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FORMATION OF SINGULARITIES FOR A KIND OF QUASILINEAR NON-STRICTLY HYPERBOLIC SYSTEM

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Abstract

The author gets a blow-up result of C^1 solution to the Cauchy problem for a first order quasilinear non-strictly hyperbolic system in one space dimension.

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§1. Introduction and Main Result

Consider the following first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = 0, \tag{1.1}$$

where $u = (u_1, \cdots, u_n)^T$ is the unknown vector function of (t, x) and $A(u) = (a_{ij}(u))$ is an $n \times n$ matrix with suitably smooth elements $a_{ij}(u)$ $(i, j = 1, \cdots, n)$.

By hyperbolicity, for any given u on the domain under consideration, $A(u)$ has n real eigenvalues $\lambda_1(u), \cdots, \lambda_n(u)$ and a complete set of left (resp. right) eigenvectors. For $i = 1, \cdots, n$, let $l_i(u) = (l_{i1}(u), \cdots, l_{in}(u))$ (resp. $r_i(u) = (r_{i1}(u), \cdots, r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda_i(u)$:

$$l_i(u)A(u) = \lambda_i(u)l_i(u) \qquad (\text{resp.} \quad A(u)r_i(u) = \lambda_i(u)r_i(u)). \tag{1.2}$$

We have

$$\det|l_{ij}(u)| \neq 0 \quad (\text{equivallently}, \quad \det|r_{ij}(u)| \neq 0). \tag{1.3}$$

All $\lambda_i(u), l_{ij}(u)$ and $r_{ij}(u)$ $(i, j = 1, \cdots, n)$ are supposed to have the same regularity as $a_{ij}(u)$ $(i, j = 1, \cdots, n)$.

Without loss of generality, we suppose that on the domain under consideration

$$l_i(u)r_j(u) \equiv \delta_{ij} \quad (i, j = 1, \cdots, n), \tag{1.4}$$

$$r_i^T(u)r_i(u) \equiv 1 \quad (i = 1, \cdots, n), \tag{1.5}$$

where δ_{ij} stands for the Kronecker's symbol.

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In the case that system (1.1) is strictly hyperbolic, namely, $A(u)$ has n distinct real eigenvalues

$$\lambda_1(u) < \lambda_2(u) < \cdots < \lambda_n(u),$$

earlier results can be found in [4,13] and [3]. Then, by means of the concept of weak linear degeneracy, Li Ta-tsien, Zhou Yi and Kong Dexing have given a complete result on the global existence and the blow-up phenomenon of C^1 solution to the Cauchy problem for system (1.1) with small C^1 initial data with certain decaying properties as $|x| \to +\infty$ in [11, 12]. Moreover, A. Bressan[1] and Yan Ping[15] have proved the global existence of C^1 solution to the Cauchy problem with small initial total variation in the linearly degenerate case and the weakly linearly degenerate case respectively. The results in [11,12] have been generalized to the non-strictly hyperbolic system with characteristics with constant multiplicity (see [7] and [10]). However, in nonlinear elasticity, for some homogeneous, isotropic hyperelastic materials (see [2, 14]), the system of plane elastic waves is usually a quasilinear non-strictly hyperbolic system with characteristics without constant multiplicity, and the eigenvalues at $u = 0$ satisfy

$$\lambda_1(0) < \lambda_2(0) \leq \cdots \leq \lambda_5(0) < \lambda_6(0).$$

Regarding this fact, in this paper, we consider the case that at $u = 0$ the rightmost or the leftmost characteristic is simple. Under appropriate conditions we will prove that in this situation the C^1 solution to the Cauchy problem for system (1.1) with a large class of small and decaying initial data must blow up in a finite time.

For fixing the idea, without loss of generality, we suppose that, in a neighbourhood of $u = 0$, the rightmost characteristic $\lambda_n(u)$ is simple, namely,

$$\lambda_1(0) \leq \lambda_2(0) \leq \cdots \leq \lambda_{n-1}(0) < \lambda_n(0). \tag{1.6}$$

Moreover, we suppose that $\lambda_n(u)$ is not a weakly linearly degenerate characteristic such that there is an integer $\alpha \geq 0$ such that, along the n-th characteristics trajectory $u = u^{(n)}(s)$ passing through $u = 0$, defined by

$$\begin{cases} \dfrac{du}{ds} = r_n(u), \\ s = 0 : u = 0, \end{cases} \tag{1.7}$$

we have

$$\frac{d^l \lambda_n(u^{(n)}(s))}{ds^l}\Big|_{s=0} = 0 \quad (l = 1, \cdots, \alpha), \tag{1.8}$$

but

$$\frac{d^{\alpha+1} \lambda_n(u^{(n)}(s))}{ds^{\alpha+1}}\Big|_{s=0} \neq 0. \tag{1.9}$$

The main result in this paper is

Theorem 1.1. *Suppose that* $A(u)$ *is suitably smooth in a neighbourhood of* $u = 0$. *Suppose furthermore that* (1.6) *holds and there is an integer* $\alpha \geq 0$ *such that* (1.8)–(1.9) *hold. For the Cauchy problem of system* (1.1) *with the following initial data*

$$u(0, x) = \varepsilon\psi(x), \tag{1.10}$$

where $\varepsilon > 0$ *is a small parameter and* $\psi(x) \in C^1$ *satisfies*

$$\sup_{x\in\mathbb{R}}\{(1 + |x|)(|\psi(x)| + |\psi'(x)|)\} < \infty, \tag{1.11}$$

if

$$l_n(0)\psi(x) \not\equiv 0, \tag{1.12}$$

then there exists $\varepsilon_0 > 0$ so small that for any fixed $\varepsilon \in (0, \varepsilon_0]$, the first order derivatives of C^1 solution $u = u(t, x)$ to Cauchy problem (1.1) *and* (1.10) *must blow up in a finite time and there is a positive constant C independent of ε, such that the life-span $\widetilde{T}(\varepsilon)$ of $u = u(t, x)$ satisfies*

$$\widetilde{T}(\varepsilon) \le C\varepsilon^{-(1+\alpha)}. \tag{1.13}$$

Remark 1.1. It is well known that the life-span of C^1 solution $u = u(t, x)$ has the following lower bound:

$$\widetilde{T}(\varepsilon) \ge \overline{C}\varepsilon^{-1}, \tag{1.14}$$

where $\overline{C}$ is a positive constant independent of ε.

In Section 2 we will present some preliminaries, then Theorem 1.1 will be proved in Section 3.

§2. Preliminaries

By Lemma 2.5 in [12], there exists a suitably smooth invertible transformation $u = u(\widetilde{u})(u(0) = 0)$ such that in $\widetilde{u}$ space, for each $i = 1, \cdots, n$, the i-th characteristic trajectory passing through $u = 0$ coincides with the $\widetilde{u}_i$-axis at least for $|\widetilde{u}_i|$ small, namely,

$$\widetilde{r}_i(\widetilde{u}_i e_i)//e_i, \quad \forall|\widetilde{u}_i| \ \text{ small } \quad (i = 1, \cdots, n), \tag{2.1}$$

where

$$e_i = (0, \cdots, 0, \overset{(i)}{1}, 0, \cdots, 0)^T. \tag{2.2}$$

Such a transformation is called the normalized transformation and the corresponding unknown variables $\widetilde{u} = (\widetilde{u}_1, \cdots, \widetilde{u}_n)$ are called the normalized variables or normalized coordinates.

Let

$$v_i = l_i(u)u \qquad (i = 1, \cdots, n), \tag{2.3}$$

$$w_i = l_i(u)u_x \qquad (i = 1, \cdots, n). \tag{2.4}$$

By (1.4), it is easy to see that

$$u = \sum_{k=1}^{n} v_k r_k(u), \tag{2.5}$$

$$u_x = \sum_{k=1}^{n} w_k r_k(u). \tag{2.6}$$

Let

$$\frac{d}{d_i t} = \frac{\partial}{\partial t} + \lambda_i(u)\frac{\partial}{\partial x}$$

be the directional derivative with respect to t along the i-th characteristic. We have (cf. [4] or [12])

$$\frac{dv_i}{d_i t} = \sum_{j,k=1}^{n} \beta_{ijk}(u)v_j w_k \quad (i = 1, \cdots, n), \tag{2.8}$$

where

$$\beta_{ijk}(u) = (\lambda_k(u) - \lambda_i(u))l_i(u)\nabla r_j(u)r_k(u). \tag{2.9}$$

Hence

$$\beta_{iji}(u) \equiv 0, \quad \forall i, j; \tag{2.10}$$

and, in normalized coordinates,

$$\beta_{ijj}(u_j e_j) \equiv 0, \quad \forall |u_j| \text{ small}, \quad \forall i, j. \tag{2.11}$$

Noting (2.6) and (2.8), we have (cf. [8])

$$d[v_i(dx - \lambda_i(u)dt)] = \Big[\frac{\partial v_i}{\partial t} + \frac{\partial(\lambda_i(u)v_i)}{\partial x}\Big] dt \wedge dx = \sum_{j,k=1}^{n} B_{ijk}(u) v_j w_k dt \wedge dx, \tag{2.12}$$

where

$$B_{ijk}(u) = \beta_{ijk}(u) + \nabla\lambda_i(u) r_k(u)\delta_{ij}. \tag{2.13}$$

By (2.10), it is easy to see that

$$B_{iji}(u) \equiv 0, \qquad \forall j \neq i, \tag{2.14}$$

while

$$B_{iii}(u) = \nabla\lambda_i(u) r_i(u). \tag{2.15}$$

By (2.11), in normalized coordinates we have

$$B_{ijj}(u_j e_j) \equiv 0, \quad \forall |u_j| \ \text{ small}, \quad \forall j \neq i. \tag{2.16}$$

On the other hand, we have (cf. [4] or [12])

$$\frac{dw_i}{d_i t} = \sum_{j,k=1}^{n} \gamma_{ijk}(u) w_j w_k \quad (i = 1, \cdots, n), \tag{2.17}$$

where

$$\gamma_{ijk}(u) = \frac{1}{2}\{(\lambda_j(u) - \lambda_k(u)) l_i(u) \nabla r_k(u) r_j(u) - \nabla\lambda_k(u) r_j(u)\delta_{ik} + (j|k)\}, \tag{2.18}$$

where $(j|k)$ stands for all terms obtained by changing j and k in the previous terms. Hence

$$\gamma_{ijj}(u) \equiv 0, \quad \forall j \neq i, \tag{2.19}$$

$$\gamma_{iii}(u) = -\nabla\lambda_i(u) r_i(u) \quad (i = 1, \cdots, n). \tag{2.20}$$

We have (cf. [8])

$$d[w_i(dx - \lambda_i(u)dt)] = \Big[\frac{\partial w_i}{\partial t} + \frac{\partial(\lambda_i(u)w_i)}{\partial x}\Big] dt \wedge dx = \sum_{j,k=1}^{n} \Gamma_{ijk}(u) w_j w_k dt \wedge dx, \tag{2.21}$$

where

$$\Gamma_{ijk}(u) = \frac{1}{2}(\lambda_j(u) - \lambda_k(u)) l_i(u)[\nabla r_k(u) r_j(u) - \nabla r_j(u) r_k(u)]. \tag{2.22}$$

Hence

$$\Gamma_{ijj}(u) \equiv 0, \qquad \forall i, j. \tag{2.23}$$

Lemma 2.1.[3,8] *Suppose that $u = u(t, x)$ is a C^1 solution to system* (1.1), *τ_1 and τ_2 are two C^1 arcs which are never tangent to the i-th characteristic direction, and D is the domain bounded by τ_1, τ_2 and two i-th characteristic curves L_i^- and L_i^+* (*see* Fig. 1). *Then* *we have*

$$\int_{\tau_1} |v_i(dx-\lambda_i(u)dt)| \le \int_{\tau_2} |v_i(dx-\lambda_i(u)dt)| + \iint_D \Big|\sum_{j,k=1}^{n} B_{ijk}(u)v_jw_k\Big| dtdx, \tag{2.24}$$

$$\int_{\tau_1} |w_i(dx-\lambda_i(u)dt)| \le \int_{\tau_2} |w_i(dx-\lambda_i(u)dt)| + \iint_D \Big|\sum_{\substack{j,k=1\\ j\neq k}}^{n} \Gamma_{ijk}w_jw_k\Big| dtdx. \tag{2.25}$$

Fig. 1

Lemma 2.2.[9] *Suppose that* $w=w(t)$ *is a* C^1 *solution to the following ordinary differential equation*

$$\frac{dw}{dt} = a_0(t)w^2 + a_1(t)w + a_2(t) \tag{2.26}$$

on the interval $[0,T]$, *where* $T>0$ *is a given number,* $a_i(t)$ $(i=0,1,2)$ *are continuous functions on* $[0,T]$ *and*

$$a_0(t) \ge 0, \quad \forall t \in [0,T]. \tag{2.27}$$

Let

$$K = \int_0^T |a_2(t)| \exp\Big(-\int_0^t a_1(s)ds\Big)dt. \tag{2.28}$$

If

$$w(0) > K, \tag{2.29}$$

then

$$\int_0^T a_0(t)\exp\Big(\int_0^t a_1(s)ds\Big)dt < (w(0)-K)^{-1}. \tag{2.30}$$

§3. Proof of Theorem 1.1

Without loss of generality, we will prove Theorem 1.1 in normalized coordinates. Moreover, as in [11], we may assume that $\lambda_1(0)>0$.

By (1.6), there exist positive constants δ and δ_0 so small that

$$\lambda_n(u) - \lambda_i(v) \ge 4\delta_0, \quad \forall |u|,|v| \le \delta \quad (i=1,\cdots,n-1), \tag{3.1}$$

$$|\lambda_i(u) - \lambda_i(v)| \le \frac{\delta_0}{2}, \quad \forall |u|,|v| \le \delta \quad (i=1,\cdots,n). \tag{3.2}$$

Let $x=x_n(t)$ be the n-th characteristic passing through the origin:

$$\begin{cases} \dfrac{dx_n(t)}{dt} = \lambda_n(u(t,x_n(t))), \\ t=0: x_n=0, \end{cases} \tag{3.3}$$

and denote

$$x_n^+(t) = (\lambda_n(0) + \delta_0)t. \tag{3.4}$$

For $\delta \geq 0$ suitably small, $x = x_n(t)$ must lie to the left of the straight line $x = x_n^+(t)$.

Fig. 2

For any fixed $T > 0$, let (see Fig. 2)

$$D_n^T = \{(t,x)| \quad 0 \leq t \leq T, \quad x_n(t) \leq x \leq x_n^+(t)\}, \tag{3.5}$$

$$D_+^T = \{(t,x)| \quad 0 \leq t \leq T, \quad x \geq x_n^+(t)\}. \tag{3.6}$$

In what follows we will consider the Cauchy problem on the maximum determinate domain $D_n^T \cup D_+^T$ corresponding to the initial data on $x \geq 0$.

For any fixed $T > 0$, let

$$V(D_+^T) = \max_{i=1,\cdots,n} \|(1+|x|)v_i(t,x)\|_{L^\infty(D_+^T)}, \tag{3.7}$$

$$W(D_+^T) = \max_{i=1,\cdots,n} \|(1+|x|)w_i(t,x)\|_{L^\infty(D_+^T)}, \tag{3.8}$$

$$W_\infty^c(T) = \max_{i=1,\cdots,n-1} \sup_{(t,x)\in D_n^T} \{(1+|x-\lambda_i(0)t|)|w_i(t,x)|\}, \tag{3.9}$$

$$W_1(T) = \sup_{0\leq t\leq T} \int_{x_n(t)}^{x_n^+(t)} |w_n(t,x)|dx, \tag{3.10}$$

$$\widetilde{W}_1(T) = \max_{1\leq i\leq n-1} \int_{c_i} |w_n(t,x)|dt, \tag{3.11}$$

where c_i denotes any given i-th characteristic on D_n^T,

$$U_\infty(T) = \max_{1\leq i\leq n} \sup_{(t,x)\in D_n^T\cup D_+^T} |u_i(t,x)|, \tag{3.12}$$

$$V_\infty(T) = \max_{1\leq i\leq n} \sup_{(t,x)\in D_n^T\cup D_+^T} |v_i(t,x)|. \tag{3.13}$$

For the time being we assume that on any given existence domain $D_n^T \cup D_+^T$ of the C^1 solution $u = u(t,x)$, we have

$$|u(t,x)| \leq \delta. \tag{3.14}$$

At the end of the proof of Lemma 3.3, we will explain that this hypothesis is reasonable.

Lemma 3.1. *For $i = 1, \cdots, n-1$, on the domain D_n^T we have*

$$\kappa_1 t \leq x - \lambda_i(0)t \leq \kappa_2 t, \tag{3.15}$$

$$\kappa_1 x \leq x - \lambda_i(0)t \leq \kappa_2 x, \tag{3.16}$$

where κ_1 and κ_2 are two positive constants independent of T.

Proof. For any given point $(t,x) \in D_n^T$, when δ is suitably small, by the definition of D_n^T and (3.2) it is easy to see that

$$(\lambda_n(0) - \delta_0)t \leq x \leq (\lambda_n(0) + \delta_0)t; \tag{3.17}$$

then, noting (3.2) again, (3.15)–(3.16) follow immediately.

Similarly to Lemma 3.2 in [11], we have

Lemma 3.2. *Suppose that* (1.6) *holds and* $A(u) \in C^2$ *in a neighbourhood of* $u = 0$. *Suppose furthermore that the initial data satisfy* (1.11). *Then there exists* $\varepsilon_0 > 0$ *so small that for any fixed* $\varepsilon \in (0, \varepsilon_0]$, *on any given existence domain* D_+^T *of the* C^1 *solution* $u = u(t,x)$ *to Cauchy problem* (1.1) *and* (1.10), *we have the following uniform a priori estimates*

$$V(D_+^T), \quad W(D_+^T) \leq \kappa_3 \varepsilon, \tag{3.18}$$

where κ_3 *is a positive constant independent of* ε *and* T.

Lemma 3.3. *Under the assumptions of Lemma* 3.2, *there existes* $\varepsilon_0 \geq 0$ *so small that for any fixed* $\varepsilon \in (0, \varepsilon_0]$,*on any given existence domain* $D_n^T \cup D_+^T$ *of the* C^1 *solution* $u = u(t,x)$ *to Cauchy problem* (1.1) *and* (1.10), *there exist positive constants* κ_i $(i = 4, 5, 6)$ *independent of* ε *and* T, *such that we have the following uniform a priori estimates*

$$W_\infty^c(T) \leq \kappa_4 \varepsilon, \tag{3.19}$$

$$\widetilde{W}_1(T), \quad W_1(T) \leq \kappa_5 \varepsilon |\log \varepsilon|, \tag{3.20}$$

$$V_\infty(T), \quad U_\infty(T) \leq \kappa_6 \varepsilon |\log \varepsilon|, \tag{3.21}$$

where

$$T\varepsilon^{2+\alpha} \leq 1. \tag{3.22}$$

Proof. We first estimate $\widetilde{W}_1(T)$.

On D_n^T, we draw arbitrarily an i-th characteristic curve $c_i : x = x_i(t)$ $(1 \leq i \leq n-1)$ which intersects $x = x_n(t)$ and $x = x_n^+(t)$ at points A and B respectively (see Fig.3).

Fig. 3

In order to estimate $\int_{c_i} |w_n(t,x)| dt$, we apply (2.25) in Lemma 2.1 on the domain AOB

(in which one of the n-th characteristic shrinks to the point B) to get

$$\begin{aligned}
&\int_{c_i} |w_n(t,x)||\lambda_i(u(t,x)) - \lambda_n(u(t,x))|dt \\
\le& \int_{OB} |w_n(t,x)||\lambda_n(0) + \delta_0 - \lambda_n(u(t,x))|dt + \iint_{AOB} \Big| \sum_{\substack{j,k=1\\ j\neq k}}^{n} \Gamma_{ijk}(u) w_j w_k \Big| dtdx \\
\le& \int_{OB} |w_n(t,x)||\lambda_n(0) + \delta_0 - \lambda_n(u(t,x))|dt + \iint_{AOB} \Big| \sum_{\substack{j,k=1\\ j\neq k}}^{n-1} \Gamma_{ijk}(u) w_j w_k \Big| dtdx \\
&+ \iint_{AOB} \Big| \sum_{j=1}^{n-1} (\Gamma_{njn}(u) + \Gamma_{nnj}(u)) w_j w_n \Big| dtdx. \qquad (3.23)
\end{aligned}$$

Then, by Lemma 3.1 and Lemma 3.2 and noting (3.1) and (3.14), it is easy to get

$$\begin{aligned}
\int_{c_i} |w_n(t,x)|dt \le& C_1 W(D_+^T) \int_0^T (1+t)^{-1} dt + C_2 \Big\{ (W_\infty^c(T))^2 \int_0^T \int_0^T (1+t)^{-1}(1+\xi)^{-1} dtd\xi \\
&+ W_\infty^c(T) W_1(T) \int_0^T (1+t)^{-1} dt \Big\} \\
\le& C_3 \{\varepsilon + (W_\infty^c(T))^2 \log(1+T) + W_\infty^c(T) W_1(T) \log(1+T)\}, \qquad (3.24)
\end{aligned}$$

henceforth $C_i (i = 1, 2, \cdots)$ will denote positive constants independent of ε and T. Then, noting (3.22), we get

$$\widetilde{W}_1(T) \le C_4 \{\varepsilon + (W_\infty^c(T))^2 |\log\varepsilon| + W_\infty^c(T) W_1(T)\} |\log\varepsilon|. \qquad (3.25)$$

Similarly, we have

$$W_1(T) \le C_5 \{\varepsilon + (W_\infty^c(T))^2 |\log\varepsilon| + W_\infty^c(T) W_1(T)\} |\log\varepsilon|. \qquad (3.26)$$

We next estimate $W_\infty^c(T)$.

Passing through any given point $(t,x) \in D_n^T$, we draw the i-th characteristic $\xi = x_i(s;t,x)$ $(i \neq n)$ which intersects $x = x_n^+(t)$ at a point (t_0, y) (see Fig. 4).

Noting (3.2), for $\delta \ge 0$ suitably small, we have

$$\frac{x-y}{t-t_0} \le \lambda_i(0) + \frac{\delta_0}{2}. \qquad (3.27)$$

Then, noting (3.17), it is easy to get

$$C_6 t \le t_0 \le t. \qquad (3.28)$$

Integrating (2.17) along $\xi = x_i(s;t,x)$ from t_0 to t yields

$$w_i(t,x) = w_i(t_0,y) + \int_{t_0}^t \sum_{j,k=1}^n \gamma_{ijk}(u) w_j w_k (s, x_i(s;t,x)) ds. \qquad (3.29)$$

Fig.4

By Lemma 3.2 and noting (3.28), we have

$$|w_i(t_0,y)| \le \kappa_1\varepsilon(1+y)^{-1} \le C_7\varepsilon(1+t_0)^{-1} \le C_8\varepsilon(1+t)^{-1}. \tag{3.30}$$

Then, noting (2.19) and (3.14) and using Lemma 3.1 and (3.28), it follows from (3.29) that

$$|w_i(t,x)| \le \{C_8\varepsilon + C_9(W_\infty^c(T))^2\log(1+T) + C_{10}W_\infty^c(T)\widetilde{W}_1(T)\}(1+t)^{-1}. \tag{3.31}$$

Thus, using Lemma 3.1 and noting (3.22), we get

$$\begin{aligned} W_\infty^c(T) &\le C_{11}\{\varepsilon + (W_\infty^c(T))^2\log(1+T) + W_\infty^c(T)\widetilde{W}_1(T)\} \\ &\le C_{12}\{\varepsilon + (W_\infty^c(T))^2|\log\varepsilon| + W_\infty^c(T)\widetilde{W}_1(T)\}. \end{aligned} \tag{3.32}$$

We now prove (3.19) and (3.20).

Noting (1.11) and (3.14), by (2.4) it is easy to see that

$$|w_i(0,x)| \le C_{13}\varepsilon(1+|x|)^{-1} \quad (i=1,\cdots,n). \tag{3.33}$$

Then, we have

$$W_\infty^c(0) \le C_{13}\varepsilon, \tag{3.34}$$

$$\widetilde{W}_1(0) = W_1(0) = 0. \tag{3.35}$$

By continuity, there exists a positive number τ_0 such that there exist positive constants κ_4 and κ_5 independent of ε, such that (3.19)–(3.20) hold on $0 \le T \le \tau_0$. Thus, in order to prove (3.19)–(3.20), it suffices to show that we can choose κ_4 and κ_5 in such a way that for any fixed $T_0 \in [0,T]$ (in which T satisfies (3.22)), when $\varepsilon_0 \ge 0$ is suitably small, if

$$W_\infty^c(T_0) \le 2\kappa_4\varepsilon, \tag{3.36}$$

$$\widetilde{W}_1(T_0), \quad W_1(T_0) \le 2\kappa_5\varepsilon|\log\varepsilon|, \tag{3.37}$$

then

$$W_\infty^c(T_0) \le \kappa_4\varepsilon, \tag{3.38}$$

$$\widetilde{W}_1(T_0), \quad W_1(T_0) \le \kappa_5\varepsilon|\log\varepsilon|. \tag{3.39}$$

For this purpose, substituting (3.36), (3.37) into the right-hand side of (3.25), (3.26) and (3.32) (in which we take $T=T_0$), we get

$$\widetilde{W}_1(T_0) \le C_4\varepsilon|\log\varepsilon|\{1+4(\kappa_4^2+\kappa_4\kappa_5)\varepsilon|\log\varepsilon|\}, \tag{3.40}$$

$$W_1(T_0) \le C_5\varepsilon|\log\varepsilon|\{1+4(\kappa_4^2+\kappa_4\kappa_5)\varepsilon|\log\varepsilon|\}, \tag{3.41}$$

$$W_\infty^c(T_0) \le C_{12}\varepsilon\{1+4(\kappa_4^2+\kappa_4\kappa_5)\varepsilon|\log\varepsilon|\}. \tag{3.42}$$

Then, it is easy to see that, for $\varepsilon_0 \geq 0$ suitably small, we have

$$\widetilde{W}_1(T_0) \leq 2C_4\varepsilon|\log\varepsilon|, \tag{3.43}$$

$$W_1(T_0) \leq 2C_4\varepsilon|\log\varepsilon|, \tag{3.44}$$

$$W_\infty^C(T_0) \leq 2C_{12}\varepsilon. \tag{3.45}$$

Hence, taking

$$\kappa_4 \geq 2C_{12} \quad \text{and} \quad \kappa_5 \geq 2\max(C_4, C_5),$$

we get (3.38), (3.39). This proves (3.19), (3.20).

We now prove (3.21).

By (2.3) and (3.14) it is easy to see that $V_\infty(T)$ is equivalent to $U_\infty(T)$.

For any given point $(t,x) \in D_n^T$, we have (see Fig. 5)

$$u(t,x) = u(t,x_0) - \int_x^{x_0} u_\xi(t,\xi)d\xi, \tag{3.46}$$

where (t,x_0) is located on the straight line $x = x_n^+(t)$.

Fig. 5

Noting (3.14) and (3.18)–(3.20) and using (2.5), (2.6) and Lemma 3.1, we have

$$\begin{aligned}|u(t,x)| &= \sum_{k=1}^n |v_k(t,x_0)| + \sum_{k=1}^n \int_x^{x_0} |w_k(t,\xi)|d\xi \\ &\leq C_{13}\{V(D_+^T) + W_\infty^c(T)\log(1+T) + W_1(T)\} \leq C_{14}\varepsilon|\log\varepsilon|.\end{aligned} \tag{3.47}$$

Then, taking $\kappa_6 \geq C_{14}$ and noting Lemma 3.2, we get (3.21).

Finally, we point out that when $\varepsilon_0 > 0$ is suitably small, we have

$$U_\infty(T) \leq \kappa_4\varepsilon|\log\varepsilon| \leq \frac{1}{2}\delta.$$

This implies the validity of hypothesis (3.14). The proof of Lemma 3.3 is finished.

Let

$$V_\infty^c(T) = \max_{1\leq i\leq n-1} \sup_{(t,x)\in D_n^T} \{(1+|x-\lambda_i(0)t|)|v_i(t,x)|\}, \tag{3.48}$$

$$U_\infty^c(T) = \max_{1\leq i\leq n-1} \sup_{(t,x)\in D_n^T} \{(1+|x-\lambda_i(0)t|)|u_i(t,x)|\}, \tag{3.49}$$

$$V_1(T) = \sup_{0\leq t\leq T} \int_{x_n(t)}^{x_n^+(t)} |v_n(t,x)|dx, \tag{3.50}$$

$$\widetilde{V}_1(T) = \max_{1\leq i\leq n-1} \int_{c_i} |v_n(t,x)|dt, \tag{3.51}$$

where c_i denotes any given i-th characteristic on D_n^T,

$$W_\infty(T) = \max_{1\le i\le n} \sup_{(t,x)\in D_n^T\cup D_+^T} |w_i(t,x)|. \tag{3.52}$$

Lemma 3.4. *Under the assumptions of Theorem* 1.1, *there exists* $\varepsilon_0 > 0$ *so small that for any fixed* $\varepsilon \in (0,\varepsilon_0]$, *on any given existence domain* $D_n^T \cup D_+^T$ *of the* C^1 *solution* $u = u(t,x)$ *to Cauchy problem* (1.1) *and* (1.10) *there exist positive constants* κ_i $(i = 7,\cdots,10)$ *independent of* ε *and* T, *such that the following uniform a priori estimates hold:*

$$V_\infty^c(T) \le \kappa_7\varepsilon, \tag{3.53}$$

$$V_1(T), \widetilde{V}_1(T) \le \kappa_8\varepsilon|\log\varepsilon| + \kappa_9(\varepsilon|\log\varepsilon|)^{2+\alpha}T, \tag{3.54}$$

where T *satisfies* (3.22),

$$W_\infty(T) \le \kappa_{10}\varepsilon, \tag{3.55}$$

where

$$T\varepsilon^{\upsilon+\alpha} \le 1 \quad (\upsilon \in (0,1) \quad \textit{is} \quad \textit{a} \quad \textit{constant}). \tag{3.56}$$

Proof. We first prove that

$$U_\infty^c(T) \le C_{15}V_\infty^c(T). \tag{3.57}$$

For any given point $(t,x) \in D_n^T$, by Hadamard's formula and noting that (2.1) holds in normalized coordinates, we have

$$\begin{aligned} u_i(t,x) &= \sum_{k=1}^{n} v_k r_k^T(u)e_i = \sum_{k=1}^{n-1} v_k r_k^T(u)e_i \\ &\quad + v_n\Big(\int_0^1 \sum_{l\ne n} \frac{\partial r_n^T}{\partial u_l}(\tau u_1,\cdots,\tau u_{n-1},u_n)u_l d\tau\Big)e_i \quad (i = 1,\cdots,n-1). \end{aligned} \tag{3.58}$$

Then, using Lemma 3.1 and noting (3.14) and (3.21), we get

$$U_\infty^c(T) \le C_{16}\{V_\infty^c(T) + \varepsilon|\log\varepsilon|U_\infty^c(T)\}. \tag{3.59}$$

Hence, when $\varepsilon_0 > 0$ is suitably small, (3.57) holds.

Similarly to the proof of Lemma 3.3, we first estimate $\widetilde{V}_1(T)$.

Similarly to (3.23), we apply (2.24) in Lemma 2.1 on the domain AOB (see Fig.3) to get

$$\begin{aligned} &\int_{c_i} |v_n(t,x)||\lambda_i(u(t,x)) - \lambda_n(u(t,x))|dt \\ &\le \int_{OB} |v_n(t,x)||\lambda_n(0) + \delta_0 - \lambda_n(u(t,x))|dt + \iint_{AOB} \Big|\sum_{j,k=1}^{n} B_{njk}(u)v_jw_k\Big|dtdx \\ &\le \int_{OB} |v_n(t,x)||\lambda_n(0) + \delta_0 - \lambda_n(u(t,x))|dt + \iint_{AOB} \Big|\sum_{\substack{j,k=1\\ j\ne k\\ or j=k\ne n}}^{n} B_{njk}(u)v_jw_k\Big|dtdx \\ &\quad + \iint_{AOB} |(B_{nnn}(u) - B_{nnn}(u_ne_n))v_nw_n|dtdx \\ &\quad + \iint_{AOB} |B_{nnn}(u_ne_n)v_nw_n|dtdx \quad (i = 1,\cdots,n-1). \end{aligned} \tag{3.60}$$

By (2.15) and (1.8), (1.9) and noting that $u = (u_1, \cdots, u_n)$ are normalized coordinates, we have

$$|B_{nnn}(u_n e_n)| = \Big|\frac{\partial \lambda_n(0, \cdots, 0, u_n)}{\partial u_n}\Big| \le C_{17}|u_n|^{\alpha}. \tag{3.61}$$

Then, noting Lemma 3.1 and (3.57) and using Hadamard's formula, when $\varepsilon_0 > 0$ is small enough, similarly to (3.24) we have

$$\begin{aligned}&\int_{c_i} |v_n(t, x)| dt\\ &\le C_{18}\{V(D_+^T)\log(1 + T) + W_\infty^c(T)V_\infty^c(T)(\log(1 + T))^2\\ &\quad + W_\infty^c(T)V_1(T)\log(1 + T) + W_1(T)V_\infty^c(T)\log(1 + T) + (V_\infty(T))^{1+\alpha}W_1(T)T\}.\end{aligned} \tag{3.62}$$

Hence, using Lemmas 3.2–3.3 and noting (3.22), we get

$$\widetilde{V}_1(T) \le C_{19}\varepsilon|\log\varepsilon|\{1 + |\log\varepsilon|V_\infty^c(T) + V_1(T) + (\varepsilon|\log\varepsilon|)^{1+\alpha}T\}. \tag{3.63}$$

Similarly, we have

$$V_1(T) \le C_{20}\varepsilon|\log\varepsilon|\{1 + |\log\varepsilon|V_\infty^c(T) + V_1(T) + (\varepsilon|\log\varepsilon|)^{1+\alpha}T\}. \tag{3.64}$$

Moreover, integrating (2.8) along the i-th characteristic $\xi = x_i(s; t, x)$ in Fig. 4 from t_0 to t, we obtain

$$v_i(t, x) = v_i(t_0, y) + \int_{t_0}^{t} \sum_{j,k=1}^{n} \beta_{ijk}(u)v_j w_k(s, x_i(s; t, x))ds. \tag{3.65}$$

Noting (2.11) and Lemmas 3.1–3.3, similarly to (3.32) we get

$$\begin{aligned}V_\infty^c(T) &\le C_{21}\{\varepsilon + W_\infty^c(T)V_\infty^c(T)\log(1 + T) + V_\infty^c(T)\widetilde{W}_1(T) + W_\infty^c(T)\widetilde{V}_1(T)\}\\ &\le C_{22}\varepsilon\{1 + |\log\varepsilon|V_\infty^c(T) + \widetilde{V}_1(T)\}.\end{aligned} \tag{3.66}$$

Thus, repeating the procedure of proving (3.19), (3.20), (3.53), (3.54) follow from (3.63), (3.64) and (3.66).

We next prove (3.55).

Noting that, when $(t, x) \in D_+^T$, we have

$$|w_i(t, x)| \le W(D_+^T) \quad (i = 1, \cdots, n), \tag{3.67}$$

while, when $(t, x) \in D_n^T$, we have

$$|w_i(t, x)| \le W_\infty^c(T) \quad (i = 1, \cdots, n - 1), \tag{3.68}$$

by Lemmas 3.2–3.3, we need only to estimate $\sup\limits_{(t,x)\in D_n^T} |w_n(t, x)|$.

Integrating (2.17) along the n-th characteristic $\xi = x_n(s; t, x)$ passing through any given point $(t, x) \in D_n^T$, we get

$$\begin{aligned}w_n(t, x) = w_n(t_0, y) &+ \int_{t_0}^{t} \sum_{\substack{j,k=1\\ j\ne k}}^{n} \gamma_{njk}(u)w_j w_k ds + \int_{t_0}^{t}(\gamma_{nnn}(u) - \gamma_{nnn}(u_n e_n))w_n^2 ds\\ &+ \int_{t_0}^{t} \gamma_{nnn}(u_n e_n)w_n^2 ds,\end{aligned} \tag{3.69}$$

where (t_0, y) is the intersection point of $\xi = x_n(s; t, x)$ with $x = x_n^+(t)$.

Noting (2.20), similarly to (3.62) we have

$$|\gamma_{nnn}(u_n e_n)| \le C_{23}|u_n|^{\alpha}. \tag{3.70}$$

Then, using Lemmas 3.1–3.3 and noting Hadamard's formula, we get

$$\begin{aligned}|w_n(t,x)| &\le C_{24}\{W(D_+^T) + (W_\infty^c(T))^2 + W_\infty^c(T)W_\infty(T)\log(1+T)\\ &\quad + V_\infty^c(T)(W_\infty(T))^2\log(1+T) + (V_\infty(T))^\alpha (W_\infty(T))^2 T\}\\ &\le C_{25}\{\varepsilon + \varepsilon|\log\varepsilon| W_\infty(T) + \varepsilon|\log\varepsilon|(W_\infty(T))^2 + (\varepsilon|\log\varepsilon|)^\alpha (W_\infty(T))^2 T\},\end{aligned} \tag{3.71}$$

where T satisfies (3.22).

Thus, by (3.67), (3.68) and using Lemmas 3.2–3.3, we get

$$W_\infty(T) \le C_{26}\{\varepsilon + \varepsilon|\log\varepsilon|W_\infty(T) + (\varepsilon|\log\varepsilon| + (\varepsilon|\log\varepsilon|)^\alpha T)(W_\infty(T))^2\}. \tag{3.72}$$

Then, noting (3.56) and by continuity, completely repeating the procedure of proving (3.19), (3.20) gives (3.55). The proof of Lemma 3.4 is finished.

Finally, we prove Theorem 1.1.

First of all, noting that $u = (u_1, \cdots, u_n)$ are supposed to be normalized variables, the corresponding initial condition should be written as (cf. [11])

$$t = 0 : u = \varepsilon\psi(x) + O(\varepsilon^2). \tag{3.73}$$

By (1.11), it is easy to see that there exists $x_0 \in \mathbb{R}$ such that

$$\begin{aligned}M_0 &= \Big(-\frac{1}{\alpha!}\frac{\partial^{\alpha+1}\lambda_n}{\partial {u_n}^{\alpha+1}}(0)(l_n(0)\psi(x_0))^\alpha l_n(0)\psi'(x_0)\Big)^{-1}\\ &= \Big(\sup_{x\in\mathbb{R}}\Big\{-\frac{1}{\alpha!}\frac{d^{\alpha+1}\lambda_n(u^{(n)}(s))}{ds^{\alpha+1}}\Big|_{s=0}(l_n(0)\psi(x))^\alpha l_n(0)\psi'(x)\Big\}\Big)^{-1}.\end{aligned} \tag{3.74}$$

Without loss of generality, we may assume that $x_0 > 0$ (otherwise, we take a translation of the t-axis).

Let

$$a = -\frac{1}{\alpha!}\frac{\partial^{\alpha+1}\lambda_n}{\partial u_n^{\alpha+1}}(0). \tag{3.75}$$

By (1.11) and (3.74), we have

$$a(l_n(0)\psi(x_0))^\alpha l_n(0)\psi'(x_0) > 0. \tag{3.76}$$

Without loss of generality, we assume that

$$a(l_n(0)\psi(x_0))^\alpha > 0 \quad \text{and} \quad l_n(0)\psi'(x_0) > 0. \tag{3.77}$$

By Lemmas 3.2-3.4, when $\varepsilon > 0$ is suitably small, the Cauchy problem (1.1) and (1.10) admits a unique C^1 solution $u = u(t,x)$ on the maximum determinate domain $D_n^T \cup D_+^T$, where

$$T \overset{\triangle}{=} \varepsilon^{-(\upsilon+\alpha)} \le \widetilde{T}(\varepsilon) - 1 \overset{\triangle}{=} \overline{T}. \tag{3.78}$$

For the time being we suppose that

$$\widetilde{T}(\varepsilon)\varepsilon^{2+\alpha} \le 1. \tag{3.79}$$

At the end of the proof of Theorem 1.1 we will show that this hypothesis is reasonable.

Let $x = x_n(t, x_0)$ be the n-th characteristic passing through the point $(0, x_0)$. By the definition of D_n^T, it is easy to see that this characteristic must enter D_n^T at a finite time $t_0 > 0$ and stay in D_n^T for $t > t_0$ (cf. [9] or [11]).

On the existence domain $D_n^T \cup D_+^T$ of the C^1 solution $u = u(t,x)$, by (2.17), along $x = x_n(t,x_0)$ we have

$$\frac{dw_n}{d_n t} = a_0(t)w_n^2 + a_1(t)w_n + a_2(t), \tag{3.80}$$

where

$$\begin{cases} a_0(t) = \gamma_{nnn}(u), \\ a_1(t) = \displaystyle\sum_{j=1}^{n-1}(\gamma_{nnj}(u) + \gamma_{njn}(u))w_j, \\ a_2(t) = \displaystyle\sum_{\substack{j,k=1 \\ j\neq k}}^{n-1} \gamma_{njk}(u)w_j w_k. \end{cases} \tag{3.81}$$

For $\varepsilon_0 > 0$ suitably small, by (3.78) we have

$$t_0 < T_0 \triangleq \varepsilon^{-\alpha} < T. \tag{3.82}$$

Noting (2.10) and (3.78), (3.79) and using Lemmas 3.2–3.4, integrating (2.8) along $x = x_n(t,x_0)$ gives

$$\begin{aligned} |v_n(t,x_n(t,x_0)) - v_n(0,x_0)| &\le \Big|\int_0^t \sum_{\substack{j,k=1 \\ k\neq n}}^{n} \beta_{njk}(u)v_j w_k(s,x_n(s,x_0))ds\Big| \\ &\le C_{27}\{V(D_+^t)W(D_+^t) + V_\infty^c(t)W_\infty^c(t) + V_\infty(t)W_\infty^c(t)\log(1+t)\} \\ &\le C_{28}\varepsilon^2(\log\varepsilon)^2, \quad \forall t \in [0,\overline{T}]. \end{aligned} \tag{3.83}$$

Noting that, on the existence domain $D_n^T \cup D_+^T$ of the C^1 solution $u = u(t,x)$, we have

$$|u_n(t,x) - v_n(t,x)| = \Big|\sum_{k=1}^{n} v_k(r_k(u) - r_k(u_k e_k))^T e_n\Big| \le C_{29}\varepsilon^2(\log\varepsilon)^2, \tag{3.84}$$

and noting (3.73) we have

$$|u_n(t,x_n(t,x_0)) - \varepsilon l_n(0)\psi(x_0)| \le C_{30}\varepsilon^2(\log\varepsilon)^2, \quad \forall t \in [0,\overline{T}]. \tag{3.85}$$

On the other hand, on the existence domain $(D_n^T \cup D_+^T) \cap \{(t,x)| \quad T_0 \le t \le \overline{T}\}$ of the C^1 solution $u = u(t,x)$, by Hadamard's formula and Lemma 3.4, along $x = x_n(t,x_0)$ (which stays in D_n^T now) we have

$$|\gamma_{nnn}(u) - \gamma_{nnn}(u_n e_n)| \le C_{31}(1+t)^{-1}V_\infty^c(\overline{T}) \le C_{32}\varepsilon^{1+\alpha}. \tag{3.86}$$

Moreover, by (2.20) we have

$$\gamma_{nnn}(u_n e_n) = a(\varepsilon l_n(0)\psi(x_0))^\alpha + a[(u_n(t,x_n(t,x_0)))^\alpha - (\varepsilon l_n(0)\psi(x_0))^\alpha] + O(|u_n|^{1+\alpha}), \tag{3.87}$$

where a is given by (3.75). Then, using (3.21), it follows from (3.85)–(3.87) that

$$a_0(t) = \gamma_{nnn}(u) = a(\varepsilon l_n(0)\psi(x_0))^\alpha + O(\varepsilon^{1+\alpha}|\log\varepsilon|^{1+\alpha}), \forall t \in [T_0,\overline{T}]. \tag{3.88}$$

Therefore, for $\varepsilon_0 \ge 0$ suitably small, we get

$$a_0(t) \ge \frac{1}{2}a(\varepsilon l_n(0)\psi(x_0))^\alpha, \quad \forall t \in [T_0,\overline{T}]. \tag{3.89}$$

Noting (3.21) and (3.55), similarly to (3.84) we have

$$\begin{aligned}\Big|\frac{\partial u_n}{\partial x}(t,x)-w_n(t,x)\Big| &= \Big|\sum_{k=1}^{n} w_k(r_k(u)-r_k(u_ke_k))^T e_n\Big| \\ &\le C_{33}\varepsilon^2|\log\varepsilon|, \quad \forall (t,x)\in D_n^{T_0}\cup D_+^{T_0}. \end{aligned} \tag{3.90}$$

Similarly to (3.83), using Lemmas 3.1–3.4 and noting (3.82) we have

$$\begin{aligned}|w_n(t,x_n(t,x_0))-w_n(0,x_0)| &\le C_{34}\{(W(D_+^{T_0}))^2+W_\infty(T_0)W_\infty^c(T_0)\log(1+T_0) \\ &\quad +(W_\infty(T_0))^2V_\infty^c(T_0)\log(1+T_0)+(V_\infty(T_0))^\alpha(W_\infty(T_0))^2T_0\} \\ &\le C_{35}\varepsilon^2|\log\varepsilon|^{\max(1,\alpha)}, \quad \forall t\in[0,T_0]. \end{aligned} \tag{3.91}$$

Then, noting (3.73), it follows from (3.90), (3.91) that

$$w_n(T_0,x_n(T_0,x_0))=\varepsilon l_n(0)\psi'(x_0)+O(\varepsilon^2|\log\varepsilon|^{\max(1,\alpha)}). \tag{3.92}$$

Using Lemma 3.1 and Lemma 3.3, it is easy to see that

$$\int_{T_0}^{\overline{T}}|a_1(t)|dt\le C_{36}W_\infty^c(\overline{T})\int_{T_0}^{\overline{T}}(1+t)^{-1}dt\le C_{37}\varepsilon|\log\varepsilon|, \tag{3.93}$$

$$\int_{T_0}^{\overline{T}}|a_2(t)|dt\le C_{38}(W_\infty^c(\overline{T}))^2\int_{T_0}^{\overline{T}}(1+t)^{-2}dt\le C_{39}\varepsilon^2. \tag{3.94}$$

Let

$$K=\int_{T_0}^{\overline{T}}|a_2(t)|\exp\Big(-\int_{T_0}^{t}a_1(s)ds\Big)dt.$$

We have

$$K\le\int_{T_0}^{\overline{T}}|a_2(t)|dt\cdot\exp\Big(\int_{T_0}^{T}|a_1(t)|dt\Big)\le C_{40}\varepsilon^2. \tag{3.95}$$

Hence, for $\varepsilon_0>0$ suitably small, noting (3.77) we have

$$w_n(T_0,x_n(T_0,x_0))>K. \tag{3.96}$$

Applying Lemma 2.2 to the initial value problem for ordinary differential equation (3.80) with the initial data

$$t=T_0: \quad w_n=w_n(T_0,x_n(T_0,x_0)), \tag{3.97}$$

we obtain

$$\begin{aligned}\int_{T_0}^{\overline{T}}a_0(t)dt\cdot\exp\Big(-\int_{T_0}^{\overline{T}}|a_1(t)|dt\Big) &\le \int_{T_0}^{\overline{T}}a_0(t)\exp\Big(\int_{T_0}^{t}a_1(s)ds\Big)dt \\ &\le (w_n(T_0,x_n(T_0,x_0))-K)^{-1}. \end{aligned} \tag{3.98}$$

Thus, noting (3.78), (3.88), (3.92), (3.93) and (3.95), (3.96), it is easy to get that there exists a positive constant C independent of ε and T, such that

$$\widetilde{T}(\varepsilon)\le C\varepsilon^{-(1+\alpha)}, \tag{3.99}$$

and $C>M_0$.

By (3.99), when $\varepsilon_0>0$ is suitably small, we get

$$\widetilde{T}(\varepsilon)\varepsilon^{2+\alpha}\le C\varepsilon<1. \tag{3.100}$$

This proves (3.79). The proof of Theorem 1.1 is complete.

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