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THERMAL EQUILIBRIUM AND KMS CONDITION AT THE PLANCK SCALE

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Abstract

Considering the expected thermal equilibrium characterizing the physics at the Planck scale, it is here stated, for the first time, that, as a system, the space-time at the Planck scale must be considered as subject to the Kubo-Martin-Schwinger (KMS) condition. Consequently, in the interior of the KMS strip, i.e. from the scale $\mathcal{B} = 0$ to the scale $\mathcal{B} = \ell_{Planck}$, the fourth coordinate g_{44} must be considered as complex, the two real poles being $\mathcal{B} = 0$ and $\mathcal{B} = \ell_{planck}$. This means that within the limits of the KMS strip, the Lorentzian and the Euclidean metric are in a "quantum superposition state" (or coupled), this entailing a "unification" (or coupling) between the topological (Euclidean) and the physical (Lorentzian) states of space-time.

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§1. Introduction

Much have been recently said regarding the physical state of the universe at the vicinity of the Planck scale. String theory, non commutative geometry, supergravity or quantum gravity have contributed, independently of each other, to establish on solid basis the data of a "transition phase" in the physical content and the geometric structures of the (pre)space-time at such a scale. But what is the nature of this dramatic change? In the present paper, we propose a novel approach, based on one of the most natural and realistic physical condition predicted by the Standard Model for the (pre)universe. In agreement with some well-established results[1−4] and, more recently, the approach of C. Kounnas et al.[5,6], we argue that at the Planck scale, the "space-time system" is in a thermodynamical equilibrium state[2]. This notion of equilibrium state at the Planck scale has been recently stated with some new interesting arguments in [7–9]. As a consequence, according to [10], we suggest hereafter that the (pre)universe should be considered as subject to the Kubo-Martin-Schwinger (KMS) condition at such a scale[11]. Surprisingly, the well known KMS and modular theories[12] have never been applied to the study of the metric properties in the context of quantum cosmology; however, the KMS condition might have dramatic consequences onto Planck scale physics. For which reasons ? Because, when applied to quantum space-time, the KMS statistics are such that, within the limits of the "KMS strip" (i.e.

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between the scale zero and the Planck scale), the time like direction of the system should be considered as complex: $t \mapsto t_c = t_r + it_i$. We have showed in [10] that at the scale zero, the theory is projected onto the pure imaginary boundary $t \mapsto t_c = it_i$ of the KMS strip. Namely, there exists, around $\beta \to 0$, a non-trivial topological limit of quantum field theory, dual to the usual topological limit associated with $\beta \to \infty$ in the partition function (1.2). Such a topological state of the (pre)space-time can be described by the topological invariant

$$\underset{\beta\to 0}{Z} = T_r(-1)^s \tag{1.1}$$

given by the $\beta \to 0$ limit of the partition function (1.2). S is here the instanton number of the theory. We have demonstrated[10] that this topological index is isomorphic to the first Donaldson Invariant[13]. This suggests that at zero scale, the observables O_i must be replaced by the homology cycles $H_i \subset \mathcal{M}^{(k)}_{\text{mod}}$ in the moduli space of gravitational instantons[14]. We get then a deep correspondence—a symmetry of duality[14–16], between physical theory and topological field theory. Conversely, on the (classical) infrared limit$\beta \geqslant \ell_{\text{planck}}$, the imaginary component cancels and the time like direction becomes pure real $t \mapsto t_c = t_r$ So, within the limits of the KMS strip (i.e. for $0 < \beta < \ell_{\text{planck}}$) the Lorentzian and the Euclidean metric are in a "quantum superposition state" (or coupled). Such a superposition state, described in detail in [10], is entailing a "unification" (or coupling) between the topological (Euclidean) and the physical (Lorentzian) states of space-time. The (Lorentzian / Euclidean) states of the metric $g_{\mu\nu}$ are given by the partition function

$$Z = T_r(-1)^s e^{-\beta H} \tag{1.2}$$

with $\beta = \frac{1}{kT}$, as usual n being the "metric number" of the theory. To avoid any difficulty of interpretation, let us remark that the scale parameter β admits two possible interpretations: (i) either $\beta = \frac{\hbar}{kT}$ can be seen as a real time parameter of a Lorentaian $(3+1)$-dimensional theory[17] or (ii) $\beta = it$ can be interpreted as the fourth space-like direction of a Riemannian 4-dimensional Euclidean theory (e.g. $\beta = \ell_{planck}$) (see [18]). In the second case, β is a periodic imaginary time interval[5,6]. Considering the hypothesis of holomorphicity formulated in Section 4, we use here the two interpretations. In such a context, the KMS state of the (pre)space-time may be considered as a transition phase from the Euclidean topological phase ($\mathcal{B} = 0$) to the Lorentzian physical phase, beyond the Planck scale[10–19].

The present article is organized as follows. In Section 2, we recall that at the Planck scale, the "space-time system" should most likely be considered as being in a thermodynamical equilibrium state. In Section 3, we show that, as a natural consequence of this equilibrium state, the space-time must be considered as subject to the KMS condition. In Section 4, we suggest that, considering the KMS properties, the time-like direction g_{44} of the metric should be seen as complex $t_c = t_r + it_i$ within the limits of the KMS strip. In Section 5, we discuss briefly the transition from imaginary time t_i to real time t_r in terms of KMS breaking beyond the Planck scale.

§2. Thermodynamical Equilibrium of the Space-time at the Planck Scale

It is well known that at the Planck scale, one must expect a thermodynamical "phase transition", closely related to (i) the existence of an upper limit in the temperature growth—the Hagedorn temperature[1]—and (ii) the equilibrium state characterizing, most likely, the (pre)space-time at such a scale. In such a context, the seminal investigations of K. Huang and S. Weinberg[2,3] and later of several others (see [20]) have renewed the initial idea of

Hagedorn concerning the existence, at very high temperature, of a limit restricting the growth of states excitation. Already some time ago, J. J. Atik and E. Witten have shown the existence of a Hagedorn limit around the Planck scale in string theory[1]. The reason is that, as recently recalled by C. Kounnas[6], in the context of $N = 4$ superstrings, at finite temperature, the partition function $Z(\beta)$ and the mean energy $U(\beta)$ develop some power pole singularities in $\beta \equiv T^{-1}$ since the density of states of a system grows exponentially with the energy E :

$$Z(\beta) = \int dE\rho(E)e^{-\beta E} \sim \frac{1}{(\beta - b)^{(k-1)}},$$
$$U(\beta) = \frac{\partial}{\partial\beta} lnZ \sim (k-1)\frac{1}{\beta - b}. \tag{2.1}$$

Clearly, Equauion (2.1) exhibits the existence, around the Planck scale, of a critical temperature $T_H = b^{-1}$, where the (pre)space-time system must be viewed as in a thermodynamical equilibrium state. Indeed, *a(t)* being the cosmological scale factor, the global temperature T follows the well-known law

$$T(t) \approx T_p \frac{a(t_p)}{a(t)}$$

and around the Planck time, T is reaching the critical limit $T_p \approx \frac{E_p}{k_B} \approx (\frac{\hbar C^5}{G})^{\frac{1}{2}} k_B^{-1} \approx 1,4.10^{32}K$. In fact, it is currently admitted in string theory that, before the inflationary phase, the ratio between the interaction rate(Γ) of the initial fields and the (pre)space-time expansion (H) is $\frac{\Gamma}{H} << 1$, so that the H system can reasonably be considered in equilibrium state. This has been established a long time ago within some precursor works of (once again) S. Weinberg[2], E. Witten[l] and several others. It has recently been showed by C. Kounnas et al. in the superstrings context[5]. However, this natural notion of equilibrium, when viewed as a global gauge condition, has dramatic consequences regarding physics at the Planck scale. Among those consequences, unquestionably the most important one is that the (pre)space-time at the Planck scale must be considered as subject to the famous "KMS condition", a very special and interesting physical state that we are now going to describe.

§3. The (Pre)space-time in KMS State at the Planck Scale

Let us first recall on mathematical basis what an equilibrium state is.

Definition 3.1. *H being an autoadjoint operator and $\aleph$ the Hilbert space of a finite system, the equilibrium state ω of this system is described by the Gibbs condition $\omega(A) = \frac{T_{r_\aleph}(e^{-\beta H}A)}{T_{r_\aleph}(e^{-\beta H})}$ and satisfies the KMS condition.*

This well-known definition has been proposed for the first time in [11]. Now, it is usual (and natural) to oppose the notion of equilibrium to the one of evolution of a system. In fact, the famous Tomita-Takesaki modular theory has established that the "intrinsic" dynamic of a quantum system corresponds, in a unique manner, to the strongly continuous one parameter * - automorphism group a_t of some von Neumann C^*-algebra $\mathbf{A}$ (see [12])

$$a_t(A) = e^{iHt}Ae^{-iHt}. \tag{3.1}$$

This one parameter group describes the time evolution of the observables of the system and corresponds to the well-known Heisenberg algebra. Nothing is mysterious at this stage. However, we are here brought to find the remarkable discovery of Takesaki and Winnink, connecting the evolution group $a_t(A)$ of a system (more precisely the modular group $M = \Delta^{it}M\Delta^{-it}$) with the equilibrium state $\varphi(A) = \frac{T_r(Ae^{-\beta H})}{T_r(e^{-\beta H})}$ of this system[11,12]. With this more or less unexpected relation between evolution $a_t(A)$ and equilibrium $\varphi(A)$, we now meet the famous "KMS condition". More exactly, in the frame of quantum statistical mechanics, the KMS condition provides a rigorous mathematical formulation about the coexistence of different possible equilibrium states at the *same* given temperature T.

Let us recall now how such a relation between equilibrium state and evolution of a system is realized by the KMS condition. It has been clearly established[11] that a state ω on the C*-algebra **A** and the continuous one parameter automorphism group of A at the temperature $\mathcal{B} = 1/kT$ verify the KMS condition if, for any pair A, B of the $*$-sub-algebra of **A**, a_t-invariant and of dense norm, there exists a $f(t_c)$ function holomorphic in the strip $\{t_c = t + i\beta \in \mathbb{C}, Imt_c \in [0, \mathcal{B}]\}$ such that:

(i) $f(t) = \varphi(Aa_t(B))$,

(ii) $f(t + i\beta) = \varphi(a_t(B)A), \forall i \in \mathbb{R}$. (3.2)

Moreover, a state φ on the C^*-algebra **A** is a separator if the given algebraic representation is a von Neumann algebra W* endowed with a separator and a cyclic vector. The sets

$$I_l = \{A \in \mathbf{A} : \varphi(\mathbf{A}^*\mathbf{A}) = 0\},$$
$$I_r = \{A \in \mathbf{A} : \varphi(\mathbf{A}\mathbf{A}) = 0\}$$

are forming a left and right ideal in **A**. For any KMS state, we have $I_l = I_r$.

The above definition expresses the bijective relation between equilibrium state, holomorphic state of the measure parameters and KMS state.

Now, considering the general properties raised by the KMS condition, if we admit that around the Planck scale, the (pre)space-time system is in a thermal equilibrium state, then we are also bound to admit that this system is in a KMS state. Indeed, it has been shown a long time ago[11] that if a state of a system ω satisfies the equilibrium condition $\int_{-\infty}^{+\infty} \omega([h, a_t(A)])dt = 0, \forall A \in U$, then ω satisfies the KMS condition. So, there is a biunivoque relation between equilibrium state and KMS state. So, if we admit that around ℓ_{planck} the (pre)space-time system is in a thermal equilibrium state, then according to [11], we are also bound to admit that this system is in a KMS state.

Next, let us push forwards the consequences raised by the holomorphicity of the KMS strip.

§4. Holomorphic Time Flow at the Planck Scale

As a critical consequence of the KMS condition, we are induced to consider that the time-like coordinate g_{00} becomes holomorphic within the limits of the KMS strip. Indeed, as demonstrated in details in [10-19], within the KMS strip, we necessarily should have

$$t \to t_c = t_r + it_i. \tag{4.1}$$

In the same way, the physical (real) temperature should also be considered as complex at the Planck scale:

$$T \to T_c = T_r + iT_1 \tag{4.2}$$

as proposed by Atick and Witten in another context[1]. This unexpected effect is simply due to the fact that, given a von Neumann algebra W^* and two elements A, B of W^*, then there exists a function $f(t_c)$ holomorphic in the strip $\{t_c \in \mathbb{C}, \text{ im } t_c \in [0, \hbar\beta]\}$ such that

$$f(t) = \varphi(Aa_t(B))$$

and

$$f(t_r + \hbar\beta) = \varphi(a_t(B)A), \forall t \in \mathbb{R}. \tag{4.3}$$

Here, t is the usual time parameter of the 3D theory, like $\hbar\beta = \hbar/kT$. So in our case, within the limits of the KMS strip, i.e. from the scale zero ($\beta = 0$) to the Planck scale ($\beta = \ell_{\text{planck}}$), the "time-like" direction of the system must be extended to the complex variable

$$t_c = t_r + it_i \in \mathbb{C}, Imt_c \in [it_i, t_r]. \tag{4.4}$$

Of course, the holomorphicity of the time like direction of the space-time is induced in a natural manner by the fact that in our approach, the thermodynamical system is the space-time itself. Such a situation has been investigated in details in [10] in the context of "quantum groups" and non-commutative geometry.

Indeed, according to Tomita's modular theory[21], the KMS condition, when applied to the space-time as a system, allows, within the KMS strip, the existence of an "extended" (holomorphic) automorphism "group of evolution", which depends, in the classification of factors[12] on a "type $III\lambda$ factor" $\boldsymbol{M}_q$ (a factor is a special type of von Neumann algebra, whose center is reduced to the scalars $a \in \mathbb{C}$). The extended automorphism group has the following form

$$\boldsymbol{M}_q \mapsto \sigma_{\beta_c}(\boldsymbol{M}_q) = e^{H\beta_c}\boldsymbol{M}_q e^{-H\beta_c} \tag{4.5}$$

with the $\beta_c = \beta_r + i\hbar\beta_i$ parameter being formally complex and able to be interpreted as a complex time t and / or temperature $T \to T_c = T_r + iT_i$. So, the KMS condition suggests the existence at the Planck scale, of an effective one loop potential coupled, in $N = 2$ supergravity, to the complex dilaton + axion field $\varphi = \frac{1}{g^2} + ia$ and yielding the dynamical form $\eta_{\mu\nu} = \text{diag}(1, 1, 1, e^{i\theta})$ for the metric. The signature of $\eta_{\mu\nu}$ is Lorentzian (i.e. physical) for $\theta = \pm\pi$ and can become Euclidean (topological) for $\theta = 0$. Consequently, the "KMS signature" of the metric is $(+++\pm)$. This is as it should be since., considering the quantum fluctuations of $g_{\mu\nu}$, there is no more invariant measure on the non commutative metric. Therefore, according to von Neumann algebra theory, the "good factor" addressing those constraints is uniquely a non commutative traceless algebra, i.e. the type $III\lambda$ factor $\boldsymbol{M}_q$, of the general form constructed by Connes[12]:

$$\boldsymbol{M}_q = M^{0,1}_{\text{Top}} \succ\!\lhd_\theta \mathbb{R}^*_+/\beta\mathbb{Z} \equiv \boldsymbol{M}^{0,1}_{\text{Top}} \succ\!\lhd_\theta \beta S_1, \tag{4.6}$$

$\boldsymbol{M}^{0,1}_{\text{Top}}$ being a type II_∞ factor and $\mathbb{R}^*_+/\beta\mathbb{Z}$ the group acting periodically on $\boldsymbol{M}^{0,1}_{\text{Top}}$. The relation between the periods λ and β is such that $\lambda = \frac{2\pi}{\beta}$, so that when $\beta \to \infty$, we get $\lambda \to 0$ (the periodicity is suppressed).

At the Lie group level, this "superposition state" can simply be given by the symmetric homogeneous space constructed in [10]:

$$\sum h = \frac{SO(3,1) \bigotimes SO(4)}{SO(3)} \tag{4.7}$$

to which corresponds, at the level of the underlying metric spaces involved, the topological quotient space:

$$\sum top = \frac{\mathbb{R}^{3,1} \bigoplus \mathbb{R}^4}{SO(3)}. \tag{4.8}$$

In the non commutative context, G. Bogdanoff has constructed, again in [10], the "cocyle bicrossproduct":

$$U_{q(so(4))^{(op)}} \ {}^{\psi} \triangleright\!\blacktriangleleft U_{q(so(3,1))}, \tag{4.9}$$

where $U_{q(so(4))^{(op)}}$ and $U_{q(so(3,1))}$ are Hopf algebras (or "quantum groups"[21]) and ψ is a 2-cocycle of q-deformation. The bicrossproduct (4.2) suggests an unexpected kind of "unification" between the Lorentzian and the Euclidean Hopf algebras at the Planck scale and yields the possibility of a "q- deformation" of the signature from the Lorentzian (physical) mode to the Euclidean (topological) mode[10−19].

Now, starting from Equation (4.5), it appears clearly that the Tomita-Takesaki modular automorphisms group $\sigma_{\beta c}(\boldsymbol{M}_q)$ corresponds to the "unification" given by equ.(4.9) and induces, within the KMS field, the existence of two dual flows.

(i) On the boundary $\beta_i \geq \ell_{\text{planck}}$, the first possible flow, of the form

$$\sigma_t(M_q) = e^{iH\beta} M_q e^{-iH\beta} \tag{4.10}$$

corresponds to the algebra of observables of the system and to the Lorentzian flow in real time. In this perspective, this flow is a "physical flow", which we call $\underset{\beta>0}{\mathbf{P}^f}$. This scale represents the physical part of the light cone and, consequently, the notion of (Lebesgue) measure is fully defined. Therefore, the (commutative) algebra involved is endowed with a hyperfinite trace and is given on the infinite Hilbert space $\mathcal{L}(\mathfrak{h})$, with $\mathfrak{h} = L^2(\mathbb{R})$. Then $\mathcal{L}(L^2(\mathbb{R}))$ is a type $\mathbf{I}_\infty$ factor, indexed by the real group $\mathbb{R}$, which we call $\boldsymbol{M}_{\text{phys}}$, Of course, at this scale, the theory is Lorentzian, controlled by $SO(3,1)$.

(ii) On the "zero scale" $\beta_i = 0$ limit, the second flow takes necessarily the non unitary form:

$$\sigma_{i\beta}(\boldsymbol{M}_q) = e^{\beta_r H} \boldsymbol{M}_q e^{-\beta_r H} \tag{4.11}$$

giving on $\boldsymbol{M}_q$ the semi-group of unbounded and non-stellar operators. This initial topological scale corresponds to the imaginary vertex of the light cone, i.e. a zero-size gravitational instanton. All the measures performed on the Euclidean metric being ρ-equivalent up to infinity, the system is ergodic. As shown by A. Connes, any ergodic flow for an invariant measure in the Lebesgue measure class gives a unique type II_∞ hyperfinite factor[12]. This strongly suggests that the singular 0-scale should be described by a type II_∞ factor, endowed with a hypcrfinite trace noted T_{r_∞}. We have called $\boldsymbol{M}^{0,1}_{\text{Top}}$ such a "topological" factor, which is an infinite tensor product $\bigotimes^\infty$ of matrices algebra (ITPFI) of the $R_{0,1}$ Araki-Woods type[23]. Since $\boldsymbol{M}_q = \boldsymbol{M}^{0,1}_{Top} \succ\!\lhd_\theta \beta S_1$, on the $\beta_i = 0$ limit, we get $\boldsymbol{M}_q \equiv \boldsymbol{M}^{0,1}_{\text{Top}}$. With respect to the analytic continuation between () and (), $\sigma_{i\beta}(\boldsymbol{M}_q)$ represents a "current in imaginary time". We have stated in [19] that this current is another way to interpret the "flow of weights" of the algebra $\boldsymbol{M}_q$ (see [24]). Clearly, according to [24], the flow of weights of M_q is an ergodic flow, which represents an invariant of M_q. Then, $\sigma_{i\beta}(\boldsymbol{M}_q)$ yields a pure topological amplitude [25] and, as such, "propagates" in imaginary time from zero to infinity. $\sigma_{i\beta}(\boldsymbol{M}_q)$ is not defined on the whole algebra $\boldsymbol{M}_q$ but on an ideal $\{\Im\}$ of $\boldsymbol{M}_q$. One can demonstrate that in this case, the theory is Riemannian, the isometrics of the metric being given by $SO(4)$. We have showed[10] that this scale zero corresponds to the first Donaldson invariant $\mathbf{I} = \sum_i (-1)^{n_i}$ and can be described by the topological quantum field theory proposed by E. Witten in [25].

To finish, let us observe that the topological flow does not commute with the physical flow. Again, this is a direct and natural consequence of the KMS condition.

§5. Discussion

(i) It is interesting to remark that in the totally different context of superstrings, J.J. Atick and E. Witten were the first to propose such an extension of the real temperature towards a complex domain[1]. Recently, in $N = 4$ supersymmetric string theory, I. Antoniadis, J.P. Deredinger and C. Kounnas[5] have also suggested to shift the real temperature to imaginary one by identification with the inverse radius of a compactified Euclidean time on S^1, with

$R = 1/2\pi T$. Consequently, one can introduce a complex temperature in the thermal moduli space, the imaginary part coming from the $B_{\mu\nu}$ antisymmetric field under type $\boldsymbol{IIA} \stackrel{S/T/U}{\longleftrightarrow}$ type $\boldsymbol{IIB} \stackrel{S/T/U}{\longleftrightarrow} \boldsymbol{Heterotic}$ string-string dualities. More precisely, in Antoniadis *and al* approach, the field controlling the temperature comes from the product of the real parts of three complex fields: $s =$Re $\mathbf{S}$, $t =$Re $\mathbf{T}$ and $u =$Re $\mathbf{U}$. Within our KMS approach, the imaginary parts of the moduli **S, T, U** can be interpreted in term of Euclidean temperature. Indeed, from our point of view, a good reason to consider the temperature as complex at the Planck scale is that a system in thermodynamical equilibrium state must be considered as subject to the KMS condition[11].

(ii) On the other hand, according to most of the models, supergravity is considered as broken for scales greater than the Planck scale[22−24]. But supersymmetry breaking is also closely connected to the cancellation of the thermodynamic equilibrium state. C. Kounnas has recently demonstrated that a five-dimensional ($N = 4$) supersymmetry can be described by a four-dimensional theory in which supersymmetry is spontaneously broken by finite thermal effects[6]. This scenario may be applied to our setting. As a matter of fact, the end of the thermal equilibrium phase at the Planck scale might bring about the breaking of KMS state and of supersymmetry $N = 4$. This corresponds exactly, in our case, to the decoupling between imaginary time and real time.

To sum up, the chain of events able to explain the transition from the topological phase to the physical phase of the space-time might be the following:

{thermodynamical equilibrium breaking}→{KMS state breaking}→{imaginary time/real time decoupling}→{topological state / physical state decoupling}→{Super symmetry breaking}.

Such a transition has been detailed in [19]. Likewise, the supersymmetry is broken in [5,6] by the finite temperature, which corresponds in our view to the decoupling between real and imaginary (topological) temperature (the topological temperature being identified, in Kounnas model, with the inverse radius of a compactified Euclidean time on S^1: $2\pi T = 1/R$). Applying this representation, the partition function in our case is given by the (super)trace over the thermal spectrum of the theory in 4 dimensions. According to this, supersymmetry breaking and transition from topological state to physical state are deeply connected.

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