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SOME REMARKS ABOUT THE *R*-BOUNDEDNESS**

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Abstract

Let X, Y be UMD-spaces that have property (α), $1 < p < \infty$ and let $\mathcal{M}$ be an R-bounded subset in $\mathcal{L}(X, Y)$. It is shown that $\{T_{(M_k)_{k\in\mathbb{Z}}} : M_k, k(M_{k+1} - M_k) \in \mathcal{M}$ for $k \in \mathbb{Z}\}$ is an R-bounded subset of $\mathcal{L}(L^p(0, 2\pi; X), L^p(0, 2\pi; Y))$, where $T_{(M_k)_{k\in\mathbb{Z}}}$ denotes the L^p-multiplier given by the sequence $(M_k)_{k\in\mathbb{Z}}$. This generalizes a result of Venni [10]. The author uses this result to study the strongly L^p-well-posedness of evolution equations with periodic boundary condition. Analogous results for operator-valued L^p-multipliers on $\mathbb{R}$ are also given.

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Recent developments of operator-valued Fourier multipliers (on $[0, 2\pi]$ or $\mathbb{R}$) show that one can not expect to generalize the classical Fourier multiplier theorems to the operator-valued case without using the notion of R-boundedness. More precisely, let X, Y be Banach spaces and let $1 < p < \infty$. If $(M_k)_{k\in\mathbb{Z}} \subset \mathcal{L}(X, Y)$ is an L^p-multiplier, then the sequence $(M_k)_{k\in\mathbb{Z}}$ must be R-bounded (see [1, Proposition 1.11]), where we denote by $\mathcal{L}(X, Y)$ the set of all bounded linear operators from X to Y. Conversely if X, Y are UMD-spaces (see [3] for the definition and further properties concerning this notion), $1 < p < \infty$ and if both $(M_k)_{k\in\mathbb{Z}}$ and $(k(M_{k+1} - M_k))_{k\in\mathbb{Z}}$ are R-bounded, then the sequence $(M_k)_{k\in\mathbb{Z}}$ is an L^p-multiplier (see [1, Theorem 1.3]). One has the same phenomenon for operator-valued Fourier multipliers on $\mathbb{R}$ (see e.g. [4] or [11]). Such kind of results can be applied to the study of the strongly L^p-well-posedness of evolution equations with Dirichlet or periodic boundary conditions [1, 11].

In this paper we show that when X, Y are UMD-spaces, $1 < p < \infty$ and assume that $\mathcal{M} \subset \mathcal{L}(X, Y)$ is R-bounded, then $\{T_{(M_k)_{k\in\mathbb{Z}}} : M_k, k(M_{k+1} - M_k) \in \mathcal{M}\ (k \in \mathbb{Z})\}$ is R-bounded in $\mathcal{L}(L^p(0, 2\pi; X), L^p(0, 2\pi; Y))$, where $T_{(M_k)_{k\in\mathbb{Z}}}$ denotes the bounded linear operator from $L^p(0, 2\pi; X)$ to $L^p(0, 2\pi; Y)$ defined by the multiplier $(M_k)_{k\in\mathbb{Z}}$. This generalizes our previous result (see [1, Theorem 1.3]) and a result of A. Venni, where A. Venni has only considered the case $X = Y$ and $\mathcal{M} = \Omega I$ which is trivially R-bounded, where Ω is a bounded subset of $\mathbb{C}$ and I denotes the identity of X (see [10]). We also establish similar results for

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operator-valued L^p-multipliers on $\mathbb{R}$. We then apply the obtained results to the study of the strongly L^p-well-posedness of evolution equations with different boundary conditions.

First we recall some notions. Let X be a complex Banach space and $1 \le p < \infty$. We consider the Banach space $L^p(0, 2\pi; X)$ with norm

$$\|f\|_p := \Big(\int_0^{2\pi} \|f(t)\|^p dt \Big)^{\frac{1}{p}}.$$

For $f \in L^p(0, 2\pi; X)$, we denote by

$$\hat{f}(k) := \frac{1}{2\pi} \int_0^{2\pi} e^{-ikt} f(t) dt$$

the k-th Fourier coefficient of f, where $k \in \mathbb{Z}$. For $k \in \mathbb{Z}$, $x \in X$ we let $e_k(t) = e^{ikt}$ and $(e_k \otimes x)(t) = e_k(t)x$ $(t \in \mathbb{R})$. Then for $x_k \in X$, $k = -m, -m+1, \cdots, m$, $f = \sum\limits_{k=-m}^{m} e_k \otimes x_k$ is the X-valued trigonometric polynomial given by $f(t) = \sum\limits_{k=-m}^{m} e^{ikt} x_k$ $(t \in \mathbb{R})$. Then $\hat{f}(k) = 0$ if $|k| > m$. The space $\mathcal{T}(X)$ of all X-valued trigonometric polynomials is dense in $L^p(0, 2\pi; X)$.

Let X, Y be Banach spaces and let $\mathcal{L}(X, Y)$ be the set of all bounded linear operators from X to Y. If $(M_k)_{k \in \mathbb{Z}} \subset \mathcal{L}(X, Y)$ is a sequence, we consider the associated linear mapping $M : \mathcal{T}(X) \to \mathcal{T}(Y)$ given by

$$M\Big(\sum_k e_k \otimes x_k \Big) = \sum_k e_k \otimes M_k x_k.$$

We say that the sequence $(M_k)_{k \in \mathbb{Z}}$ is an L^p-multiplier, if there exists a constant C such that

$$\Big\| \sum_k e_k \otimes M_k x_k \Big\|_p \le C \Big\| \sum_k e_k \otimes x_k \Big\|_p$$

for all X-valued trigonometric polynomials $\sum\limits_k e_k \otimes x_k$. This is equivalent to say that there exists a unique operator $\widetilde{M} \in \mathcal{L}(L^p(0, 2\pi; X),\ L^p(0, 2\pi; Y))$ extending the operator M.

When $X = Y = \mathbb{C}$, the classical Marcinkiewicz multiplier theorem states that when $(m_k)_{k \in \mathbb{Z}} \subset \mathbb{C}$ is such that

$$\sup_{k \in \mathbb{Z}} |m_k| < \infty \quad \text{and} \quad \sup_{j \in \mathbb{N}} \sum_{2^j \le |k| < 2^{j+1}} |m_{k+1} - m_k| < \infty,$$

then $(m_k)_{k \in \mathbb{Z}}$ is an L^p-multiplier whenever $1 < p < \infty$. When $(M_k)_{k \in \mathbb{Z}} \subset \mathcal{L}(X, Y)$, the R-boundedness of the multiplier is required. Recall that a family $\mathcal{T} \subset \mathcal{L}(X, Y)$ is called Rademacher bounded (R-bounded, in short), if there exists $c_q \ge 0$ such that

$$\Big\| \sum_{j=1}^{n} \gamma_j \otimes T_j x_j \Big\|_q \le c_q \Big\| \sum_{j=1}^{n} \gamma_j \otimes x_j \Big\|_q$$

for all $T_1, T_2, \cdots, T_n \in \mathcal{T}$, $x_1, x_2, \cdots, x_n \in X$ and $n \in \mathbb{N}$, where $1 \le q < \infty$ and γ_j is the j-th Rademacher functions on $[0, 1]$ given by $\gamma_j(t) = \text{sgn}(\sin(2^n \pi t))$ (see [7]). Note that for $j \ge 1$ and $x \in X$, we denote by $\gamma_j \otimes x$ the X-valued function $\gamma_j x$ (see [1, 2, 5, 11, 12]). By

Kahane's inequality (see [7, Theorem 1.e.13]), if such constant c_q exists for some $1 \leq q < \infty$, then there also exists such constant for all $1 \leq q < \infty$. We denote by $R_q(\mathcal{T})$ the smallest constant c_q. $R_q(\mathcal{T})$ is called the R-bounded of $\mathcal{T}$. It is known that R-boundedness is strictly stronger than the boundedness in norm unless X is of cotype 2 and Y is of type 2 (see [1, Proposition 1.13]).

The classical Marcinkiewicz multiplier theorem has been generalized in the operator-valued case in the following way (see [1, Theorem 1.3]):

Theorem 1. *Let X, Y be UMD-spaces and let $(M_k)_{k\in\mathbb{Z}} \subset \mathcal{L}(X,Y)$. Assume that both sets $\{M_k : k \in \mathbb{Z}\}$ and $\{k(M_{k+1} - M_k) : k \in \mathbb{Z}\}$ are R-bounded, then $(M_k)_{k\in\mathbb{Z}}$ is an L^p-multiplier whenever $1 < p < \infty$.*

Notice that it has been shown that the R-boundedness of the sequence $(M_k)_{k\in\mathbb{Z}}$ is a necessary condition for $(M_k)_{k\in\mathbb{Z}}$ to be an L^p-multiplier (see [1, Proposition 1.11]).

We will use the subsequent geometric property of Banach spaces introduced by Pisier [9] and later used by Clément-de Pagter-Sukochev-Witvliet [5] in the study of the interplay between R-boundedness and unconditional Schauder decompositions. A Banach space X has the property (α), if there exists a constant $C \geq 0$ such that

$$\Big(\int_0^1\int_0^1 \Big\|\sum_{i,j=1}^n \alpha_{ij}\gamma_i(t)\gamma_j(s)x_{ij}\Big\|^2 dtds\Big)^{1/2} \leq C\Big(\int_0^1\int_0^1 \Big\|\sum_{i,j=1}^n \gamma_i(t)\gamma_j(s)x_{ij}\Big\|^2 dtds\Big)^{1/2}$$

for all $x_{ij} \in X$, $\alpha_{ij} = \pm 1$ $(i,j = 1,2,\cdots,n)$ and for all $n \in \mathbb{N}$.

An unconditional Schauder decomposition of a Banach space X is a family $(\Lambda_k)_{k\geq 0}$ of bounded linear projections on X such that

(a) $\Delta_k\Delta_\ell = 0$ if $k \neq \ell$,

(b) $\sum\limits_{k=0}^{\infty} \Delta_{\pi(k)}x = x$ for all $x \in X$ and for each permutation $\pi : \mathbb{N}_0 \to \mathbb{N}_0$, where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

It is well known that when X is a UMD-space, $1 < p < \infty$, for $k \in \mathbb{N}$ if we define

$$\Delta_k f = \sum_{2^{k-1}\leq|m|<2^k} e_m \otimes \hat{f}(m) \qquad \text{and} \qquad \Delta_0 f = e_0 \otimes \hat{f}(0), \quad f \in L^p(0,2\pi;X),$$

then $(\Delta_k)_{k\in\mathbb{N}_0}$ is an unconditional Schauder decomposition of $L^p(0,2\pi;X)$ (see [3] for a proof).

We will use the following result of Clément-de Pagter-Sukochev-Witwiliet which gives the interplay between unconditional Schauder decomposition and the R-boundedness (see [5, Theorem 3.14]).

Theorem 2. *Let X be a Banach space that has property (α), let $\mathcal{M} \subset \mathcal{L}(X)$ be an R-bounded subset and let $(\Delta_k)_{k\geq 0}$ be an unconditional Schauder decomposition of X. Then*

$$\mathcal{S} = \Big\{\sum_{k=0}^{\infty} T_k\Delta_k : T_k \in \mathcal{T},\ \Delta_k T_k = T_k\Delta_k \ \text{ for all } k \geq 0\Big\}$$

is an R-bounded subset of $\mathcal{L}(X)$.

Notice that it has been shown by Clément-de Pagter-Sukochev-Witwiliet that if $(T_k)_{k\geq 0} \subset \mathcal{L}(X)$ is R-bounded and if $(\Delta_k)_{k\geq 0}$ is an unconditional Schauder decomposition of X satisfying $T_k\Delta_k = \Delta_k T_k$ for $k \geq 0$, then the sum $\sum\limits_{k=0}^{\infty} T_k\Delta_k$ converges and defines a bounded linear operator on X (see [5, Theorem 3.4]). One aim of this paper is to use Theorem 2 to obtain the following result which may be considered as a generalization of Theorem 1.

Theorem 3. *Let X, Y be UMD-spaces that have property (α), let $\mathcal{M}$ be an R-bounded subset in $\mathcal{L}(X,Y)$. Then $\{T_{(M_k)_{k\in\mathbb{Z}}} : M_k, k(M_{k+1}-M_k) \in \mathcal{M}$ for $k \in \mathbb{Z}\}$ is an R-bounded subset of $\mathcal{L}(L^p(0,2\pi;X), L^p(0,2\pi;Y))$, where $1<p<\infty$ and $T_{(M_k)_{k\in\mathbb{Z}}}$ denotes the bounded linear operator from $L^p(0,2\pi;X)$ to $L^p(0,2\pi;Y)$ defined by the multiplier $(M_k)_{k\in\mathbb{Z}}$.*

Proof. First assume that $X=Y$. Let $Z=\{f\in L^p(0,2\pi;X) : \hat{f}(k)=0 \text{ for all } k\leq 0\}$. Since X is a UMD-space, the Riesz projection from $L^p(0,2\pi;X)$ to Z is bounded. Hence to show the theorem, it suffices to show that $\{T_{(M_k)_{k\in\mathbb{Z}}} : M_k = 0 \text{ for all } k\leq 0$ and $M_k, k(M_{k+1}-M_k)\in\mathcal{M}$ for $k\in\mathbb{Z}\}$ is an R-bounded subset of $\mathcal{L}(L^p(0,2\pi;X))$.

Let $(M_k)_{k\in\mathbb{Z}}$ be such that $M_k=0$ for $k\leq 0$ and $M_k, k(M_{k+1}-M_k)\in\mathcal{M}$ for all $k\geq 0$. If A is a bounded linear operator on X, we denote by J_A the bounded linear operator on $L^p(0,2\pi;X)$ defined by $(J_Af)(t)=A(f(t))$ for $t\in[0,2\pi]$ and $f\in L^p(0,2\pi;X)$. Then it is easy to verify that both $\{J_{M_k} : k\in\mathbb{Z}\}$ and $\{k(J_{M_{k+1}}-J_{M_k}) : k\in\mathbb{Z}\}$ are subsets of $\{J_M : M\in\mathcal{M}\}$ which is R-bounded by assumption and Fubini's Theorem. Now the proof of Theorem 1 given in [1] implies that

$$\begin{aligned} T_{(M_k)_{k\in\mathbb{Z}}} &= \sum_{n\geq 1} J_{M_{2^{n-1}}} P_{2^{n-1}}\Delta_n + \sum_{n\geq 1} J_{M_{2^n-1}} P_{2^{n-1}}\Delta_n \\ &\quad + \sum_{n\geq 1}\Big[\sum_{k=2^{n-1}+1}^{2^n-1}(J_{M_k}-J_{M_{k-1}})P_k\Big]\Delta_n \\ &= T^{(1)}_{(M_k)_{k\in\mathbb{Z}}} + T^{(2)}_{(M_k)_{k\in\mathbb{Z}}} + T^{(3)}_{(M_k)_{k\in\mathbb{Z}}}, \end{aligned}$$

where P_k is the bounded linear projection on $L^p(0,2\pi;X)$ defined by

$$P_k\Big(\sum_{l\in\mathbb{Z}} e_l\otimes x_l\Big) = \sum_{l\geq k} e_l\otimes x_l,$$

and $(\Delta_n)_{n\geq 0}$ is the unconditional Schauder decomposition of $L^p(0,2\pi;X)$ given by

$$\Delta_n f = \sum_{2^{n-1}\leq |k|<2^n} e_k\otimes\hat{f}(k) \quad \text{for } n\geq 1 \qquad \text{and} \qquad \Delta_0 f = e_0\otimes\hat{f}(0).$$

Since $\{P_l : l\in\mathbb{Z}\}$ is R-bounded (see [1, Lemma 1.10]) and $\{J_{M_k} : k\in\mathbb{Z}\}$ is a subset of $\{J_M : M\in\mathcal{M}\}$ which is R-bounded, by Theorem 2,

$$\{T^{(1)}_{(M_k)_{k\in\mathbb{Z}}} + T^{(2)}_{(M_k)_{k\in\mathbb{Z}}} : M_k=0,\ n\in\mathbb{N}, \text{ for all } k\leq 0 \text{ and } M_k, k(M_{k+1}-M_k)\in\mathcal{M} \text{ for } k\in\mathbb{Z}\}$$

is R-bounded. We should mention that when X has property (α) and $1<p<\infty$, the space $L^p(0,2\pi;X)$ has also property (α), and

$$\mathcal{M}_1\mathcal{M}_2 = \{TS : T\in\mathcal{M}_1,\ S\in\mathcal{M}_2\}$$

is R-bounded whenever $\mathcal{M}_1$ and $\mathcal{M}_2$ are R-bounded subsets. For the R-boundedness of the third part in the decomposition of $T_{(M_k)_{k\in\mathbb{Z}}}$, by Theorem 2 we need to show that the set

$$\begin{aligned} \Big\{ \sum_{k=2^{n-1}+1}^{2^n-1} (J_{M_k}-J_{M_{k-1}})P_k : n\in\mathbb{N},\ M_k=0 \text{ for all } k\leq 0 \\ \text{and} \qquad M_k, k(M_{k+1}-M_k)\in\mathcal{M} \qquad \text{for} \qquad k\in\mathbb{Z}\Big\} \end{aligned}$$

is R-bounded. This follows from the fact that the complex absolute convex hull of an R-bounded subset is still R-bounded (see [5, Lemma 3.3]) and the trivial estimate

$$\sum_{k=2^{n-1}+1}^{2^n-1} 1/k \leq 1.$$

Now we consider the general case. Since X, Y are UMD-spaces that have property (α), also $X \oplus Y$ is a UMD-space and has property (α). Define $M_k' \in \mathcal{L}(X \otimes Y)$ by

$$M_k'(x, y) = (0, M_k x).$$

It follows from the first part of the proof that

$$\{T_{(M_k')_{k\in\mathbb{Z}}} : M_k, k(M_{k+1} - M_k) \in \mathcal{M} \text{ for } k \in \mathbb{Z}\}$$

is R-bounded. Then this implies that

$$\{T_{(M_k)_{k\in\mathbb{Z}}} : M_k, k(M_{k+1} - M_k) \in \mathcal{M} \text{ for } k \in \mathbb{Z}\}$$

is R-bounded. The proof is completed.

The above result can be also considered as a generalization of a result of A. Venni [10, Theorem 3], where he has established the result in the case $\mathcal{M} = \Omega I$ with Ω a bounded subset of $\mathbb{C}$ and I denotes the identity of X, in this case $\mathcal{M}$ is trivially R-bounded.

We have shown that each bounded subset in $\mathcal{L}(X, Y)$ is R-bounded if and only if X is of cotype 2 and Y is of type 2 (see [1, Proposition 1.13]). One immediate application of this result and Theorem 3 is the following

Corollary 1. *Let X, Y be UMD-spaces that have property (α). Assume that X is of cotype 2 and Y is of type 2. Then*

$$\Big\{T_{(M_k)_{k\in\mathbb{Z}}} : M_k \in \mathcal{L}(X, Y),\ \sup_{k\in\mathbb{Z}} \|M_k\| \leq 1,\ \sup_{k\in\mathbb{Z}} \|k(M_{k+1} - M_k)\| \leq 1\Big\}$$

is R-bounded subset in $\mathcal{L}(L^p(0, 2\pi; X), L^p(0, 2\pi; Y))$.

Next we give one application of Theorem 3 to the study of the strongly L^p-well-posedness of evolution equation with periodic boundary condition. Let X be a complex Banach space and let $1 \leq p < \infty$. We denote

$$H^{1,p}_{\text{per}} = \Big\{f \in L^p(0, 2\pi; X) : \text{ there exists } g \in L^p(0, 2\pi; X) \text{ and } x \in X \text{ such that}$$
$$f(t) = x + \int_0^t g(s)ds \qquad \text{for} \quad t \in [0, 2\pi] \qquad \text{and} \qquad \int_0^{2\pi} g(s)ds = 0\Big\}$$

the periodic Sobolev space. Each function in $H^{1,p}_{\text{per}}$ can be identified with a continuous function and it is a.e. differentiable.

Now let A be a closed operator on X. For $1 \leq p < \infty$ and $f \in L^p(0, 2\pi; X)$, we consider the problem

$$\mathrm{P}_{\text{per}} \qquad \begin{cases} u'(t) = Au(t) + f(t), & t \in [0, 2\pi], \\ u(0) = u(2\pi). \end{cases}$$

By a strong L^p-solution we understand a function $u \in H^{1,p}_{\text{per}}$ such that $u(t) \in D(A)$ and $u'(t) = Au(t) + f(t)$ for almost all $t \in [0, 2\pi]$. We say that the problem P_{per} is strongly

L^p-well-posed if such solution exists and is unique for each $f \in L^p(0, 2\pi; X)$. In [1], the following characterization based on Theorem 1 of the strongly L^p-well-posedness for $\mathrm{P_{per}}$ was given.

Theorem 4. *Assume that X is a UMD-space and $1 < p < \infty$. Then the following assertions are equivalent:*

(i) *The problem $\mathrm{P_{per}}$ is strongly L^p-well-posed;*
(ii) *$i\mathbb{Z} \subset \varrho(A)$ and $(kR(ik, A))_{k\in\mathbb{Z}}$ is an L^p-multiplier;*
(iii) *$i\mathbb{Z} \subset \varrho(A)$ and the sequence $(kR(ik, A))_{k\in\mathbb{Z}}$ is R-bounded.*

Let X be a complex Banach space and $1 \leq p < \infty$. We denote by $\mathrm{Rad}_p(X)$ the closure of

$$\Big\{ \sum_{j=1}^{n} \gamma_j \otimes x_j : x_j \in X, n \in \mathbb{N} \Big\}$$

in $L^p(0, 1; X)$. By Kahane's inequality (see [7, Theorem 1.e.13]), $\mathrm{Rad}_p(X)$ is independent from the choice of $1 \leq p < \infty$ and the norms induced by $L^p(0, 1; X)$ on $\mathrm{Rad}_p(X)$ are all equivalent, so we will denote $\mathrm{Rad}_p(X)$ simply by $\mathrm{Rad}(X)$ equipped with the norm induced by $L^2(0, 1; X)$. It is known that

$$\mathrm{Rad}(X) = \Big\{ \sum_{j=1}^{\infty} \gamma_j \otimes x_j : \text{ the series } \sum_{j=1}^{\infty} \gamma_j \otimes x_j \text{ converges in } L^2(0, 1; X) \Big\}$$

(see [2, Section 3]). Let A be a closed operator on X and assume that $i\mathbb{Z} \subset \varrho(A)$. Define the operator on $\mathrm{Rad}(X)$ by

$$\mathcal{D}(\mathcal{A}) = \Big\{ \sum_{j=1}^{\infty} \gamma_j \otimes x_j : x_j \in D(A),\ \sum_{j=1}^{\infty} \gamma_j \otimes \frac{1}{j} A x_j \text{ converges in } L^2(0, 1; X) \Big\},$$

$$\mathcal{A}\Big(\sum_{j=1}^{\infty} \gamma_j \otimes x_j \Big) = \sum_{j=1}^{\infty} \gamma_j \otimes \frac{1}{j} A x_j.$$

It is straightforward to verify that $\mathcal{A}$ is a closed linear operator on $\mathrm{Rad}(X)$.

Another consequence of Theorem 3 is the following

Theorem 5. *Let X be a UMD-space that has property (α) and let $1 < p < \infty$. Assume that A is a closed operator on X such that $i\mathbb{Z} \subset \varrho(A)$. Then the problem $\mathrm{P_{per}}$ is strongly L^p-well-posed if and only if the problem $\mathrm{P_{per}}$ associated with the operator $\mathcal{A}$ is strongly L^p-well-posed.*

Proof. Since X is a UMD-space that has property (α) and $1 < p < \infty$, the space $L^2(0, 1; X)$ is also a UMD-space and has property (α). Hence $\mathrm{Rad}(X)$ is a UMD-space and has property (α).

First assume that the problem $\mathrm{P_{per}}$ is strongly L^p-well-posed. Then by Theorem 1, $i\mathbb{Z} \subset \varrho(A)$ and $\{ik(ik - A)^{-1} : k \in \mathbb{Z}\}$ is R-bounded. It is easy to see that $i\mathbb{Z} \subset \varrho(\mathcal{A})$ and

$$\sup_{k\in\mathbb{Z}} \|ik(ik - \mathcal{A})^{-1}\| \leq R_2(\{ik(ik - A)^{-1} : k \in \mathbb{Z}\}) < \infty.$$

For fixed $k \geq 1$, consider the multiplier on $L^p(0, 2\pi; X)$ defined by the sequence $M_k = (M_{k,n})$, where $M_{k,n} = in(in - \frac{A}{k})^{-1}$ for $k \geq 1$ and $n \in \mathbb{Z}$. It is clear that $M_{k,n} \in \{im(im - A)^{-1} : m \in \mathbb{Z}\} := \mathcal{M}$ and for $k \geq 1$ and $n \in \mathbb{Z}$,

$$n(M_{k,n+1} - M_{k,n}) = ink(ink - A)^{-1} A (i(n + 1)k - A)^{-1}$$

is in $\mathcal{M}\mathcal{N} := \{ST : S \in \mathcal{M}, T \in \mathcal{N}\}$ which is R-bounded (see [5, Lemma 3.3]), where $\mathcal{N} := \{A(im - A)^{-1} : m \in \mathbb{Z}\}$ is clearly R-bounded. By Theorem 3, if we denote by T_k the bounded linear operator on $L^p(0, 2\pi; X)$ defined by the multiplier M_k, then $\{T_k : k \geq 1\}$ is an R-bounded subset of $\mathcal{L}(L^p(0, 2\pi; X))$. Note that if $\sum_j e_j \otimes x_{j,k}$ are X-valued trigonometric polynomials, one has

$$T_k\Big(\sum_n e_n \otimes x_{n,k}\Big) = \sum_n e_n \otimes in\Big(in - \frac{A}{k}\Big)^{-1} x_{n,k}.$$

There exists $C \geq 0$ depending only on X, p and A such that

$$\Big\|\sum_k \gamma_k T_k\Big(\sum_n e_n \otimes x_{n,k}\Big)\Big\|_{L^p(0,1;L^p(0,2\pi;X))} \leq C\Big\|\sum_k \gamma_k\Big(\sum_n e_n \otimes x_{n,k}\Big)\Big\|_{L^p(0,1;L^p(0,2\pi;X))}.$$

On the other hand, by Fubini's Theorem, we have

$$\begin{aligned}
&\Big\|\sum_n e_n \otimes in(in - \mathcal{A})^{-1}\Big(\sum_k \gamma_k \otimes x_{n,k}\Big)\Big\|_{L^p(0,2\pi;L^p(0,1;X))}\\
&= \Big\|\sum_n e_n \otimes \Big(\sum_k \gamma_k \otimes in\Big(in - \frac{A}{k}\Big)^{-1} x_{n,k}\Big)\Big\|_{L^p(0,2\pi;L^p(0,1;X))}\\
&= \Big\|\sum_k \gamma_k \otimes \Big(\sum_n e_n \otimes in\Big(in - \frac{A}{k}\Big)^{-1} x_{n,k}\Big)\Big\|_{L^p(0,1;L^p(0,2\pi;X))}\\
&= \Big\|\sum_k \gamma_k \otimes T_k\Big(\sum_n e_n \otimes x_{n,k}\Big)\Big\|_{L^p(0,1;L^p(0,2\pi;X))}\\
&\leq C\Big\|\sum_k \gamma_k \otimes \sum_n e_n \otimes x_{n,k}\Big\|_{L^p(0,1;L^p(0,2\pi;X))}\\
&= C\Big\|\sum_n e_n \otimes \sum_k \gamma_k \otimes x_{n,k}\Big\|_{L^p(0,2\pi;L^p(0,1;X))}.
\end{aligned}$$

This shows that the sequence $(in(in - \mathcal{A})^{-1})_{n\in\mathbb{Z}}$ is an L^p-multiplier. By Theorem 4, the problem $\mathrm{P}_{\mathrm{per}}$ associated with $\mathcal{A}$ is strongly L^p-well-posed.

Now assume that the problem $\mathrm{P}_{\mathrm{per}}$ associated with $\mathcal{A}$ is strongly L^p-well-posed, then by Theorem 4 we have $i \in \varrho(\mathcal{A})$. This implies that there exists $C \geq 0$ such that for $\sum_k \gamma_k \otimes x_k \in \mathrm{Rad}(X)$,

$$\begin{aligned}
&\Big\|\sum_k \gamma_k \otimes ik(ik - A)^{-1} x_k\Big\|_{L^p(0,1;\mathrm{Rad}(X))}\\
&= \Big\|i(i - \mathcal{A})^{-1}\Big(\sum_k \gamma_k \otimes x_k\Big)\Big\|_{L^p(0,1;L^p(0,2\pi;X))} \leq C\Big\|\sum_k \gamma_k \otimes x_k\Big\|_{L^p(0,1;\mathrm{Rad}(X))}.
\end{aligned}$$

This means that the set $\{ik(ik - A)^{-1} : k \in \mathbb{Z}\}$ is R-bounded, so the problem $\mathrm{P}_{\mathrm{per}}$ is strongly L^p-well-posed by Theorem 4. The proof is completed.

In the second part of this paper, we study the L^p-multipliers on $\mathbb{R}$. Let X be a Banach space and consider the Banach space $L^p(\mathbb{R}; X)$ for $1 < p < \infty$. We denote by $\mathcal{D}(\mathbb{R}; X)$ the space of all X-valued C^∞-functions with compact support. $\mathcal{S}(\mathbb{R}; X)$ will be the X-valued Schwartz space equipped with the locally convex topology generated by the seminorms

$$\|f\|_k = \sup_{x\in\mathbb{R},\alpha\leq k} (1 + |x|)^k \|f^{(\alpha)}(x)\|_X, \qquad k \in \mathbb{Z}$$

and we let

$$\mathcal{S}'(\mathbb{R};X) := \mathcal{L}(\mathcal{S}(\mathbb{R}),X),$$

where $\mathcal{S}(\mathbb{R})$ denotes the $\mathbb{C}$-valued Schwartz space. Let Y be another Banach space. Then given $M \in L^1_{\mathrm{loc}}(\mathbb{R};\mathcal{L}(X,Y))$, we may define an operator $T : \mathcal{F}^{-1}\mathcal{D}(\mathbb{R};X) \to \mathcal{S}'(\mathbb{R};Y)$ by means of

$$T\phi := \mathcal{F}^{-1}M\mathcal{F}\phi \qquad \text{for all } \mathcal{F}\phi \in \mathcal{D}(\mathbb{R};X),$$

where $\mathcal{F}$ denotes the Fourier transform. Since $\mathcal{F}^{-1}\mathcal{D}(\mathbb{R};X)$ is dense in $L^p(\mathbb{R};X)$, we see that T is well defined on a dense subset of $L^p(\mathbb{R};X)$. We say that M is an L^p-multiplier if T can be extended to a bounded linear operator from $L^p(\mathbb{R};X)$ to $L^p(\mathbb{R};Y)$.

It is known that the R-boundedness of the set $\{M(x) : x \neq 0\}$ is necessary for the function $M \in C^1(\mathbb{R} \setminus \{0\};\mathcal{L}(X,Y))$ to be a multiplier from $L^p(\mathbb{R};X)$ to $L^p(\mathbb{R};Y)$ (see [4, Proposition 1]).

The following result is the operator-valued version of vector-valued Mikhlin theorem of Bourgain [3], McConnell [8] and Zimmermann [13], it is due to Weis [11] (see [4] for another proof).

Theorem 6. *Suppose that X,Y are UMD-spaces, $1 < p < \infty$, let $M \in C^1(\mathbb{R} \setminus \{0\};\mathcal{L}(X,Y))$ be such that the subsets $\{M(x) : x \neq 0\}$ and $\{xM'(x) : x \neq 0\}$ are R-bounded. Then M is an L^p-multiplier from $L^p(\mathbb{R};X)$ to $L^p(\mathbb{R};Y)$.*

Let X be a UMD-space, $1 < p < \infty$ and let $M \in C^1(\mathbb{R}\setminus\{0\};\mathcal{L}(X))$ such that both sets $\{M(x) : x \neq 0\}$ and $\{xM'(x) : x \neq 0\}$ are R-bounded. We will denote by T_M the associated bounded linear operator on $L^p(\mathbb{R};X)$. The proof of Theorem 6 given by Clément and Prüss in [4] combined with Theorem 2 gives the following result which is somehow the analogue of our Theorem 3 in the case of L^p-multipliers on $\mathbb{R}$.

Theorem 7. *Let X,Y be UMD-spaces that have property (α) and let $1 < p < \infty$. Assume that $\mathcal{M}$ is an R-bounded subset of $\mathcal{L}(X,Y)$. Then the set $\{T_M : M \in C^1(\mathbb{R} \setminus \{0\};\mathcal{L}(X,Y)), M(x), xM'(x) \in \mathcal{M}$ for $x \neq 0\}$ is R-bounded in $\mathcal{L}(L^p(\mathbb{R};X),L^p(\mathbb{R};Y))$.*

Proof. Since the Riesz projection on $L^p(\mathbb{R};X)$ is bounded as X is a UMD-space and $1 < p < \infty$, we only need to show that

$$\{T_M : M \in C^1((0,\infty);\mathcal{L}(X,Y)), M(x), xM'(x) \in \mathcal{M} \ \text{ for all } x > 0\}$$

is R-bounded.

Let $M \in C^1((0,\infty);\mathcal{L}(X,Y))$, and $M(x), xM'(x) \in \mathcal{M}$ for all $x > 0$. Consider the dyadic decomposition $[2^j, 2^{j+1})$ $(j \in \mathbb{Z})$ of $(0,\infty)$ and a fixed subdecomposition

$$A^k_{j,l} = [2^j + (l-1)2^{j-k}, 2^j + l2^{j-k}), \qquad l = 1,2,\cdots,2^k.$$

Consider the multiplier defined by

$$M_{j,k}(t) = M(2^j + l2^{j-k}) \qquad \text{for } \ t \in A^k_{j,l}.$$

Let R be the Riesz projection and for $\rho \in \mathbb{R}$, let $R_\rho = e^{i\rho\cdot}Re^{-i\rho\cdot}$. Let

$$\begin{aligned}
\Delta_j &= R_{2^j} - R_{2^{j+1}}, && j \in \mathbb{Z},\\
D^k_{j,l} &= R_{2^j+(l-1)2^{j-k}} - R_{2^j+l2^{j-k}}, && j \in \mathbb{Z},\ l = 1,2,\cdots,2^k,\\
P^k_{j,l} &= \sum_{r=l}^{2^k} D^k_{j,r}, && j \in \mathbb{Z},\ l = 1,2,\cdots,2^k.
\end{aligned}$$

Then $(\Delta_j)_{j\in\mathbb{Z}}$ is an unconditional Schauder decomposition of $L^p(\mathbb{R};X)$ and

$$\mathcal{Q} = \{P^k_{j,l} :\ j\in\mathbb{Z},\ l=1,2,\cdots,2^k\}$$

is R-bounded. By [4], we have

$$T_{M_{j,k}} = \sum_{j\in\mathbb{Z}} M(2^j)\Delta_j + \sum_{j\in\mathbb{Z}} \Big[\sum_{r=1}^{2^k}(M(2^j+r2^{j-k}) - M(2^j+(r-1)2^{j-k}))P^k_{j,r}\Big]\Delta_j.$$

By Theorem 2, the set

$$\Big\{\sum_{j\in\mathbb{Z}} M(2^j)\Delta_j : M\in C^1((0,\infty);\mathcal{L}(X,Y)), M(x), xM'(x)\in\mathcal{M} \text{ for all } x>0\Big\}$$

is R-bounded in $\mathcal{L}(L^p(\mathbb{R};X),L^p(\mathbb{R};Y))$ and its R-bound is independent of j and k. To show that the set

$$\Big\{ \sum_{j\in\mathbb{Z}} \Big[\sum_{r=1}^{2^k}(M(2^j+r2^{j-k}) - M(2^j+(r-1)2^{j-k}))P^k_{j,r}\Big]\Delta_j :$$
$$M\in C^1((0,\infty);\mathcal{L}(X,Y)), M(x), xM'(x)\in\mathcal{M} \ \text{ for all }\ x>0\Big\}$$

is R-bounded in $\mathcal{L}(L^p(\mathbb{R};X),L^p(\mathbb{R};Y))$ and its R-bound is independent of j and k, it suffices to find an R-bounded set $\mathcal{N}$ in $\mathcal{L}(L^p(\mathbb{R};X),L^p(\mathbb{R};Y))$ such that

$$\sum_{r=1}^{2^k}(M(2^j+r2^{j-k}) - M(2^j+(r-1)2^{j-k}))P^k_{j,r}\in\mathcal{N}.$$

We remark that

$$\begin{aligned} &M(2^j+r2^{j-k}) - M(2^j+(r-1)2^{j-k})\\ &= \int_0^1 \frac{d}{ds}M(2^j+(r-1)2^{j-k}+s2^{j-k})ds\\ &= 2^{-k}\int_0^1 2^jM'(2^j+(r-1)2^{j-k}+s2^{j-k})ds.\end{aligned}$$

Hence if we let $\mathcal{P}=\mathcal{M}\mathcal{Q}=\{TS : T\in\mathcal{M}, S\in\mathcal{Q}\}$ which is R-bounded as both $\mathcal{M}$ and $\mathcal{Q}$ are R-bounded. Then we can take $\mathcal{N}$ as the complex absolute convex hull of $\mathcal{P}$.

We have shown that

$$\{T_{M_{j,k}} : M\in C^1((0,\infty);\mathcal{L}(X,Y)), M(x), xM'(x)\in\mathcal{M} \ \text{ for all } x\neq 0\}$$

is R-bounded and its R-bound is independent of j and k. Applying a similar argument as in the second step of the proof of Theorem 1 in [4] and letting $k\to\infty$, this implies that the set

$$\{T_M : M\in C^1((0,\infty);\mathcal{L}(X,Y)), M(x), xM'(x)\in\mathcal{M} \ \text{ for all } x\neq 0\}$$

is R-bounded. This completes the proof.

The above result can be also considered as a generalization of a result of Venni [10, Theorem 9], where he has established the result in the case $\mathcal{M}=\Omega I$ with Ω a bounded subset of $\mathbb{C}$ and I denotes the identity of X, in this case the set $\mathcal{M}$ is trivially R-bounded.

Since each bounded subset in $\mathcal{L}(X,Y)$ is R-bounded if and only if X is of cotype 2 and Y is of type 2 (see [1, Proposition 1.13]). One immediate application of Theorem 7 is the following result which is the analogue of our Corollary 1 in the case of L^p-multipliers on $\mathbb{R}$.

Corollary 2. *Let X, Y be UMD-spaces that have property (α). Assume that X is of cotype 2 and Y is of type 2. Then*

$$\Big\{T_M : M \in C^1(\mathbb{R}\setminus\{0\};\mathcal{L}(X,Y)),\ \sup_{x\neq 0}\|M(x)\|\leq 1,\ \sup_{x\neq 0}\|xM'(x)\|\leq 1\Big\}$$

is R-bounded in $\mathcal{L}(L^p(\mathbb{R};X), L^p(\mathbb{R};Y))$.

In the last part, we give an application of Theorem 7. Let X be a complex Banach space and let A be the generator of a bounded analytic C_0-semigroup T_t on X. Consider the abstract Cauchy problem

$$\mathrm{P_c} \qquad \begin{cases} u'(t) = Au(t) + f(t), & t \in [0, +\infty), \\ u(0) = 0, \end{cases}$$

where $f \in L^p(0,+\infty;X)$ is given and $1 < p < \infty$. We say that $\mathrm{P_c}$ is strongly L^p-well-posed if for every $f \in L^p(0,\infty;X)$, the mild solution given by the formulae $u(t) = \int_0^t T_{t-s}f(s)ds$ $(t \in (0,+\infty))$ satisfies

$$\|Au\|_p + \|u'\|_p \leq C\|f\|_p$$

for some constant independent of f.

One important application of Theorem 6 is the interesting characterization of the strongly L^p-well-posedness of $\mathrm{P_c}$ in term of R-boundedness: when X is a UMD-space, $\mathrm{P_c}$ is strongly L^p-well-posed if and only if the set $\{is(is-A)^{-1} : s \in \mathbb{R}\setminus\{0\}\}$ is R-bounded (see [11]).

Now let $(p_j)_{j\geq 1}$ be a positive sequence. We introduce one operator $\mathcal{A}$ on $\mathrm{Rad}(X)$ in the following way:

$$\mathcal{D}(\mathcal{A}) = \Big\{\sum_{j=1}^{\infty}\gamma_j\otimes x_j : x_j \in D(A),\ \sum_{j=1}^{\infty}\gamma_j\otimes p_jAx_j \text{ converges in } L^p(0,1;X)\Big\},$$

$$\mathcal{A}\Big(\sum_{j=1}^{\infty}\gamma_j\otimes x_j\Big) = \sum_{j=1}^{\infty}\gamma_j\otimes p_jAx_j.$$

Then $\mathcal{A}$ is a densely defined closed operator. We can use similar method as in [2] to show that if the problem $\mathrm{P_c}$ is strongly L^p-well-posed, then $\mathcal{A}$ generates a bounded analytic C_0-semigroup on $\mathrm{Rad}(X)$. We have actually the following result.

Theorem 8. *Let X be a UMD-space that has the property (α), $1 < p < \infty$. Assume that A is the generator of a bounded analytic C_0-semigroup on X and that the associated problem $\mathrm{P_c}$ is strongly L^p-well-posed. Then the problem $\mathrm{P_c}$ associated with $\mathcal{A}$ is strongly L^p-well-posed.*

Proof. Since X is a UMD-space that has property (α) and $1 < p < \infty$, the space $L^2(0,1;X)$ is also a UMD-space which has property (α). Hence $\mathrm{Rad}(X)$ is a UMD-space that has property (α).

For fixed $k \geq 1$, consider the multiplier on $L^p(0, 2\pi; X)$ defined by the function $M_k(x) = p_k A(ix - p_k A)^{-1}$ $(x \neq 0)$. Let $\mathcal{M} = \{A(is - A)^{-1} : s \neq 0\}$. Then $\mathcal{M}$ is R-bounded as the problem $\mathrm{P_c}$ associated with A is strongly L^p-well-posed. For $x \neq 0$, we have

$$M_k(x) = A\Big(\frac{ix}{p_k} - A\Big)^{-1} \in \mathcal{M},$$

$$xM_k'(x) = -M_k(x) - M_k(x)^2 \in -\mathcal{M} - \mathcal{M}^2.$$

Since the set $-\mathcal{M} - \mathcal{M}^2$ is R-bounded, by Theorem 7, the set $\{T_{M_k} : k \geq 1\}$ is R-bounded in $\mathcal{L}(L^p(\mathbb{R}; X))$. Noting that for $f \in L^p(\mathbb{R}_+; X)$, we have

$$T_{M_k}(f)(t) = \int_0^t p_k A e^{(t-s)p_k A} f(s) ds.$$

By the R-boundedness of $\{T_{M_k} : k \geq 1\}$, there exists $C \geq 0$ such that for $n \in \mathbb{N}$ and $f_j \in L^p(\mathbb{R}_+; X)$ $(j = 1, 2, \cdots, n)$,

$$\Big\| \sum_{j=1}^n \gamma_j \otimes T_{M_k} f_j \Big\|_{L^p(0,1;L^p(\mathbb{R}_+;X))} \leq C \Big\| \sum_{j=1}^n \gamma_j \otimes f_j \Big\|_{L^p(0,1;L^p(\mathbb{R}_+;X))}.$$

Note that the operator $\mathcal{A}$ is the generator of the bounded analytic C_0-semigroup on $\mathrm{Rad}(X)$ defined by

$$\mathcal{T}_t\Big(\sum_{j=1}^{+\infty} \gamma_j \otimes x_j\Big) = \sum_{j=1}^{+\infty} \gamma_j \otimes T_{p_j t} x_j.$$

Hence for $\mathcal{F} = \sum\limits_{j=1}^N \gamma_j \otimes f_j \in L^p(\mathbb{R}_+; \mathrm{Rad}(X))$, we have by Fubini's Theorem,

$$\begin{aligned}
&\int_{\mathbb{R}_+} \Big\| \mathcal{A} \int_0^t e^{(t-s)\mathcal{A}_2} \mathcal{F}(s) ds \Big\|^p_{\mathrm{Rad}_p(X)} dt \\
&= \int_{\mathbb{R}_+} \int_0^1 \Big\| \sum_{j=1}^\infty \gamma_j(x) p_j A \int_0^t e^{(t-s)p_j A} f_j(s) ds \Big\|_X^p dxdt \\
&= \int_0^1 \int_{\mathbb{R}_+} \Big\| \sum_{j=1}^\infty \gamma_j(x) p_j A \int_0^t e^{(t-s)p_j A} f_j(s) ds \Big\|_X^p dtdx \\
&= \Big\| \sum_{j=1}^n \gamma_j \otimes T_{M_k} f_j \Big\|_{L^p(0,1;L^p(\mathbb{R}_+,X))} \\
&\leq C \Big\| \sum_{j=1}^n \gamma_j \otimes f_j \Big\|_{L^p(0,1;L^p(\mathbb{R}_+,X))} \\
&= C \|\mathcal{F}\|^p_{L^p(\mathbb{R}_+;\mathrm{Rad}(X))}.
\end{aligned}$$

This shows that the problem $\mathrm{P_c}$ associated with $\mathcal{A}$ is strongly L^p-well-posed.

Now let (Ω, Σ, μ) be a measure space and let $1 < p < \infty$. Consider the Banach space $X = L^p(\Omega, \Sigma, \mu)$. Let A be the generator of a bounded analytic semigroup T on X and let $(p_j)_{j\geq 1}$ be a positive sequence. We define an operator on $L^p(\Omega, \Sigma, \mu; l^2(\mathbb{N}))$ by

$$\mathcal{D}(\mathcal{A}) = \Big\{ (f_j)_{j\geq 1} : f_j \in D(A),\ \Big(\sum_{j=1}^\infty |q_j A f_j|^2 \Big)^{1/2} \in L^p(\Omega, \Sigma, \mu) \Big\}, \tag{1}$$

$$\mathcal{A}((f_j)_{j\geq 1}) = (q_j A f_j)_{j\geq 1}. \tag{2}$$

Then $\mathcal{A}$ generates the bounded analytic semigroup $\mathcal{T}$ on $L^p(\Omega, \Sigma, \mu; l^2(\mathbb{N}))$ defined by

$$\mathcal{T}((f_j)_{j\geq 1}) = (T_{p_j t} f_j)_{j\geq 1}.$$

We recall the well-known Khintchine's inequality: there exists $C_1, C_2 > 0$ such that for $f_j \in L^p(\Omega, \Sigma, \mu)$, we have

$$C_1 \Big\| \Big(\sum_{j\geq 1} |f_j|^2 \Big)^{1/2} \Big\|_p \leq \Big\| \sum_{j\geq 1} \gamma_j \otimes f_j \Big\|_{\mathrm{Rad}(X)} \leq C_2 \Big\| \Big(\sum_{j\geq 1} |f_j|^2 \Big)^{1/2} \Big\|_p.$$

One interesting consequence of Khintchine's inequality is that $\mathrm{Rad}(X)$ can be identified in a natural way with $L^p(\Omega, \Sigma, \mu; l^2(\mathbb{N}))$. The above consideration and Theorem 8 imply the following

Corollary 3. *Let (Ω, Σ, μ) be a measure space, let $1 < p < \infty$ and let $X = L^p(\Omega, \Sigma, \mu)$. Assume that A is the generator of a bounded analytic semigroup T on X, let $(p_j)_{j\geq 1}$ be a positive sequence and let $\mathcal{A}$ be the operator defined by* (1) *and* (2)*. If the problem* $\mathrm{P_c}$ *associated with A is strongly L^p-well-posed. Then the problem* $\mathrm{P_c}$ *associated with $\mathcal{A}$ is strongly L^p-well-posed.*

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