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ON THE SPACES OF THE MAXIMAL POINTS***

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Abstract

For a continuous domain D, some characterizations that the convex powerdomain CD is a domain hull of $\text{Max}(CD)$ is given in terms of compact subsets of D. And in the case, it is proved that the set of the maximal points $\text{Max}(CD)$ of CD with the relative Scott topology is homeomorphic to the set of all Scott compact subsets of $\text{Max}(D)$ with the topology induced by the Hausdorff metric derived from a metric on $\text{Max}(D)$ when $\text{Max}(D)$ is metrizable.

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§1. Introduction

It is an interesting and active research direction to deal with some problems in topology by employing appropriate domain environment. In [8], J. D. Lawson proved that each Polish space can be arise as the set of maximal points of an ω-continuous domain; K. Martin[13] obtained the similar results by virtue of introducing Lebesgue measurement on continuous domains, and he investigated relations between the maximal points of D and those of the convex powerdomain CD. In this paper a characterization that the convex powerdomain CD is a domain hull of $\text{Max}(CD)$ is given in terms of Scott compact subsets of D. And in this case, it is proved that the set of the maximal points $\text{Max}(CD)$ of CD with the relative Scott topology is homeomorphic to the set of all Scott compact subsets of $\text{Max}(D)$ with the topology induced by the Hausdorff metric if $\text{Max}(D)$ is metrizable.

A dcpo D is a partially ordered set such that every directed set E of D has a least upper bound in D, denoted by $\vee E$. For $x, y \in D, x \ll y$ implies that for each directed set

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$E \subseteq D$ with $y \leq \vee E$, there exists some $e \in E$ such that $x \leq e$. For each $x \in D$, we denote $\Downarrow x = \{y \in D : y \ll x\}$ and $\Uparrow x = \{y \in D : x \ll y\}$. A dcpo D is called a continuous domain if $\Downarrow x$ is directed and $x = \vee \Downarrow x$ for each $x \in D$. For a set $A \subseteq D$, we denote $\uparrow A = \{y \in D : \exists a \in A, a \leq y\}$. A is called an upper set if $\uparrow A = A$. $\downarrow A$ and the lower set can be defined dually. Also $\Uparrow A, \Downarrow A$ can be given similarly.

A subset U of a dcpo D is Scott open provided U is an upper set and $E \cap U \neq \emptyset$ for each directed set $E \subseteq D$ with $\vee E \in U$. The topology $\sigma(D)$ formed by all the Scott open sets of D is called the Scott topology. The topology generated by taking $\sigma(D) \cup \{D \backslash \uparrow x : x \in D\}$ as a subbase is called the Lawson topology, denoted by $\lambda(D)$.

Definition 1.1.[1] *An abstract basis is given by a set B with a transitive order $\prec$ such that*

$$M \prec x \Longrightarrow \exists y \in B, M \prec y \prec x$$

for all $x \in B$ and all nonempty finite sets $M \subseteq B$

Obviously, $(D, \ll)$ is an abstract basis for a continuous domain D.

A subset I of an abstract basis $(B, \prec)$ is called an ideal if I is a directed lower set with respect to the transitive order $\prec$. The collection of ideals of $(B, \prec)$ ordered by set-theoretical inclusion is a continuous domain, denoted by $Id(B, \prec)$ (see [1]).

Let D be a continuous domain and $A, B \subseteq D$. We define relations as follows:

$$A \ll_L B \Longleftrightarrow \forall a \in A, \exists b \in B, a \ll b,$$

$$A \ll_U B \Longleftrightarrow \forall b \in B, \exists a \in A, a \ll b,$$

$$A \ll_{EM} B \Longleftrightarrow A \ll_L B A \ll_U B.$$

Similarly we can define the relations $\leq_L, \leq_U$ and $\leq_{EM}$.

Let Fin(D) be the collection of all nonempty finite subsets of D. It is easily to see that $(Fin(D), \ll_{EM})$ is an abstract basis.

Definition 1.2.[1,13] *let D be a continuous domain.*

(1) $Id(Fin(D), \ll_{EM})$ is called the convex powerdomain of D, written CD for short.

(2) $A^* = \{F \in Fin(D) : F \ll_{EM} A\}$ for each nonempty set $A \subseteq D$.

For a dcpo D, let $\mathrm{Max}(D)$ denote the set of all maximal points of D and $Com(\mathrm{Max}(D))$ the collection of all Scott compact subsets of $\mathrm{Max}(D)$.

Proposition 1.1.[12,13] (1) *$K^* \in CD$ for each Scott compact subset of D.*

(2) *$\forall F \in Fin(D), I \in CD, F \in I \Leftrightarrow F^* \ll I$.*

(3) *$K_I = \cap\{\uparrow F : F \in I\}$ is a Scott compact upper set for each $I \in CD$, and $K_I^* \subseteq I$ for each $I \in \mathrm{Max}(CD)$.*

Definition 1.3.[8] *A continuous domain D is called a domain hull of* $\mathrm{Max}(D)$ *if equation $\lambda(D)\mid_{\mathrm{Max}(D)} = \sigma(D)\mid_{\mathrm{Max}(D)}$ holds, where $\lambda(D)\mid_{\mathrm{Max}(D)}$ and $\sigma(D)\mid_{\mathrm{Max}(D)}$ are the relative Lawson topology and Scott topology respectively.*

Theorem 1.1.[12] *For a continuous domain D, the following are equivalent:*

(1) *D is a domain hull of* $\mathrm{Max}(D)$*;*

(2) *For each $x \in D$, there is a Scott closed set A_x of D such that $\uparrow x \cap \mathrm{Max}(D) = A_x \cap \mathrm{Max}(D)$;*

(3) *For each* $x \in D$ *and each* $y \in \mathrm{Max}(D), x \not\leq y$ *implies that there exist* $x_0 \ll x, y_0 \ll y$ *such that* $\uparrow x_0 \cap \uparrow y_0 = \emptyset$.

§2. Characterization of Max(CD)

In [12], we obtained the following results:

Theorem 2.1.[12] Let D be a continuous domain. Then

(1) $K = K_{K^*} = \cap\{\uparrow F : F \in K^*\}$ for each Scott compact upper set K of D

(2) If D is a domain hull of $\mathrm{Max}(D)$, and $K \in Com(\mathrm{Max}(D))$, then $K^* \in \mathrm{Max}(CD)$.

Lemma 2.1. *If* D *is a domain hull of* $\mathrm{Max}(D)$ *and* $K_I \subseteq \mathrm{Max}(D)$ *for each* $I \in \mathrm{Max}(CD)$*, then* $K \cap \mathrm{Max}(D)$ *is Scott compact for each Scott compact upper set* K *of* D.

Proof. Let K_0 be an arbitrary Scott compact upper set of D. Then $K_0^* \in CD$ and hence there exists a $J \in \mathrm{Max}(CD)$ such that $K_0^* \subseteq J$. Thus by Theorem 2.1(1) we have $K_J \subseteq K_0 \cap \mathrm{Max}(D)$. To complete the proof, it suffices to show $K_J = K_0 \cap \mathrm{Max}(D)$.

Suppose that there is a $k_0 \in K_0 \cap \mathrm{Max}(D) \setminus K_J$. Then $k_0 \notin \uparrow F_0$ for some $F_0 \in J$. Note that there is a $G_a \in K_0^*$ such that $a \in G_a$ for each $a \ll k_0$. We can take an $F_a \in J$ such that

$$F_0 \ll_{EM} F_a, \quad G_a \ll_{EM} F_a,$$

and hence $a \ll x_a$ for some $x_a \in F_a$. Again by Theorem 2.1,$K_J^* = J$, and hence $x_a \ll m_a$ for some $m_a \in K_J$. Thus the net $\{m_a : a \ll k_0\}$ in $\mathrm{Max}(D)$ has an cluster $m_0 \in K_J$ with respect to the relative Scott topology on $\mathrm{Max}(D)$ as K_J is Scott compact. Note that $\mathrm{Max}(D)$ with the relative Scott topology is Hausdorff as D is a domain hull of $\mathrm{Max}(D)$, there exist $u_0 \ll k_0, v_0 \ll m_0$ such that $\Uparrow u_0 \cap \Uparrow v_0 \cap \mathrm{Max}(D) = \emptyset$. On the other hand , we can take a u_1 with $u_0 \ll u_1 \ll k_0$ such that $m_{u_1} \in \Uparrow v_0 \cap \mathrm{Max}(D)$ as m_0 is a cluster of the net $\{m_a : a \ll k_0\}$ and $\Uparrow v_0 \cap \mathrm{Max}(D)$ is a neighborhood of m_0. By $u_1 \ll x_{u_1} \ll m_{u_1}$, then $m_{u_1} \in \Uparrow u_0 \cap \mathrm{Max}(D)$, which is contradiction. Thus the proof is completed.

Theorem 2.2. *If continuous domain* D *is a domain hull of* $\mathrm{Max}(D)$*, then the following statements are equivalent.*

(1) CD *is a domain hull of* $\mathrm{Max}(CD)$*;*

(2) $K_I \subseteq \mathrm{Max}(D)$ *for each* $I \in \mathrm{Max}(CD)$*;*

(3) $K \cap \mathrm{Max}(D)$ *is Scott compact for each Scott compact upper set* K *of* D *and* $I = K_I^*$ *for each* $I \in \mathrm{Max}(CD)$.

Proof. (1)⇒(2): Suppose $K_I \not\subseteq \mathrm{Max}(D)$ for some $I \in \mathrm{Max}(CD)$.Then there exists a $k_0 \in K_I \backslash \mathrm{Max}(D)$. We can take $m_0 \in K_I \cap \mathrm{Max}(D)$ with $k_0 < m_0$. For each $k \in K_I \backslash \downarrow k_0$, take an a_k with $a_k \ll k$ and $a_k \not\leq k_0$, and for each $s \in \downarrow k_0 \cap K_I$, take an arbitrary b_s with $b_s \ll s$. We obtain a Scott open cover $\{\Uparrow a_k : k \in K_I \backslash \downarrow k_0\} \cup \{\Uparrow b_s : s \in \downarrow k_0 \cap K_I\}$ of K_I, hence there is a finite subcover $\{\Uparrow a_{k_i} : i = 1, 2, \cdots, n_1\} \cup \{\Uparrow b_{s_j} : j = 1, 2, \cdots, n_2\}$. Then

$$G = \{a_{k_i} : i = 1, 2, \cdots, n_1\} \cup \{b_{s_j} : j = 1, 2, \cdots, n_2\} \ll_{EM} K_I.$$

Again we take a b_0 with $\{b_{s_j} : j = 1, 2, \cdots, n_2\} \ll b_0 \ll m_0$ and $b_0 \not\leq k_0$ and let $F = \{a_{k_i} : i = 1, 2, \cdots, n_1\} \cup \{b_0\}$. Then it is easy to see $F^* \not\subseteq I$. By Theorem 1.1, it suffice to show that $\uparrow F^* \cap \uparrow H^* \neq \emptyset$ for each $H \in I$.

For each $H \in I$, take an $\overline{H} \in I$ with $G \ll \overline{H}$ and $H \ll \overline{H}$. Since $k_0 \in K_I$,

$$\overline{H}_1 = \{h \in \overline{H} : \exists k \in K_I \cap \downarrow m_0, h \ll k\} \neq \emptyset.$$

If $\overline{H}_1 = \overline{H}$, then it is not difficult to show $\{m_0\}^* \in \uparrow F^* \cap \uparrow H^* \neq \emptyset$ and hence the proof is completed.

We now suppose $\overline{H} \backslash \overline{H}_1 \neq \emptyset$. For each $h \in \overline{H} \backslash \overline{H_1}$, then there is a $k_h \in K_I \backslash \downarrow m_0$ such that $h \ll k_h$. Note $G \ll_{EM} K_I$, then $A_h = \{a_{k_i} \in G : a_{k_i} \ll k_h\} \neq \emptyset$ and we can take a u_h such that $A_h \cup \{h\} \ll u_h \ll k_h$. For each $h \in \overline{H}_1$, take a u_h with $\{h, b_0\} \ll u_h \ll m_0$. Thus we can show $H \ll_{EM} U_H$ and $F \ll_{EM} U_H$ for $U_H = \{u_h : h \in \overline{H}\}$, and hence $U_H^* \in \uparrow F^* \cap \uparrow H^* \neq \emptyset$.

(2) $\Rightarrow$ (3): Follows from Lemma 2.1, Proposition 1.1 and Theorem 2.1.

(3) $\Rightarrow$ (1): Take an arbitrary $I \in CD, J \in \text{Max}(CD)$ with $I \not\subseteq J$. It suffice to show that there exists a G, $G \in J$ such that $\Uparrow I \cap \Uparrow G^* = \emptyset$ by Theorem 1.1 and Proposition 1.1. By $I \not\subseteq J$ and Proposition 1.1(3), we can take an $F \in I \backslash J$ with $F \not\ll_{EM} K_J$, which implies $F \not\ll_U K_J$ or $F \not\ll_L K_J$.

(i) If $F \not\ll_L K_J$, then there exists an $x_1 \in F$ such that $x_1 \not\leq m$ for each $m \in K_J$, thus by Theorem 1.1 we can take $x_m \ll x_1, \overline{a_m} \ll m$ such that $\uparrow x_m \cap \uparrow \overline{a_m} = \emptyset$. For each $m \in K_J$, take an a_m satisfying $\overline{a_m} \ll a_m \ll m$, then obtain an open cover $\{\Uparrow a_m : m \in K_J\}$ of K_J. Suppose that $\{\Uparrow a_{m_i} : i = 1, 2, \cdots, n\}$ is a finite subcover of K_J. Then $G = \{a_{m_i} : i = 1, 2, \cdots, n\} \ll_{EM} K_J$, and hence $G \in J$ by Proposition 1.1(3). In the following we show that $\Uparrow I \cap \Uparrow G^* = \emptyset$.

Firstly take an $\overline{x_1}$ with $\{x_{m_i} : i = 1, 2, \cdots, n\} \ll \overline{x_1} \ll x_1$, then take an arbitrary y_x with $y_x \ll x$ for each $x \in F \backslash \{x_1\}$. Then $\overline{F} = \{y_x : x \in F \backslash \{x_1\}\} \cup \{\overline{x_1}\} \ll_{EM} F$, hence $\overline{F} \in I$.

Suppose $I_0 \in \Uparrow I \cap \Uparrow G^*$, then $\overline{F} \in I_0$ and $\overline{G} = \{\overline{a_{m_i}} : i = 1, 2, \cdots, n\} \in I_0$, hence there is an $H \in I_0$ such that $\overline{G} \ll_{EM} H, \overline{F} \ll_{EM} H$. Thus there is an h, $h \in H$ such that $\overline{x_1} \ll h$ and $\overline{a_{m_i}} \ll h$ for some $\overline{a_{m_i}} \in \overline{G}$, which contradicts to $\uparrow x_1 \cap \uparrow \overline{a_{m_i}} = \emptyset$. Hence $\Uparrow I \cap \Uparrow G^* = \emptyset$.

(ii) If $F \not\ll_U K_J$, then there exists an $m_0 \in K_J$ such that $x \not\leq m_0$ for each $x \in F$. From Theorem 1.1, it follows that there are $a_x \ll x, b_x \ll m_0$ such that $\uparrow a_x \cap \uparrow b_x = \emptyset$ for each $x \in F$. now we can take b_F and $\overline{b_F}$ with $\{b_x : x \in F\} \ll \overline{b_F} \ll b_F \ll m_0$ by the finiteness of F, and take a $G \in J$ with $b_F \in G$. For each $y \in G \backslash \{b_F\}$, take an arbitrary $\overline{b_y}$ with $\overline{b_y} \ll y$, then

$$\overline{G} = \{\overline{b_y} : y \in G \backslash \{b_F\} \cup \{\overline{b_F}\} \ll_{EM} G.$$

Suppose $I_0 \in \Uparrow I \cap \Uparrow G^*$. Then $F_1 = \{a_x : x \in F\} \in I_0, \overline{G} \in I_0$. Take an $H \in I_0$ with $\{F_1, \overline{G}\} \ll_{EM} H$, it will, similarly to the proof of (i), induce a contradiction.

In view of the above, the proof is completed.

§3. Metric Topology on Max(CD)

From the characterization theorem above and Theorem 2.1(1), it follows that the mapping

$$g : \text{Com}(\text{Max}(D)) \to \text{Max}(CD),$$
$$K \longmapsto K^*$$

is a bijection when D and CD are domain hulls of $\mathrm{Max}(D)$ and $\mathrm{Max}(CD)$ respectively. In addition if $\mathrm{Max}(D)$ with the relative Scott topology is metrizable, then a interesting question is posed[13]:

Is $\mathrm{Max}(CD)$ with the relative Scott topology homeomorphic to $Com(\mathrm{Max}(D)$ with the topology induced by the Hausdorff metric derived from a metric on $\mathrm{Max}(D)$?

In the following we will show that the answer to the question is yes. Now suppose that d is a metric on $\mathrm{Max}(D)$, then the Hausdorff metric $\overline{d}$ on $Com(\mathrm{Max}(D))$ derived from d as follows:

$$\overline{d}(K_1, K_2) = \inf\{r : K_1 \subseteq B^r_d(K_2), K_2 \subseteq B^r_d(K_1)\},$$

where $B^r_d(K_1) = \{x \in \mathrm{Max}(D) : d(x, K_1) < r\}$ and $d(x, K_1) = inf\{d(x, y) : y \in K_1\}$.

Theorem 3.1. *Let continuous domain D and its convex powerdomain CD be domain hulls of* $\mathrm{Max}(D)$ *and* $\mathrm{Max}(CD)$ *respectively, and* $\mathrm{Max}(D)$ *metrizable. Then*

$$g : (Com(\mathrm{Max}(D), T_{\overline{d}}) \to (\mathrm{Max}(CD), \sigma(CD)|\mathrm{Max}(CD))$$

is a continuous and open mapping and hence is a homeomorphism, where $T_{\overline{d}}$ is the topology induced by the Hausdorff metric $\overline{d}$.

Proof. (i) g is continuous. Suppose that $J \in \mathrm{Max}(CD)$ and $\Uparrow I \cap \mathrm{Max}(CD)$ is an arbitrary Scott open neighborhood of J. By $I \ll J$, there are $G_1, G_2 \in Fin(D)$ with $G_1 \ll_{EM} G_2$ such that $I \subseteq G_1^* \subseteq G_2^* \subseteq J$, which implies $G_1 \ll_{EM} K_J$, and hence $K_J \subseteq \Uparrow G_1$ by Theorem 2.3(3). Since K_J is compact subset of metric space $\mathrm{Max}(D)$, there exists a positive real number r such that for each $k \in K_J$,

$$B^r_d(k) \subseteq \Uparrow x_{i_k} \cap \mathrm{Max}(D)$$

for some $x_{i_k} \in G_1$. By $G_1 \ll_{EM} K_J$, for each $x \in G_1$ there is a $k_x \in K_J$ such that

$$B^{r_x}_d(k_x) \subseteq \Uparrow x \cap \mathrm{Max}(D) \subseteq \Uparrow x$$

for some positive real number r_x. Let $r_0 = \min\{r, r_x : x \in G_1\}/2$, we claim

$$B^{r_0}_{\overline{d}}(K_J) = \{K \in \mathrm{Com}(\mathrm{Max}(D) : \overline{d}(K_J, K) < r_0\} \subseteq g^{-1}[\Uparrow I \cap \mathrm{Max}(CD)],$$

hence g is continuous.

In fact, from $K \in B^{r_0}_{\overline{d}}(K_J)$, it follows that for each $k \in K$ there is a $y_k \in K_J$ such that $k \in B^{r_0}_d(y_k) \subseteq \Uparrow G_1$, hence $G_1 \ll_U K$. Now let $x \in G_1$. From $K_J \subseteq B^{r_0}_d(K)$, it follows that there is a $u_x \in K$ such that $d(k_x, u_x) < r_0$, which implies $u_x \in B^{r_0}_d(k_x) \subseteq \Uparrow x$. Thus we have $x \ll u_x$, and hence $G_1 \ll_L K$. By $G_1 \ll_{EM} K$ and $I \subseteq G_1^*$, we know that $K \in g^{-1}[\Uparrow I \cap \mathrm{Max}(CD)]$. Hence g is continuous.

(ii) To prove that g is an open mapping, it suffices to prove that for each $K \in \mathrm{Max}(D)$ and for $r > 0, g[B^r_{\overline{d}}(K)]$ is an open set. Suppose $K_1 \in B^r_{\overline{d}}(K)$, then $K_1 \subseteq B^{r_0}_d(K), K \subseteq B^{r_0}_d(K_1)$ for some $r_0 < r$. For each $k \in K_1$ there is an $x_k \in K$ such that $d(k, x_k) < r_0$. We can take an s_k with $0 < s_k < \inf\{(r - r_0)/3, r_0\}$ such that $B^{s_k}_d(k) \subseteq B^{r_0}_d(x_k)$, and take an $a_k \ll k$ such that

$$k \in \Uparrow a_k \cap \mathrm{Max}(D) \subseteq B^{s_k}_d(k).$$

Thus we obtain an open cover $\{\Uparrow a_k : k \in K_1\}$ of K_1 which has a finite subcover $\{\Uparrow a_{k_i} :$

$i = 1, 2, \cdots, n\}$, then $F = \{a_{k_1}, a_{k_2}, \cdots, a_{k_n}\} \in K_1^*$, i.e. $K_1^* \in U = \Uparrow F^* \cap \mathrm{Max}(CD)$. To complete the proof, it suffices to show $U \subseteq g[B_{\overline{d}}^r(K)]$.

Suppose $K_2^* \in U$, then $F \ll_{EM} K_2$ and hence

$$\begin{aligned} K_2 \subseteq \Uparrow F \cap \mathrm{Max}(D) &\subseteq \cup\{B_d^{s_{k_i}}(k_i) : i = 1, 2, \cdots, n\} \\ &\subseteq \cup\{B_d^{r_0}(x_{k_i}) : i = 1, 2, \cdots, n\} \\ &\subseteq B_d^{r_0}(K). \end{aligned}$$

For each $u \in K$, again by $K \subseteq B_d^{r_0}(K_1)$ and $F \ll_{EM} K_1$, there is a $y_u \in K_1$ such that $d(u, y_u) < r_0$, and there is a $a_{k_{i_u}} \in F$ such that $a_{k_{i_u}} \ll y_u$, which means $d(y_u, k_{i_u}) < s_{k_{i_u}}$. Again by $F \ll_{EM} K_2$, there is a $z_u \in K_2$ such that $a_{k_{i_u}} \ll z_u$, which implies $d(k_{i_u}, z_u) < s_{k_{i_u}}$. Thus we have

$$\begin{aligned} d(u, z_u) &\leq d(u, y_u) + d(y_u, k_{i_u}) + d(k_{i_u}, z_u) \\ &< r_0 + 2s_{k_{i_u}} < r_0 + 2(r - r_0)/3 < r, \end{aligned}$$

which means $\overline{d}(K, K_2) < r$, hence $K_2^* \in g[B_{\overline{d}}^r(K)]$.

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