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ON PERIODIC DYNAMICAL SYSTEMS***

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Abstract

The authors investigate the existence and the global stability of periodic solution for dynamical systems with periodic interconnections, inputs and self-inhibitions. The model is very general, the conditions are quite weak and the results obtained are universal.

Keywords Neural network, Dynamical systems, Periodic solution, Global convergence
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§ 1. Introduction

Recurrently connected neural networks, sometimes called Grossberg-Hopfield neural networks, are described by the following differential equations

$$\frac{du_i(t)}{dt} = -d_i u_i(t) + \sum_{j=1}^{n} a_{ij} g_j(u_j(t)) + I_i \qquad (i = 1, \cdots, n), \tag{1.1}$$

where $g_j(x)$ are activation functions, d_i, a_{ij} are constants and I_i are constant inputs.

In practice, however, the interconnections contain asynchronous terms in general, and the interconnection weights a_{ij}, b_{ij}, self-inhibition d_i and inputs I_i should depend on time, often periodically. Therefore, we need to discuss the following dynamical systems with time-varying delays

$$\begin{aligned}\frac{du_i}{dt} = {} & -d_i(t)u_i(t) + \sum_{j=1}^{n} a_{ij}(t) g_j(u_j(t)) \\ & + \sum_{j=1}^{n} b_{ij}(t) f_j(u_j(t - \tau_{ij}(t))) + I_i(t) \qquad (i = 1, 2, \cdots, n),\end{aligned} \tag{1.2}$$

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or its particular case

$$\frac{du_i}{dt} = -d_i(t)u_i(t) + \sum_{j=1}^{n} a_{ij}(t)g_j(u_j(t)) + \sum_{j=1}^{n} b_{ij}(t)f_j(u_j(t-\tau_{ij})) + I_i(t) \qquad (i = 1, 2, \cdots, n), \tag{1.3}$$

and the systems with distributed delays

$$\frac{du_i(t)}{dt} = -d_i(t)u_i(t) + \sum_{j=1}^{n} a_{ij}(t)g_j(u_j(t)) + \sum_{j=1}^{n} b_{ij}(t)\int_0^\infty k_{ij}(s)f_j(u_j(t-\tau_{ij}(t)-s))ds + I_i(t) \qquad (i = 1, 2, \cdots, n), \tag{1.4}$$

where $d_i(t) > d_i > 0$, $a_{ij}(t), b_{ij}(t), \tau_{ij}(t) > 0, I_i(t) : \mathbf{R}^+ \to \mathbf{R}$ are continuously periodic functions with period $\omega > 0$, $i, j = 1, 2, \cdots, n$. For references, see [1–4, 6, 7] and the papers cited therein.

To unify models (1.2) and (1.4), we discuss the following general model

$$\frac{du_i}{dt} = -d_i(t)u_i(t) + \sum_{j=1}^{n} a_{ij}(t)g_j(u_j(t)) + \sum_{j=1}^{n} \int_0^\infty f_j(u_j(t-\tau_{ij}(t)-s))d_sK_{ij}(t,s) + I_i(t) \qquad (i = 1, 2, \cdots, n), \tag{1.5}$$

where $d_sK_{ij}(t,s)$, for any fixed $t \geq 0$, are Lebesgue-Stieljies measures and satisfy $d_sK_{ij}(t+\omega, s) = d_sK_{ij}(t,s)$, $d_i(t) > 0$, $a_{ij}(t), b_{ij}(t), I_i(t), \tau_{ij}(t) > 0 : \mathbf{R}^+ \to \mathbf{R}$ are continuously periodic functions with period $\omega > 0$.

The initial condition is

$$u_i(s) = \phi_i(s) \qquad \text{for} \quad s \in (-\infty, 0], \tag{1.6}$$

where $\phi_i \in C(-\infty, 0]$, $i = 1, \cdots, n$.

It is easy to see that if $d_sK_{ij}(t,0) = b_{ij}(t)$ and $d_sK_{ij}(t,s) = 0$ for $s \neq 0$, then (1.5) reduces to (1.2); In addition, if $\tau_{ij}(t) = \tau_{ij}$ are constants, then it reduces to (1.3). Instead, if $d_sK_{ij}(t,s) = b_{ij}(t)k_{ij}(s)ds$, then (1.5) reduces to (1.4).

As a precondition, we assume that for the system (1.5), there exists a unique solution with the initial condition (1.6) and the solution continuously depends on the initial data.

§ 2. Main Results

For the convenience, throughout this letter, we make the following two assumptions.

Assumption 2.1. *$|g_j(s)| \leq G_j|s| + C_j$, $|f_j(s)| \leq F_j|s| + D_j$, where $G_j > 0$, $F_j > 0$, C_j and D_j are constants $(j = 1, \cdots, n)$.*

Assumption 2.2. *$|g_i(x+h) - g(x)| \leq G_i|h|$ and $|f_i(x+h) - f(x)| \leq F_i|h|$ $(i = 1, \cdots, n)$.*

Main Theorem. *Suppose that Assumption* 2.1 *is satisfied. If there exist positive constants* $\xi_1, \xi_2, \cdots, \xi_n$ *such that for all* $t > 0$,

$$-\xi_i d_i(t) + \sum_{j=1}^{n} \xi_j G_j |a_{ij}(t)| + \sum_{j=1}^{n} \xi_j F_j \int_0^\infty |d_s K_{ij}(t,s)| < -\eta < 0 \qquad (i = 1, 2, \cdots, n), \tag{2.1}$$

then the system (1.5) *has at least an* ω*-periodic solution* $x(t)$*. In addition, if Assumption* 2.2 *is satisfied and there exists a constant* α *such that for all* $t > 0$,

$$\begin{aligned} -\xi_i(d_i(t) - \alpha) &+ \sum_{j=1}^{n} \xi_j G_j |a_{ij}(t)| \\ &+ \sum_{j=1}^{n} \xi_j F_j e^{\alpha \tau_{ij}(t)} \int_0^\infty e^{\alpha s} |d_s K_{ij}(t,s)| \le 0 \qquad (i = 1, 2, \cdots, n), \end{aligned} \tag{2.2}$$

then for any solution $u(t) = [u_1(t), \cdots, u_n(t)]$ *of* (1.5),

$$||u(t) - x(t)|| = O(e^{-\alpha t}), \qquad t \to \infty. \tag{2.3}$$

Proof. Pick a constant M satisfying $M > \frac{J}{\eta}$, where

$$J = \max_i \max_t \Big\{ \sum_{j=1}^{n} |a_{ij}(t)| C_j + \sum_{j=1}^{n} D_j \int_0^\infty |d_s K_{ij}(t,s)| + |I_i(t)| \Big\}. \tag{2.4}$$

And let $C = C((-\infty, 0], R^n)$ be the Banach space with norm

$$\|\phi\| = \sup_{-\infty < \theta \le \omega} \|\phi(\theta)\|_{\{\xi,\infty\}},$$

where

$$\|\phi(\theta)\|_{\{\xi,\infty\}} = \max_{i=1,\cdots,n} \xi^{-1} |\phi_i(\theta)|.$$

Denote

$$\Omega = \{x(\theta) \in C : \|x(\theta)\| \le M, \|\dot{x}(\theta)\| \le N\}, \tag{2.5}$$

where

$$N = (\alpha + \beta + \gamma)M + c$$

and

$$\begin{aligned} \alpha &= \max_i \sup_t |d_i(t)| \xi_i^{-1}, \\ \beta &= \max_{i,j} \sup_t |a_{ij}(t)| \xi_i^{-1} G_j, \\ \gamma &= \max_{i,j} \sup_t \int_0^\infty |d_s K_{ij}(t,s)| F_j \xi_i^{-1}, \\ c &= \max_i \sup_t |I_i(t)| \xi_i^{-1}. \end{aligned}$$

It is easy to check that Ω is a convex compact set.

Now, define a map T from Ω to C by

$$T : \phi(\theta) \longrightarrow x(\theta+\omega, \phi),$$

where $x(t) = x(t, \phi)$ is the solution of the system (1.5) with the initial condition $x_i(\theta) = \phi_i(\theta)$ for $\theta \in (-\infty, 0]$ and $i = 1, \cdots, n$.

In the following, we will prove that $T\Omega \subset \Omega$, i.e. if $\phi \in \Omega$, then $x \in \Omega$. To do that, we define the following function

$$M(t) = \sup_{s\in(-\infty,0]} \|x(t+s)\|_{\{\xi,\infty\}}, \tag{2.6}$$

It is easy to see that

$$\|x(t)\|_{\{\xi,\infty\}} \le M(t). \tag{2.7}$$

Therefore, what we need to do is to prove $M(t) \le M$ for all $t > 0$.

Assume that $t_0 \ge 0$ is the smallest value such that

$$\|x(t_0)\|_{\{\xi,\infty\}} = M(t_0) = M, \tag{2.8}$$

$$\|x(t)\|_{\{\xi,\infty\}} \le M \qquad \text{if} \;\; t < t_0. \tag{2.9}$$

Let i_0 be an index such that

$$\xi_{i_0}^{-1}|x_{i_0}(t)| = \|x(t_0)\|_{\{\xi,\infty\}}. \tag{2.10}$$

Then direct calculation gives

$$\begin{aligned}
\Big\{\frac{d|x_{i_0}(t)|}{dt}\Big\}_{t=t_0} &\le \mathrm{sign}(x_{i_0}(t_0))\Big\{ - d_{i_0}(t_0)x_{i_0}(t_0) + \sum_{j=1}^{n} a_{i_0j}(t_0)g_j(x_j) \\
&\quad + \sum_{j=1}^{n}\int_0^\infty f_j(x_j(t_0-\tau_{i_0j}(t_0)-s))d_sK_{i_0j}(t_0,s) + I_{i_0}(t_0)\Big\} \\
&\le -d_{i_0}(t_0)|x_{i_0}(t_0)| + \sum_{j=1}^{n}|a_{i_0j}(t)|G_j|x_j(t_0)| \\
&\quad + \sum_{j=1}^{n}F_j\int_0^\infty |x_j(t_0-\tau_{i_0j}(t_0)-s)||d_sK_{i_0j}(t_0,s)| + J \\
&\le \Big[- d_{i_0}(t_0)\xi_{i_0} + \sum_{j=1}^{n}|a_{i_0j}(t_0)|G_j\xi_j\Big]\|x(t_0)\|_{\{\xi,\infty\}} \\
&\quad + \sum_{j=1}^{n}F_j\xi_j\int_0^\infty \|x(t_0-\tau_{i_0j}(t_0)-s)\|_{\{\xi,\infty\}}|d_sK_{i_0j}(t_0,s)| + J \\
&\le \Big[- d_{i_0}(t_0)\xi_{i_0} + \sum_{j=1}^{n}|a_{i_0j}(t_0)|G_j\xi_j + \sum_{j=1}^{n}F_j\xi_j\int_0^\infty |d_sK_{i_0j}(t_0,s)|\Big]M(t_0) + J \\
&\le -\eta M(t_0) + J = -\eta M + J < 0,
\end{aligned} \tag{2.11}$$

which means that $\|x(t)\|_{\{\xi,\infty\}}$ can never exceed M. Thus, $\|x(t)\|_{\{\xi,\infty\}} \le M(t) \le M$ for all $t > t_0$. Moreover, it is easy to see that $\|\dot{x}(\theta+\omega)\| \le N$. Therefore, $T\Omega \subset \Omega$.

By Brouwer fixed point theorem, there exists $\phi^* \in \Omega$ such that $T\phi^* = \phi^*$. Hence $x(t,\phi^*) = x(t, T\phi^*)$, i.e.,

$$x(t,\phi^*) = x(t+\omega,\phi^*), \tag{2.12}$$

which is an ω-periodic solution of the system (1.5).

Now, we prove that (2.2) implies (2.3).

Let $\bar{u}(t) = [u(t) - x(t)]$, $z(t) = e^{\alpha t}\bar{u}(t)$. We have

$$\frac{dz_i(t)}{dt} = -(d_i(t)-\alpha)z_i(t) + e^{\alpha t}\Big\{\sum_{j=1}^n a_{ij}(t)\Big[g_j(u_j(t)) - g_j(x_j(t))\Big] + \sum_{j=1}^n \int_0^\infty \Big[f_j(u_j(t-\tau_{ij}(t)-s)) - f_j(x_j(t-\tau_{ij}(t)-s))\Big] d_sK_{ij}(t,s)\Big\}. \tag{2.13}$$

Therefore

$$\begin{aligned}\Big|\frac{dz_i(t)}{dt}\Big| &\le -(d_i(t)-\alpha)|z_i(t)| + \sum_{j=1}^n |a_{ij}(t)|G_j|z_j(t)| \\ &\quad + \sum_{j=1}^n F_j e^{\alpha\tau_{ij}(t)} \int_0^\infty e^{\alpha s}|z_j(t-\tau_{ij}(t)-s)||d_sK_{ij}(t,s)| \\ &\le \Big[-\xi_i(d_i(t)-\alpha) + \sum_{j=1}^n \xi_j|a_{ij}(t)|G_j\Big]\|z(t)\|_{\xi,\infty} \\ &\quad + \sum_{j=1}^n \xi_j F_j e^{\alpha\tau_{ij}(t)} \int_0^\infty e^{\alpha s}\|z_j(t-\tau_{ij}(t)-s)\|_{\xi,\infty}|d_sK_{ij}(t,s)|. \end{aligned} \tag{2.14}$$

By the same approach used before, we can prove that $z(t)$ is bounded. Then $\bar{u}(t) = O(e^{-\alpha t})$. Main Theorem is proved.

In particular, let $d_sK_{ij}(t,0) = b_{ij}(t)$ and $d_sK_{ij}(t,s) = 0$, we have

Corollary 2.1. *Suppose that Assumption* 2.1 *is satisfied. If there exist positive constants* $\xi_1, \xi_2, \cdots, \xi_n$ *such that for all* $t > 0$,

$$-\xi_i d_i(t) + \sum_{j=1}^n \xi_j G_j|a_{ij}(t)| + \sum_{j=1}^n \xi_j F_j |b_{ij}(t)| < 0 \qquad (i = 1, 2, \cdots, n), \tag{2.15}$$

in particular, if

$$-\xi_i d_i + \sum_{j=1}^n \xi_j G_j|a^*_{ij}| + \sum_{j=1}^n \xi_j F_j|b^*_{ij}| < 0 \qquad (i = 1, 2, \cdots, n), \tag{2.16}$$

then the system (1.2) *or* (1.3) *has at least an* ω*-periodic solution* $x(t)$. *In addition, if Assumption* 2.2 *is satisfied, and*

$$(-d_i(t)+\alpha)\xi_i + \sum_{j=1}^n \xi_j G_j|a_{ij}(t)| + \sum_{j=1}^n \xi_j F_j|b_{ij}(t)|e^{\alpha\tau_{ij}} \le 0 \qquad (i = 1, 2, \cdots, n), \tag{2.17}$$

then for any solution $u(t) = [u_1(t), \cdots, u_n(t)]$ *of* (1.2) *or* (1.3), *we have*

$$\|u(t) - x(t)\| = O(e^{-\alpha t}), \qquad t \to \infty. \tag{2.18}$$

Instead, if $d_sK_{ij}(t,s)=b_{ij}(t)k_{ij}(s)ds$, then we have

Corollary 2.2. *Suppose that Assumption* 2.1 *is satisfied. If there exist positive constants* $\xi_1,\xi_2,\cdots,\xi_n$ *such that for all* $t>0$, *there hold*

$$\begin{aligned}&-\xi_i d_i(t)+\sum_{j=1}^{n}\xi_j G_j|a_{ij}(t)|\\&+\sum_{j=1}^{n}\xi_j F_j|b_{ij}(t)|\int_0^\infty|k_{ij}(s)|ds<-\eta<0\qquad(i=1,2,\cdots,n),\end{aligned}\tag{2.19}$$

then the system (1.4) *has at least an* ω*-periodic solution* $x(t)$. *In addition, if Assumption* 2.2 *is satisfied and*

$$\begin{aligned}&-\xi_i(d_i(t)-\alpha)+\sum_{j=1}^{n}\xi_j G_j|a_{ij}(t)|\\&+\sum_{j=1}^{n}\xi_j F_j e^{\alpha\tau_{ij}(t)}\int_0^\infty e^{\alpha s}|k_{ij}(t,s)|ds\le 0\qquad(i=1,2,\cdots,n),\end{aligned}\tag{2.20}$$

then for any solution $u(t)=[u_1(t),\cdots,u_n(t)]$ *of* (1.4), *we have*

$$||u(t)-x(t)||=O(e^{-\alpha t}),\qquad t\to\infty.\tag{2.21}$$

§ 3 . Comparisons

In [7], by using the Mawhin continuation theory, the authors proved the following

Theorem A. *Suppose that Assumption* 2.1 *is satisfied. If there are real constants* $\epsilon>0$, $\xi_i>0$, $0<\alpha_{ij}<1$, $0<\beta_{ij}<1$, $i,j=1,2\cdots,n$, *such that*

$$\begin{aligned}&(-d_i+\alpha)\xi_i+G_i\Big[\xi_i|a_{ii}^*|+\frac{1}{2}\sum_{j\neq i}\xi_j|a_{ji}^*|^{2\alpha_{ji}}\Big]+\frac{1}{2}\xi_i\sum_{j\neq i}G_j|a_{ij}^*|^{2(1-\alpha_{ij})}\\&+\frac{1}{2}F_i\sum_{j=1}^{n}\xi_j|b_{ji}^*|^{2\beta_{ji}}e^{\alpha\tau_{ji}}+\frac{1}{2}\xi_i\sum_{j=1}^{n}F_j|b_{ij}^*|^{2(1-\beta_{ij})}e^{\alpha\tau_{ij}}<0\qquad(i=1,2,\cdots,n),\end{aligned}\tag{3.1}$$

where $|a_{ij}^*|=\sup_{0<t\le\omega}|a_{ij}(t)|<+\infty$, $|b_{ij}^*|=\sup_{0<t\le\omega}|b_{ij}(t)|<+\infty$, *then the dynamical system* (1.3) *has at least an* ω*-periodic solution* $v(t)=[v_1(t),\cdots,v_n(t)]$. *Instead, if Assumption* 2.2 *is satisfied, then for any solution* $u(t)=[u_1(t),\cdots,u_n(t)]$ *of* (1.3),

$$\|u(t)-v(t)\|=O(e^{-\alpha t}),\qquad t\to\infty.\tag{3.2}$$

In [6], the following comparison theorem was given.

Theorem B. *If the set of inequalities* (3.1) *holds, then there exist constants* θ_i, $i=1,\cdots,n$, *such that*

$$(-d_i+\alpha)\theta_i+\sum_{j=1}^{n}\theta_j G_j|a_{ij}^*|+\sum_{j=1}^{n}\theta_j F_j|b_{ij}^*|e^{\alpha\tau_{ij}}<0.\tag{3.3}$$

But, the converse is not true.

Therefore, the conditions (3.1) are much more restrictive than (3.3). And Theorem A is a special case of Corollary 2.1.

In [3], the authors claimed that they investigate model (1.2) with time-varying delays under the assumption that $\tau_{ij}(t)$ is periodic and $0 \le \tau'_{ij}(t) < 1$. However, if $0 < \tau'_{ij}(t)$, then $\tau_{ij}(t)$ is not periodic. Thus, $\tau_{ij}(t)$ must be constants. The model reduces to the model (1.3). Therefore, they investigate only the model (1.3) with constant time delays, rather than model (1.2) with time-varying delays.

Under Assumption 2.2 with $g_j(x) = f_j(x)$ being increasing, they proved that if the following inequalities

$$-d_i + \sum_{j=1}^{n} G_j(1+d_i\omega)|a^*_{ij}| + \sum_{j=1}^{n} F_j(1+d_i\omega)|b^*_{ij}| < 0 \qquad (i = 1, 2, \cdots, n), \quad (3.4)$$

and some other inequalities hold, then the dynamical system has at least a periodic solution.

It is clear that this result is also a special case of Corollary 2.1. Moreover, their conditions are too strong.

§ 4. Numerical Example

In this section, we give a numerical example to verify our Main Theorem. Consider a delayed neural network with 3 neurons:

$$\begin{aligned}
\frac{dx_1}{dt} &= -\Big[2.51 + \frac{1}{2}\sin^2(\pi t)\Big]x_1(t) + \sin^2(2\pi t)\tanh(x_1(t)) + \cos^2(2\pi t)\tanh(x_2(t)) \\
&\quad + \sin^2(\pi t)\tanh(x_3(t)) + e^{-1}\sin^2(4\pi t)\arctan(x_1(t - |\sin(2\pi t)|)) \\
&\quad + e^{-1}\cos^2(4\pi t)\arctan\Big(x_2\Big(t - \frac{\pi}{2}|\cos(2\pi t)|\Big)\Big) \\
&\quad - \frac{e^{-1}}{2}\cos^2(\pi t)\arctan(x_3(t-1)) + \sin(2\pi t),
\end{aligned}$$

$$\begin{aligned}
\frac{dx_2}{dt} &= -[0.91 + 0.1\sin^2(\pi t) + 0.5\sin^2(4\pi t)]x_2(t) - 0.5\sin^2(2\pi t)\tanh(x_1(t)) \\
&\quad + 0.2\cos^2(4\pi t)\tanh(x_2(t)) + 0.3\sin^2(\pi t)\tanh(x_3(t)) \\
&\quad - 0.7e^{-1}\sin^2(4\pi t)\arctan(x_1(t - |\sin(2\pi t)|)) \\
&\quad + 0.5e^{-1}\cos^2(2\pi t)\arctan\Big(x_2\Big(t - \frac{\pi}{2}|\cos(2\pi t)|\Big)\Big) \\
&\quad + 0.2e^{-1}\cos^2(\pi t)\arctan(x_3(t-1)) + 2\cos(\pi t),
\end{aligned}$$

$$\begin{aligned}
\frac{dx_3}{dt} &= -[0.51 + 0.2\cos^2(\pi t) + 0.2\sin^2(2\pi t) + 0.1\sin^2(4\pi t)]x_3(t) \\
&\quad - 0.4\cos^2(\pi t)\tanh(x_1(t)) + 0.3\sin^2(2\pi t)\tanh(x_2(t)) \\
&\quad + 0.2\cos^2(4\pi t)\tanh(x_3(t)) + 0.2e^{-1}\sin^2(\pi t)\arctan(x_1(t - |\sin(2\pi t)|)) \\
&\quad + 0.1e^{-1}\cos^2(2\pi t)\arctan\Big(x_2\Big(t - \frac{\pi}{2}|\cos(2\pi t)|\Big)\Big) \\
&\quad + 0.3e^{-1}\sin^2(4\pi t)\arctan(x_3(t-1)) + 2\sin(2\pi t).
\end{aligned}$$

It is easy to see that the conditions (2.15) in Corollary 2.1 are satisfied. However, the conditions (3.1) in Theorem A used in [7] and (3.4) used in [3] are not satisfied. Fig. 1 shows that $x_i(t)$ $(i = 1, 2, 3)$, converges to a periodic function, respectively.

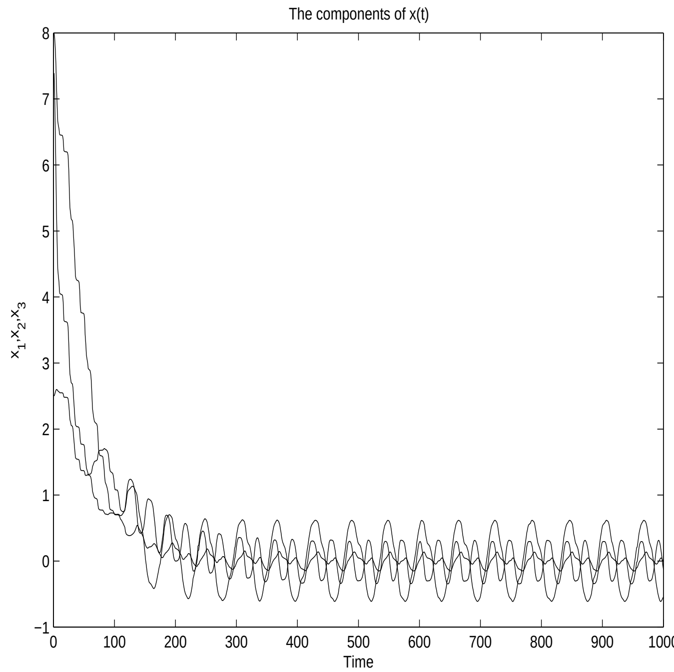

Fig. 1

§ 5 . Conclusions

In this paper, we address periodic dynamical systems. Under much weaker conditions, the existence of periodic solution and its exponential stability are proved.

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