, and not the least, was to contribute greatly to the dissemination of Soviet research in applied mathematics among Westerners.

A trip that left him with a lasting impression was the journey that he undertook in 1975 to Beijing, where he was received with great ceremony. He was in particular impressed there by the mathematical talents of Feng Kang, who had just independently rediscovered the finite element method. This was the first of a long series of trips to China, which later included three visits to Fudan University in Shanghai. In return he also invited the famous differential geometer Su Bu-chin to visit Paris in 1982 for signing an agreement between Fudan University, the INRIA, and the Ecole Polytechnique.

However, his international ventures were not limited to traditional scientific exchanges, as his talents as a lecturer, thesis adviser, and organizer produced many disciples throughout the countries he visited.

As early as the 1960s for instance, he was the adviser of Antonio Valle, the first in a long series of students from Spain and Portugal, who in turn set up numerical analysis departments at the Universities of Malaga, Sevilla, Santiago de Compostela, Lisboa, or at the Universidad Complutense de Madrid, modeled after the one he had created in Paris. In the same vein, he was in 1997 the main speaker in a European video-conference on Mathematics and the Environment, organized in Madrid by Jesus Ildefonso Diaz. He also chaired the Committee awarding the ten "Prizes for Young Mathematicians" during the Third European Congress of Mathematics held in Barcelona in 2000.

For many years, he chaired as well the Scientific Committee of the Istituto di Analisi Numerica del CNR of the University of Pavia, headed for several decades by Enrico Magenes, then by Franco Brezzi.

Together with Paul Germain, he represented France at the 1975 meeting on "Functional Analysis and Mechanics" of the IUTAM (International Union for Theoretical and Applied Mechanics) held in Luminy, where the other representatives were Klaus Kirchgässner for West Germany, Sir James Lighthill for the United Kingdom, and William Prager for the United States.

His intelligent proselytizing was not limited to Europe, however. In China for instance, he was one of the driving forces behind the creation in 1997 of the LIAMA (French-Chinese Laboratory of Computer Science, Automatics, and Applied Mathematics), an offspring of INRIA and the Chinese Academy of Sciences, housed since then in Beijing by the Institute

of Automatics of the academy. He likewise played a major role in the creation in 1998 of the ISFMA (Chinese-French Institute of Applied Mathematics), splendidly housed by the Department of Mathematics of Fudan University in Shanghai, thanks to the tireless efforts of his Director Li Ta-tsien. As President of the Scientific Committee of this institute, he attended the opening ceremony that marked its creation. He was Honorary Editor of “Chinese Annals of Mathematics”. He also established and kept close scientific ties with colleagues in Hong Kong, notably with Roderick Wong, Dean at City University of Hong Kong.

Jacques-Louis Lions was also President of the IMU (International Mathematical Union) from 1991 to 1995. During a meeting of this organization in Rio de Janeiro on 6 May, 1992, he proposed that the year 2000 be baptized “World Mathematical Year”. This proposal, which was later supported by UNESCO, turned out to be a genuine success story that significantly contributed to the improvement of the image of mathematics in the general public and helped encourage mathematical research in developing countries.

While he served as “Past-President” of the IMU from 1995 to 1999, the decision was taken that Beijing would host the twenty-third International Congress of Mathematicians in 2002, the second in Asia after that of Kyoto in 1990.

He likewise was a constant supporter of the initiatives of the Third World Academy of Sciences (TWAS), either directly or through colleagues from his group. Particularly noteworthy in this respect were his undertakings for the progress of mathematical research in Africa.

The mathematical works of Jacques-Louis Lions are immense. Alone or in collaboration, he wrote more than twenty books, most of which have become classics often translated into several foreign languages, as well as more than five hundred papers. The different themes of his work are briefly described below, in an order that approximately follows their chronology.

He had, and will continue to have for a long time, a considerable influence on mathematics and their applications, not only through his own work, but also through that of the School he created and constantly kept in touch with. This School, which numbers some fifty initial students and scores of students of students, etc., has acquired a widespread fame over the years not only in university circles, but also in industry, an accurate indication that the directions of research he envisioned and promoted were highly relevant.

If a single title were to be attached to Jacques-Louis Lions’s mathematical works, it might be with a fair degree of accuracy that of “Partial differential equations in all their aspects: Existence, uniqueness, regularity, control, homogenization, numerical analysis, etc., and the applications they model, such as fluid and solid mechanics, oceanography, climatology, etc.”.

Jacques-Louis Lions produced his first mathematical works in 1951. At the same time two major books were published, one by Laurent Schwartz on the theory of distributions and one by Sergei Sobolev on their applications to mathematical physics, as well as a founding paper by John von Neumann and Robert Richtmyer on the numerical approximation of nonlinear hyperbolic problems arising in hydrodynamics.

Inspired by these works, Lions’s first objectives were to undertake a systematic study of linear and nonlinear boundary value problems, notably by constantly using the theory of distributions, and then to find ways to numerically approximate their solutions.

He began in 1954 a series of collaborations and lasting friendships with eminent Italian

mathematicians, such as Enrico Magenes, Guido Stampacchia, Ennio de Giorgi, or Giovanni Prodi (brother of the current President of the European Union). One such collaboration resulted in an exhaustive analysis of boundary value problems posed in fractional Sobolev spaces, thanks in particular to the theory of interpolation between Banach spaces that he initiated with Jack Peetre in 1961. This analysis is the object of the celebrated three-volume treatise “Non-Homogeneous Boundary Value Problems and Applications” (1968-1970) that he wrote with Enrico Magenes. From 1965 to 1967, he also developed with Guido Stampacchia the foundations of the theory of variational inequalities, as they appear for instance in unilateral problems in elasticity.

His inclination for applications led him to propose a particularly elegant proof of Korn’s inequality, based notably on a fundamental result in distribution theory known as “Lions’s lemma” (although several other results of his bear the same name!). He further developed applications of the theory of variational equations or inequalities to mechanics by mathematically analyzing Bingham fluids, friction, viscoelasticity, or plasticity models. These applications constitute the substance of another well-known book, “Inequalities in Mechanics and Physics” (1972) that he wrote with Georges Duvaut.

He was equally interested in the numerical simulation of these problems, at a time when it was realized that the applicability of finite difference methods had reached its limits. For instance, these methods do not perform well when the problems to be approximated have rapidly varying coefficients or are posed on domains with complicated geometries. On the other hand, the finite element method, already familiar to engineers in handling these types of difficulties, remained essentially unknown to mathematicians.

With a remarkable intuition, Jacques-Louis Lions immediately foresaw that it was preferable to discretize the variational, or weak, formulations of partial differential equation problems rather than the partial differential equations themselves. Accordingly, he quickly pointed out to his group of colleagues and students the interest of studying and analyzing Galerkin methods in general and finite element methods in particular. A quite productive period ensued, to which he himself contributed with another classic, “Numerical Analysis of Variational Inequalities” (1976), co-authored with Roland Glowinski and Raymond Trémoières.

His book “Some Methods for Solving Nonlinear Boundary Value Problems” (1969) is a major contribution to the theory of nonlinear partial differential equations, which remains even today a substantial source of inspiration (it is unfortunate that this book was never translated into English). In this work, Jacques-Louis Lions introduced and systematically analyzed the so-called compactness methods, which play a key role in the existence theory for the Navier-Stokes and von Kármán equations, the monotony methods he had developed with Jean Leray, and the regularization and penalty methods, which can for instance be applied to the Schrödinger or Korteweg-de Vries equations. For the most part, the results found in this book are either due to himself or to his students, in particular Haïm Brezis and Luc Tartar.

Most of the works mentioned so far, together with many generalizations they led to, were to be assembled in the monumental treatise “Mathematical Analysis and Numerical Methods for Sciences and Technology” (1984-1985), conceived and edited by Jacques-Louis Lions and Robert Dautray. This work, which comprises almost four thousand pages, is justly regarded as the modern counterpart of the celebrated Courant-Hilbert.

His ongoing interest for problems with small parameters led him to write “Singular Per-

turbations in Boundary Value Problems and in Optimal Control” (1973), a book where he laid down the foundations of the asymptotic analysis of such problems. The methods and notions that he then introduced and analyzed, such as a priori estimates, stiff problems, boundary layers, multiple scales, and so on, were subsequently recognized as fundamental for many applications. For instance, they later played a major role in the mathematical modeling of elastic structures or “multi-structures”, made of plates, rods, or shells.

Another field where small parameters naturally arise is the modeling of composite materials, of constant use in the aerospace industry for instance. Their asymptotic analysis, which is a special case of what became known as homogenization theory, was abundantly developed and illustrated by applications in another seminal work, “Asymptotic Analysis for Periodic Structures” (1978), which he wrote with Alain Bensoussan and George Papanicolaou. In this book, a substantial number of essentially empirical formulas used in the modeling of periodic structures were rigorously justified for the first time, thanks notably to the compensated compactness method due to his students Francois Murat and Luc Tartar and to the oscillating test-functions method of Luc Tartar.

A fundamental work by Lev Semenovitch Pontryagin about the optimal control of systems governed by ordinary differential equations (the objective was to control the trajectories of artificial satellites) immediately attracted his attention in 1958. Through the contacts he already had at that time with the engineering community, he became quickly convinced that the next step was to extend optimal control to distributed systems, i.e., systems whose state is governed by partial differential equations. The inclination that he then developed for the optimal control of such systems was to always remain at the center of his mathematical interests.

A pioneer as always, he began by laying down the foundations of a general theory in yet another celebrated book, “Optimal Control of Systems Governed by Partial Differential Equations” (1968), where he notably introduced an infinite-dimensional version of the Riccati equation.

In two books co-authored with Alain Bensoussan, “Applications of Variational Inequalities to Stochastic Control” (1978) and “Impulse Control and Variational Inequalities” (1983), he continued his investigations by considering in particular the optimal control of systems that are not necessarily well-posed or that have multiple states.

After having so thoroughly analyzed the main aspects of optimal control theory, Jacques-Louis Lions shifted his interests to the study of “controlability”, a discipline that basically seeks to answer the following type of question: Given a system in an arbitrary initial state, the question is to devise a way of acting on it in such a fashion that its solution reaches a given final state in a finite time, for instance by imposing adequate boundary conditions.

During the prestigious “John von Neumann Lecture” that he gave at the SIAM Congress in Boston in 1986, he presented for the first time his now famous “HUM” (Hilbert Uniqueness Method) for the exact controlability of linear time-dependent equations. He chose this particular terminology to emphasize the fact that the feasibility of such controlability is related in an essential way to the uniqueness of the solution to the adjoint problem, typically obtained by the Holmgren or the Carleman theorem.

This lecture was the starting point of numerous works by himself or his School. In particular, he began by publishing no less than three books on the subject in the same year, “Exact Controlability, Perturbations and Stabilization of Distributed Systems” in two volumes (1988) and, with John Lagnese, “Modeling, Analysis, and Control of Thin Plates”

(1988), which contains an abundance of applications to the theory of elastic plates. In 1995, he also established with Enrique Zuazua the generic character of the controllability of the three-dimensional Stokes equations: If there is no approximate controllability for a given open set, it is always possible to find another arbitrarily close open set for which this type of controllability holds.

These works constituted yet another mark of his constant interest for real-life applications. He also had the concern of proposing numerically feasible approximation methods. These were the theme of a long article co-authored with Roland Glowinski, an article so long that its nearly three hundred pages took up two consecutive issues of "Acta Numerica" (1994–1995).

Even though the last works of Jacques-Louis Lions were in different areas, they continued to be partly influenced by the methodology he had developed for questions of controllability.

In 1990, he started to express his great interest in climatology in "El Planeta Tierra". In this book, which was directly published in Spanish, he described in a masterly fashion and in a remarkably accessible style the most important problems originating in this science, such as modeling, numerical simulation, sensitivity to initial conditions, etc. From 1994 to 1998, his last courses at the Collège de France were all about these subjects.

The models found in climatology include complex systems of partial differential equations such as those of Navier-Stokes or of thermodynamics. But these systems had never been seriously analyzed from the mathematical viewpoint, although they had been blithely used in a massive way since the 1960s for numerical simulation in weather forecasting.

In spite of the "truly diabolic complexity" (as he was fond of saying) of the combination of partial differential equations, boundary conditions, transmission conditions, nonlinearities, physical assumptions, etc., that enter these models, Jacques-Louis Lions, together with Roger Temam and Shouhong Wang, was able to study questions of existence and uniqueness of solutions, to establish the existence of attractors, and to propose numerical methods. He even succeeded in teaching these works on a blackboard, a pedagogical tour de force!

In 1995, he began with Evariste Sanchez-Palencia another series of works where they developed the theory of sensitive problems, exemplified by the boundary value problems that appear in the theory of linearly elastic membrane shells. In such problems, which in a sense constitute the antithesis of well-posed problems, arbitrarily small, yet arbitrarily smooth changes in the right-hand sides of the equations may induce "sudden" changes on the properties of their solutions. It is perhaps no coincidence that the analysis of such problems relies in particular on uniqueness theorems that bear resemblance with those needed in the HUM.

In his last works, Jacques-Louis Lions returned to the numerical analysis of parallel algorithms and domain decomposition methods, in a long series of Notes aux Comptes Rendus de l'Académie des Sciences most often co-authored with Olivier Pironneau and published from 1997 to 2001. In fact, these kinds of topics had been on his mind for a long time. As early as the 1980s, he had already been an ardent advocate of installing a parallel computer at INRIA. The main idea in these Notes is to introduce parallelism in the continuous problem rather than in the discrete one, an approach that is in fact quite general, since it applies equally well to any problem that is approximated by an iterative method, such as a fractional step method, a decomposition method into sub-problems in optimization theory, a domain decomposition method, and so forth.

One can only be impressed by these immense works, whether by the quality, diversity, and novelty of the mathematics used or by the permanent quest for new applications that

were previously believed to be inaccessible.

Like John von Neumann, whom he deeply admired, Jacques-Louis Lions was a visionary, who very quickly understood that the availability of ever-increasing computational power could revolutionize the modeling of phenomena and thereby improve our knowledge and mastery of the physical world, provided however that the required mathematics were simultaneously created and developed. He admirably contributed to this latter task.

Jacques-Louis Lions justly received numerous honors. Although he always remained modest about them, their list is truly astonishing: He was Commandor of the French Legion of Honor and Great Officer of the French National Order of Merit, a distinction he received at the hands of President Chirac on 23 February, 1999. He was a member of twenty-two foreign academies and *Honoris Causa* Doctor of nineteen universities, he received the most prestigious prizes, and he delivered the most coveted lectures.

He was in particular awarded three Prizes by the French Academy of Sciences, the John von Neumann Prize in 1986, the Harvey Prize from the Technion in 1991, and the Lagrange Prize at the ICIAM meeting in Edinburgh in 1999. Jacques-Louis Lions was proud to have had the rare honor of having shaken the hand of Emperor Akihito when he received the highly prestigious Japan Prize in 1991. It was the climax of a perfectly and meticulously organized week which had particularly impressed him!

In particular, he was an invited speaker three times at an International Congress of Mathematicians, in 1958, 1970, and 1974. He gave the John von Neumann Lecture at the SIAM meeting in Boston in 1986. He was plenary speaker at the ICIAM Congress in Hamburg and at the SIAM Congress in Philadelphia in 1995, and he held the Galileo Chair at the University of Pisa in 1996. He also had the extremely rare honor for a scientist, especially for a mathematician, of speaking before a parliament! More specifically, he delivered an address entitled “Will it ever be possible to describe, understand, and control the inanimate and animate world by means of the languages of mathematics and informatics?” in front of the Cortes who had specially gathered in Madrid on 21 January, 2000 for the occasion, as part of the celebrations of the “World Mathematical Year 2000”.

He belonged to the most famous academies, such as the USSR Academy of Sciences and the American Academy of Arts and Sciences to which he was elected in 1982 and 1986. In 1996, he was simultaneously elected to the Royal Society of the United Kingdom, to the National Academy of Sciences of the USA, and to the Third World Academy of Sciences, just before being simultaneously elected in 1998 to the Chinese Academy of Sciences and to the Accademia Nazionale dei Lincei in Rome. The Pontifical Academy of Sciences, to which he was elected in 1990, seemed to be particularly dear to him. It may perhaps be not so well-known, but it is an unusually rare honor to become one of its members!

Who was the man behind all these endeavors? What I knew of him convinced me that both his human and professional qualities were truly exceptional.

Jacques-Louis Lions wrote abundantly, very rapidly, and with an astonishing ease not only mathematics but also countless letters, which constituted his favorite means of communication. He was a master with the fax, which he used with an amazing efficiency! For

instance it was not uncommon for each one of his various collaborators at any given time to receive four or five faxes per week, sometimes of up to thirty pages if their contents were mathematical.

Even though he wrote abundantly, he confessed that he did not usually keep track of his manifold letters, perhaps because he could rely entirely on his memory, or perhaps because he preferred to spare himself a herculean archival task! Let us hope that his correspondents had the good idea to keep his letters, which could thus be later compiled.

By any measure, his abilities were astounding. For instance, he once told me that it only took him a few weeks to write the several hundred pages of the “Diplodocus” lecture notes mentioned earlier. Likewise his indifference to lack of sleep or to the most extreme jetlags and his freshness after lengthy flights were always a subject of astonishment among his travel companions.

As John Ball put it so well, “Jacques-Louis Lions was a man of considerable personal magnetism and charm, whose charisma, brilliance as a teacher, and accessibility attracted others to work with him”. And indeed it was obvious that Jacques-Louis Lions had an ample share of charisma, even if charisma is not easy to define in a rigorous manner! He was also incredibly open and displayed such amiable and simple manners that any one of his current students or collaborators felt they were at the center of his attention.

He was also very brave in the face of physical danger and suffering. Even when the pain became unbearable, he never complained, keeping on the contrary his compassion for others.

All those who met him will cherish the memory of his warm personality, the vision that he so well conveyed, and his profound intelligence.

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