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ON THE EXISTENCE OF GLOBAL ATTRACTOR FOR A CLASS OF INFINITE DIMENSIONAL DISSIPATIVE NONLINEAR DYNAMICAL SYSTEMS***

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Abstract

By means of a nonstandard estimation about the energy functional, the authors prove the existence of a global attractor for an abstract nonlinear evolution equation. As an application, the existence of a global attractor for some nonlinear reaction-diffusion equations with some distribution derivatives in inhomogeneous terms is obtained.

Keywords Global attractor, Measures of noncompactness, Reaction-diffusion equations
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§ 1 . Introduction

The main purpose of this paper is to prove the existence of the global attractor for the following abstract dynamical system in Hilbert space:

$$\frac{du}{dt} + Au + g(u) = f. \tag{1.1}$$

As an application, we consider the existence of the global attractor for the following nonlinear reaction-diffusion equation

$$\frac{\partial u}{\partial t} - \Delta u + g(u) = D_i f^i + f, \tag{1.2}$$

$$u|_{\partial\Omega} = 0. \tag{1.3}$$

Corresponding to the abstract dynamical system (1.1), we give three Hilbert spaces H, V and $D(A)$ satisfying

$$D(A) \subset V \subset H = H^* \subset V^*, \tag{1.4}$$

where the injections are all compact continuous, each space is dense in the next one, H^* and V^* are the dual spaces of H and V respectively. As usual, the norm and the scalar product on H and V are denoted by $|\cdot|$, $(\cdot,\cdot)$ and $\|\cdot\|$, $((\cdot,\cdot))$ respectively.

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We assume that the operator A is self-adjoint,

$$A : \begin{array}{l} V \to V^* \\ D(A) \to H \end{array} \quad \text{is an isomorphism} \tag{1.5}$$

and there exists $\alpha > 0$ such that

$$\langle Au, u\rangle_{V^*} \geq \alpha \|u\|^2 \qquad \text{for any } u \in V. \tag{1.6}$$

The inner product and the norm on $D(A)$ are denoted by (Au, Au) and $|Au| = (Au, Au)^{\frac{1}{2}}$ respectively.

Under these assumptions, $A^{-1} : H \to D(A) \hookrightarrow H$ is compact and there exists a complete orthonormal basis of H, $\{w_j\}_{j\in N}$, such that

$$Aw_j = \lambda_j w_j, \qquad j = 1, 2, \cdots, \tag{1.7}$$

$$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_j \leq \cdots \quad \text{and} \quad \lambda_j \to \infty \quad \text{as } j \to \infty, \tag{1.8}$$

$$(w_j, w_k) = \delta_{jk}, \quad \langle Aw_j, w_k\rangle = \lambda_j \delta_{jk}. \tag{1.9}$$

The assumptions about the nonlinear term $g(u)$ in (1.1) and (1.2) will be given in the next two sections respectively.

The problem about the existence of global attractor for the system (1.1) has widely applying background, especially in physics and mechanics, and many important results associated with this problem have been obtained in the past two decades, see [5, 8, 10–13].

Since the solutions of (1.1) or (1.2)–(1.3) have no higher regularity, we can not use the usual existence theorem and the standard estimation of the energy functional to prove the existence of global attractor for our problem.

Recently, using the concept of non-compactness measure, the authors of [9] have given a new method to obtain the existence of global attractors for some abstract semigroups as follows.

Theorem 1.1. (see [9]) *Let X be a Banach space and $\{S(t)\}_{t\geq 0}$ be a continuous semigroup on X. If the following conditions hold:*

(1) *There exists a bounded absorbing set for $\{S(t)\}_{t\geq 0}$ in X;*

(2) *For any bounded set B of X and any $\varepsilon > 0$, there exist $t(B) > 0$ and a finite dimensional subspace X_1 of X, such that*

$$\{\|PS(t)x\| \mid x \in B, t \geq t(B)\} \ \text{is bounded}$$

and

$$\|(I - P)S(t)x\| \leq \varepsilon \qquad \text{for any } x \in B,\ t \geq t(B),$$

where $P : X \to X_1$ is a bounded projector. Then there exists a global attractor for $\{S(t)\}_{t\geq 0}$ in $(X, \|\cdot\|)$. Furthermore, if X is a uniformly convex Banach space, especially a Hilbert space, then the two conditions mentioned above are also necessary.

We will use this theorem and some nonstandard estimations of the energy functional to prove the existence of global attractors for the dynamical system (1.1) and the reaction-diffusion equations (1.2)–(1.3).

Our main results are Theorem 2.1 and Theorem 3.1 which are stated and proved in Sections 2 and 3 respectively.

§ 2 . The Existence of the Global Attractor for System (1.1)

Definition 2.1. *Let X be a Banach space and $\{S(t)\}_{t\geq 0}$ be a continuous semigroup on X. We say that $\{S(t)\}_{t\geq 0}$ is uniformly continuous with respect to t, if for any $\varepsilon > 0$ and any bounded set B in X, there exists $\delta > 0$ such that*

$$\|S(t)x - x\| < \varepsilon \qquad \text{for any } x \in B,\ 0 \leq t \leq \delta. \tag{2.1}$$

Theorem 2.1. *Assume that $f \in V^*$ and the operator A in the system (1.1) satisfies all the assumptions given in Section 1. Assume furthermore that the semigroup $\{S(t)\}_{t\geq 0}$ of solutions corresponding to (1.1) is well defined for all nonnegative time and continuous in H. Then $\{S(t)\}_{t\geq 0}$ has a global attractor in H, provided that*

(1) $\{S(t)\}_{t\geq 0}$ is uniformly continuous with respect to t in H;

(2) $\{S(t)\}_{t\geq 0}$ has a bounded absorbing set B_0 in H;

(3) There exist two positive constants C_0 and C_1 such that all solutions $u(t) = S(t)u_0$ of the system (1.1) with initial data $u_0 \in B_0$ satisfy the following inequalities

$$\int_t^{t+r} \langle Au, u\rangle ds \leq C_0 r + C_1, \tag{2.2}$$

$$\int_t^{t+r} |\langle g(u), u\rangle| ds \leq C_0 r + C_1. \tag{2.3}$$

Proof. Without loss of generality, we assume $f = 0$.

Since $\{S(t)\}_{t\geq 0}$ has a bounded absorbing set in H, for any bounded set $B \subset H$, there exists $t(B) > 0$ such that

$$u(t) = S(t)u_0 \in B_0 \qquad \text{for any } t \geq t(B),\ u_0 \in B.$$

Especially, if we take $B = B_0$, then there exists $t_0 > 0$ such that

$$u(t) = S(t)u_0 \in B_0 \qquad \text{for any } t \geq t_0,\ u_0 \in B_0.$$

So there exists a positive constant C_2 such that

$$|u(t)|^2 = |S(t)u_0|^2 \leq C_2 \qquad \text{as } t \geq t_1 = t(B) + t_0,\ u_0 \in B. \tag{2.4}$$

Taking the inner product of (1.1) with $u(t)$ and integrating from t to $t + r$ with respect to s, we obtain

$$\frac{1}{2}|u(t+r)|^2 + \int_t^{t+r} \langle Au, u\rangle ds + \int_t^{t+r} \langle g(u), u\rangle ds = \frac{1}{2}|u(t)|^2 \leq C_2 \qquad \text{as } t \geq t_1,\ u_0 \in B. \tag{2.5}$$

Then, noting (2.3), we get

$$|u(t+r)|^2 + 2\int_t^{t+r} \langle Au, u\rangle ds \leq 2(C_0 r + C_1 + C_2) \qquad \text{as } t \geq t_1,\ u_0 \in B.$$

For brevity, we assume

$$|u(t+r)|^2 + \int_t^{t+r} \langle Au, u\rangle ds \leq C_0 r + C_1 + C_2 \qquad \text{as } t \geq t_1,\ u_0 \in B. \tag{2.6}$$

Now, in order to prove that $\{S(t)\}_{t\geq 0}$ has a global attractor in H, we only need to verify the second presumption in Theorem 1.1.

Let

$$H_1^m = \text{span}\{w_1, \cdots, w_m\} \quad \text{and} \quad H_2^m = (H_1^m)^{\perp},$$

where $\{w_j\}$ satisfies (1.7)–(1.9). Each u in H can be decomposed in the following form

$$u = u_1^m + u_2^m, \qquad u_1^m \in H_1^m,\ u_2^m \in H_2^m,$$

where $u_1^m = P_m u$, $P_m : H \to H_1^m$ is an orthogonal projector. By (1.9) and the properties of the operator, it follows from (2.5) that

$$|u_2^m(t+r)|^2 + \lambda_m \int_t^{t+r} |u_2^m(s)|^2 ds \leq |u_2^m(t+r)|^2 + \int_t^{t+r} \langle Au_2^m, u_2^m\rangle ds$$

$$\leq |u(t+r)|^2 + \int_t^{t+r} \langle Au, u\rangle ds \leq C_0 r + C_1 + C_2. \tag{2.7}$$

Now in order to prove our result, it is sufficient to prove that for any $\varepsilon > 0$, there exist $T > 0$ and $M > 0$ such that if $r \geq T$ and $m \geq M$, then we have

$$|u_2^m(t+r)|^2 = |(I-P_m)u(t+r)|^2 = |(I-P_m)S(t+r)u_0|^2 < \varepsilon \qquad \text{for any } u_0 \in B. \tag{2.8}$$

(2.8) will be accomplished in three steps.

Step 1. Take M_0 and T_0 so large that $T_0 \geq \frac{C_1+C_2}{C_0}$ and $\lambda_m \geq \frac{4C_0}{\varepsilon}$ as $m \geq M_0$. We claim that if $t \geq t_1$, $u_0 \in B$, t_1 being given by (2.4), then there exists $r_0 \in [0, T_0]$ such that

$$|u_2^m(t+r_0)|^2 < \frac{\varepsilon}{2}. \tag{2.9}$$

In fact, if this conclusion is not true, then there exist some $t' \geq t_1$, $u_0' \in B$, such that for any $r \in [0, T_0]$, we have

$$|u_2^m(t'+r)|^2 \geq \frac{\varepsilon}{2}. \tag{2.10}$$

However, it follows from (2.7) that

$$|u_2^m(t'+T_0)|^2 \leq C_0T_0 + C_1 + C_2 - \lambda_m \int_{t'}^{t'+T_0} |u_2^m(s)|^2 ds$$

$$\leq C_1 + C_2 - \Big(\lambda_m \frac{\varepsilon}{2} - C_0\Big)T_0 \leq C_1 + C_2 - C_0T_0 \leq 0,$$

which contradicts (2.10).

Step 2. If there exists an $r' \in [0, T_0]$ such that

$$|u_2^m(t+r')|^2 < \frac{\varepsilon}{2} \qquad \text{as } t \geq t_1,\ u_0 \in B,$$

then from the uniformly continuity of the semigroup with respect to t, there exists a constant $\delta > 0$, such that for any $r \in [0, \delta]$, we have

$$|u_2^m(t+r'+r)| \leq |u_2^m(t+r')| + |u_2^m(t+r'+r) - u_2^m(t+r')|$$

$$\leq |u_2^m(t+r')| + |u_2(t+r'+r) - u_2(t+r')|$$

$$\leq \sqrt{\frac{\varepsilon}{2}} + |S(r)\cdot u(t+r') - u(t+r')| \leq \sqrt{\frac{\varepsilon}{2}} + \sqrt{\frac{\varepsilon}{8}}.$$

Hence

$$|u_2^m(t+r'+r)|^2 \le \varepsilon. \tag{2.11}$$

Step 3. Combining the two steps above, we choose $M_1(\ge M)$ such that $\lambda_m \ge \frac{2}{\varepsilon}(C_0 + \frac{C_1+C_2}{\delta})$ *as* $m \ge M_1$. We infer that

$$|u_2^m(t+r)|^2 \le \varepsilon \qquad \text{as } t \ge t_1,\ u_0 \in B,\ r \ge T_0. \tag{2.12}$$

In fact, if there exists an $r_0 \ge T_0$ such that $|u_2^m(t+r_0)|^2 > \varepsilon$, then set

$$r_* = \sup\left\{ r \in [0, r_0] \,\Big|\, |u_2^m(t+r)|^2 = \frac{\varepsilon}{2} \right\},$$

$$r^* = \inf\left\{ r \in [r_*, r_0] \mid |u_2^m(t+r)|^2 = \varepsilon \right\},$$

which implies that

$$\frac{\varepsilon}{2} \le |u_2^m(t+r)|^2 \le \varepsilon \qquad \text{if } r \in [r_*, r^*].$$

Combining this with the proof of Step 2, we get

$$r^* - r_* \ge \delta. \tag{2.13}$$

Noting (2.7) again, we get

$$\begin{aligned}
\varepsilon = |u_2^m(t+r^*)|^2 &\le C_0(r^*-r_*) + C_1 + C_2 - \lambda_m \int_{t+r_*}^{t+r^*} |u_2^m(s)|^2 ds \\
&\le C_1 + C_2 - \Big(\lambda_m \frac{\varepsilon}{2} - C_0\Big)(r^*-r_*) \le C_1 + C_2 - \Big(\lambda_m \frac{\varepsilon}{2} - C_0\Big)\delta \le 0.
\end{aligned}$$

It is a contradiction. Therefore (2.10) holds and the proof is complete.

Analogously, if we take Au as a test function, then we can also obtain the uniformly boundedness about

$$\int_t^{t+r} \langle Au, Au\rangle ds \quad \text{and} \quad \int_t^{t+r} |\langle g(u), Au\rangle| ds$$

with respect to t, and then we have the following result.

Theorem 2.2. *Assume that $f \in H$ and the operator A in the system* (1.1) *satisfies all the assumptions given in Section* 1. *Assume furthermore that the semigroup $\{S(t)\}_{t\ge 0}$ of solutions corresponding to* (1.1) *is well defined for all nonnegative time and continuous in V. Then $\{S(t)\}_{t\ge 0}$ has a global attractor in V, provided that*

(1) *$\{S(t)\}_{t\ge 0}$ is uniformly continuous with respect to t in V;*

(2) *$\{S(t)\}_{t\ge 0}$ has a bounded absorbing set B_0 in V;*

(3) *There exist two positive constants C_0 and C_1 such that all solutions $u(t) = S(t)u_0$ of the system* (1.1) *with initial data $u_0 \in B_0$ satisfy the following inequalities*

$$\int_t^{t+r} \langle Au, Au\rangle ds \le C_0 r + C_1, \tag{2.14}$$

$$\int_t^{t+r} |\langle g(u), Au\rangle| ds \le C_0 r + C_1. \tag{2.15}$$

Remark 2.1. We can apply Theorem 2.2 to Equation (1.2) if we remove the distribution term $D_i f^i$. In (1.2), in this case we can take Au as a test function.

§ 3 . The Existence of the Global Attractor for the Nonlinear Reaction-Diffusion Equation with Distribution Derivatives

In this section, we will use Theorem 2.1 to consider the existence of a global attractor for the continuous semigroup generated by weak solutions of the following nonlinear reaction-diffusion equation

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + g(u) = D_i f^i + f, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \\ u(0,x) = u_0, & \text{in } \Omega, \end{cases} \tag{3.1}$$

where Ω is a bounded domain in R^n with smooth boundary, Δ is the Laplace operator in Ω, $D_i = \frac{\partial}{\partial x_i}$ is distribution derivative, $f^i,\ f \in L^2(\Omega)$ $(i = 1, \cdots, n)$ and g is a C^1 function with $g(0) = 0$ and satisfies the following assumptions

$$C_1|s|^p - C_0 \le g(s)s \le C_2|s|^p + C_0, \qquad p \ge 2 \tag{3.2}$$

and

$$g'(s) \ge -l \tag{3.3}$$

for all $s \in R$.

There are many results about the existence of global attractors for the reaction-diffusion equation without the inhomogeneous term $D_i f^i + f$, see [10, 11, 13]. Now, since the equation includes some distribution derivatives in inhomogeneous term, the solution has no higher regularity. Therefore we can not take $-\Delta u$ as a test function for the equation and can not use the usual method to prove that the semigroup of solution of (3.1) has a bounded absorbing set in $H_0^1(\Omega)$. However, we can use the abstract result, Theorem 2.1 , to prove that the system (3.1) has also a global attractor in H.

Let $A = -\Delta$, $H = L^2(\Omega)$, $V = H_0^1(\Omega)$ and $D(A) = H^2(\Omega) \cap H_0^1(\Omega)$, $\{\lambda_j\}$ be the eigenvalues of $-\Delta$ in $H_0^1(\Omega)$ and $\{w_j\}$ be the eigenvectors corresponding to $\{\lambda_j\}$ which form an orthonormal basis of H. The conditions about the space and operator A in Theorem 2.1 are all satisfied, and in order to verify the other conditions in Theorem 2.1, we only need to show the following four lemmas.

Lemma 3.1. *Assume that $\Omega \subset R^n$ is a bounded domain with smooth boundary, $f^i,\ f \in H$ $(i = 1, 2, \cdots, n)$ and g is a C^1 function with $g(0) = 0$ and satisfies* (3.2) *and* (3.3). *Then for any $T > 0$ and initial data $u_0 \in H$, there exists a unique weak solution u of Equation* (3.1) *which satisfies*

$$u \in L^2(0,T;V) \cap L^p(\Omega_T), \quad \Omega_T = (0,T) \times \Omega \quad \text{and} \quad u \in C([0,T];H).$$

Furthermore, the mapping $u_0 \to u(t)$ is continuous in H and if $v(t)$ is the solution with initial data v_0, then we have the following stability

$$|u(t) - v(t)| \le |u_0 - v_0| e^{lt}, \tag{3.4}$$

where l is given in (3.3).

This lemma and the next lemma are completely similar to that in [11, 13], so we omit the proofs.

Lemma 3.2. *Under the hypothesis of Lemma* 3.1*, the semigroup* $\{S(t)\}_{t\geq 0}$ *of solutions for* (3.1) *possesses a bounded absorbing set* B_0 *in* H*, and there exist two constants* C_3*,* $C_4 > 0$ *such that for any* $u_0 \in B_0$*, the corresponding solution* $u(t)$ *of* (3.1) *satisfies*

$$\int_t^{t+r} \langle Au, u\rangle ds \leq C_3 r + C_4, \tag{3.5}$$

$$\int_t^{t+r} |g(u)\cdot u| ds \leq C_3 r + C_4, \tag{3.6}$$

where $t \geq t_0$*,* $r > 0$ *is arbitrary.*

Lemma 3.3. (see [11]) *Let* $X \hookrightarrow H \hookrightarrow Y$ *be Banach spaces such that* X *is reflexive and the embedding* $X \hookrightarrow H$ *is compact. Suppose that* $\{u_n\}$ *is a uniformly bounded sequence in* $L^2(0,T;X)$ *and* $\left\{\frac{du_n}{dt}\right\}$ *is uniformly bounded in* $L^r(0,T;Y)$ *for some* $r > 1$*. Then there is a subsequence of* $\{u_n\}$*, which converges strongly in* $L^2(0,T;H)$*.*

This lemma is a compactness theorem. In our case, we take

$$X = V = H_0^1(\Omega), \quad H = L^2(\Omega), \quad Y = H^{-s}(\Omega),$$

where $s > 0$ such that

$$H^{-1}(\Omega) + L^q(\Omega) \subset H^{-s}(\Omega), \qquad \frac{1}{q} + \frac{1}{p} = 1.$$

Hence, all solutions of (3.1) with initial data in B_0 are uniformly bounded in $L^2(t_0, T_0; V)$, and $\{\frac{du(t)}{dt} \mid t \in [t_0, T_0], u_0 \in B_0\}$ is also uniformly bounded in $L^q(t_0, T_0; H^{-s})$ $(q > 1)$. Therefore, by Lemma 3.3, $\{u(t) \mid t \in [t_0, T_0], u_0 \in B_0\}$ is compact in $L^2(t_0, T_0; H)$, where $t_0 > 0$ is a constant such that $u(t) \in B_0$ as $t \geq t_0$, and $T_0 > t_0$ is fixed.

Lemma 3.4. *Under all the assumptions of Lemma* 3.1*, the semigroup* $\{S(t)\}_{t\geq 0}$ *associated with Equation* (3.1) *is uniformly continuous with respect to* t*.*

Proof. Since B_0 is a bounded absorbing set, we only need to prove that for any $\varepsilon > 0$, there exists a $\delta > 0$ such that for any $0 \leq r \leq \delta$, $t \geq t_0$ and $u_0 \in B_0$, we have

$$|S(t+r)u_0 - S(t)u_0| = |S(r)(S(t)u_0) - S(t)u_0| < \varepsilon. \tag{3.7}$$

Since $\{u(t) = S(t)u_0 \mid t \in [t_0, T_0],\ u_0 \in B_0\}$ is compact in $L^2(0,T;H)$, we can choose $u_1, \cdots, u_N$ such that for any $u(t) = S(t)u_0$, $t \in [t_0, T_0]$ and $u_0 \in B_0$, there exists some $i, 1 \leq i \leq N$, satisfying

$$\Big(\int_{t_0}^{T_0} |u(t) - u_i(t)|^2 \, dt\Big)^{\frac{1}{2}} < \frac{\varepsilon}{4}(T_0 - t_0)^{\frac{1}{2}} e^{-l(T_0 - t_0)}, \tag{3.8}$$

which implies that there exists some $t_1 \in [t_0, T_0]$, such that

$$|u(t_1) - u_i(t_1)| < \frac{\varepsilon}{3} e^{-l(T_0 - t_0)}. \tag{3.9}$$

Combining it with (3.4), we have

$$|u(t) - u_i(t)| \leq |u(t_1) - u_i(t_1)|\, e^{l|t-t_1|} \leq \frac{\varepsilon}{3} \qquad \text{for any } t \in [t_0, T_0]. \tag{3.10}$$

On the other hand, $u_i \in C([t_0, T_0]; H)$, $i = 1, 2, \cdots, n$, and then we can find a $\delta > 0$ such that for any $i, 1 \leq i \leq N$, we have

$$|u_i(t) - u_i(t')| < \frac{\varepsilon}{3} \qquad \text{as } t,\ t' \in [t_0, T_0],\ |t - t'| < \delta. \tag{3.11}$$

Therefore, it follows from (3.10) and (3.11) that

$$|S(t+r)u_0 - S(t)u_0| = |S(t_0+r)S(t-t_0)u_0 - S(t_0)S(t-t_0)u_0|$$
$$< |S(t_0+r)S(t-t_0)u_0 - u_i(t_0+r)| + |u_i(t_0+r) - u_i(t_0)| + |u_i(t_0) - S(t_0)S(t-t_0)u_0| < \varepsilon.$$

The proof is complete.

Thanks to the above four lemmas, by Theorem 2.1 we can obtain the following result immediately.

Theorem 3.1. *Assume that Ω is a bounded domain in R^n with smooth boundary, f^i, $f \in H$, $i = 1, \cdots, n$, and g is a C^1 function with $g(0) = 0$ and satisfies the constructive conditions* (3.2) *and* (3.3). *Then the semigroup $\{S(t)\}_{t \geq 0}$ associated with Equation* (3.1) *has a global attractor, which attracts uniformly any bounded set in H.*

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