Chin. Ann. Math.
28B(5), 2007, 571–582
DOI: 10.1007/s11401-006-0014-x

Chinese Annals of Mathematics, Series B

Boundary Identification for a Blast Furnace**

Yongji TAN* Rui HE*

Abstract In this paper, the authors discuss an inverse boundary problem for the axi-symmetric steady-state heat equation, which arises in monitoring the boundary corrosion for the blast-furnace. Measure temperature at some locations are used to identify the shape of the corrosion boundary.

The numerical inversion is complicated and consuming since the wear-line varies during the process and the boundary in the heat problem is not fixed. The authors suggest a method that the unknown boundary can be represented by a given curve plus a small perturbation, then the equation can be solved with fixed boundary, and a lot of computing time will be saved.

A method is given to solve the inverse problem by minimizing the sum of the squared residual at the measuring locations, in which the direct problems are solved by axi-symmetric fundamental solution method.

The numerical results are in good agreement with test model data as well as industrial data, even in severe corrosion case.

Keywords Inverse problem, Axi-Symmetric fundamental solution, Perturbation method
2000 MR Subject Classification 11R70, 11R11, 11R27

1 Introduction

The problem origins from iron production. Ilmenite ore consists of various titanium and iron oxides. After several pre-processing steps the ilmenite ore is melted in a blast furnace. Since the density of liquid titanium dioxide is less than that of liquid iron, the two substances are separated and pure iron is obtained. The blast furnace is axi-symmetric. The sidewall is cooled by water and the bottom of the furnace is cooled by air (see Figure 1).

The walls of the furnace are subject to both physical and chemical wear. Thus it is important to monitor the wearing to avoid molten metal from breaking out through the furnace wall and causing damage. So the shape of the inner wall surface must be determined. Many researches have been made in this field (see [1–5, 8]).

Some of the pioneering work was performed in Japan by Yoshikawa et al [5], who considered axi-symmetric configurations of blast furnaces. They attempted to incorporate the effects of the solidified melt layer in the inverse formulation based on the use of boundary elements method for heat conduction analysis and a shape optimization algorithm that could handle only a relatively small number of design variables.

Manuscript received January 13, 2006. Revised September 27, 2006. Published online September 13, 2007.
*School of Mathematical Sciences, Fudan University, Shanghai 200433, China.
E-mail: yjtan@fudan.edu.cn hr_bread@163.com
**Project supported by the National Natural Science Foundation of China (No. 10431030).

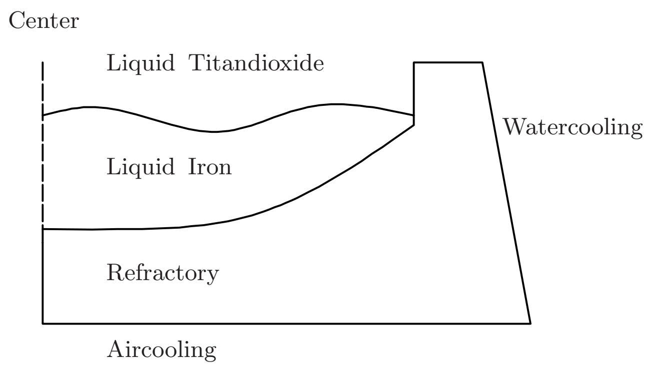


Figure 1 Vertical section of the blast furnace

Sorli and Skaar [2] from Norway reported on a very exact inverse methodology that converges quite rapidly because it utilizes an adjoint operator formulation. However, the method was demonstrated only for very simple smooth shapes of the inner surface of the wall that were not significantly different from the initially guessed wall surface configurations.

Dulikravich developed a new method in his recent paper (see [4]). The method was developed for determination of wall thickness distribution in blast furnaces by utilizing external surface measurements of temperature and heat flux and employing a Fourier series formulation of the solution of an elastic membrane model for the evolution of the furnace wall inner surface shape. But the measurement of heat flux is not easy to implement.

This paper will use a different method. Because the corrosion process of the inner wall is very slow, we assume that the wear-line can be represented by a known curve plus a small perturbation. Then we can solve the problem with the fixed boundary. It has a big advantage to use this perturbation method. In order to get the shape of the corrosion boundary, we have to solve an inverse problem by many times of iteration. Whatever method we use, FEM, BEM or FSM (see [6, 7]), we all have to solve the forward problem a large number of times. However, the boundary changes over the process. So the mesh grid must be subdivided again and again to form new coefficient matrix during the iteration process. It dose take a lot of time to do this. But if we use the perturbation method, the problem can be simplified. We can transfer the original problem to a fixed boundary value problem. The coefficient matrix will not change, and we do the decomposition only once. We just back substitute it in every iteration. Conservatively estimate, more than $\frac{2}{3}$ computing time can be saved.

In the last section of this paper, we report on some theoretical studies for the algorithm. By asymptotic expanding temperate function u, we can get the error between the exact value and the approximation value is $O(\varepsilon^2)$. The numerical results are in good agreement with both test model data and industrial data. Even in severe corrosion case, our method can still converse the corrosion boundary accurately.

2 Mathematical Model of Heat Conduction

The direct problem is to solve a stationary axi-symmetric heat conduction problem on a given domain Ω, with boundary conditions. Figure 2 is an axi-symmetric vertical section of the furnace. In the section there are 9 thermocouples installed inside the walls, and the conductivity of the furnace material is piecewise constant.

The equation is

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\Big(rk\frac{\partial u}{\partial r}\Big)+\frac{\partial}{\partial z}\Big(k\frac{\partial u}{\partial z}\Big)=0, \quad \text{in } \Omega, \tag{2.1}$$

where u is the temperature at point $(r,z)\in\Omega$, where r and z are the radial and axial coordinates, respectively. The thermal conductivity of the material, i.e., magnesia bricks, k, is considered to be temperature dependent. In practice, k is a piecewise constant, which has nothing to do with u.

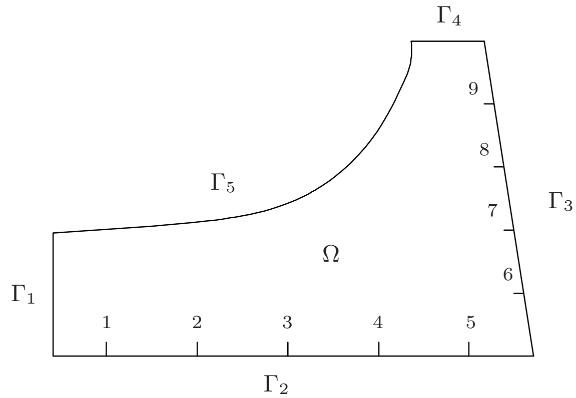


Figure 2 Domain and measurement locations

Equation (2.1) describes heat conduction in cylindrical coordinates when the angular direction component of the conduction is negligible. The boundary of Ω,Γ, is split into 5 segments, as shown in Figure 2,

$$\Gamma=\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4\cup\Gamma_5. \tag{2.2}$$

The boundary conditions for the problem are as follows. On Γ_1, the heat flux is zero, since the model is rotational symmetric,

$$\frac{\partial u}{\partial r}=0, \quad \text{on } \Gamma_1. \tag{2.3}$$

The bottom segment Γ_2 is air-cooled, so we have a mixed condition. α_2 is the heat transfer coefficient between the bottom of the furnace and air, and u_2 is the ambient air temperature,

$$-k(u)\frac{\partial u}{\partial z}+\alpha_2 u=\alpha_2 u_2, \quad \text{on } \Gamma_2. \tag{2.4}$$

The sidewall Γ_3 is water-cooled, so we have a mixed condition as well. α_3 is the heat transfer coefficient between the sidewall and water, and u_3 is the ambient water temperature,

$$k(u)\frac{\partial u}{\partial n}+\alpha_3 u=\alpha_3 u_3, \quad \text{on } \Gamma_3. \tag{2.5}$$

The upper boundary Γ_4 is also assumed to be insulated,

$$\frac{\partial u}{\partial z}=0, \quad \text{on } \Gamma_4. \tag{2.6}$$

At the wear-line boundary Γ_5, we have a Dirichlet condition. In practice, it is the isotherm with temperature 1450°C, which is the melting temperature of iron,

$$u=f(r,z), \quad \text{on } \Gamma_5. \tag{2.7}$$

3 Perturbation Method

In order to solve the inverse problem of identifying the isotherm, we have to solve many direct problems in which the boundaries are constantly changing. The corrosion of the boundary causes some inconvenience in numerical solution which makes the process very difficult. For the corrosion is very slow and the change of the shape of inner surface is very small. So we can use perturbation method (see [9, 10]) to overcome this difficulty. The wear-line will be represented by a known curve plus a small perturbation, all solution will be done on the known boundary. The boundary value problem then can be changed from flexible boundary value problem into problem with fixed boundary.

For instance, we consider a boundary value problem for an axi-symmetric Laplace equation

$$\begin{cases} \triangle u = 0, \quad \text{in } \Omega, \\ u|_{\Gamma_1} = f, \\ \Big(k\dfrac{\partial u}{\partial n} + \alpha u\Big)\Big|_{\Gamma_2} = q. \end{cases} \tag{3.1}$$

Here the operator $\triangle$ is defined as

$$\triangle = \frac{1}{r}\cdot\frac{\partial}{\partial r}\Big(r\frac{\partial}{\partial r}\Big) + \frac{\partial}{\partial z}\Big(\frac{\partial}{\partial z}\Big),$$

where Γ_2 is fixed boundary while Γ_1 is unknown and can be represented by a given curve $\overline{\Gamma}_1 = \{(r,z) \mid z = s(r)\}$ plus a small perturbation,

$$\Gamma_1 = \{(r,z) \mid z = s(r) + \varepsilon g(r)\}.$$

Formally asymptotic expand u about ε as

$$u = u_0 + \varepsilon u_1 + \cdots,$$

and take the first two terms as an approximation of u. Then the boundary condition for Γ_1 can be rewritten as

$$\begin{aligned} u|_{\Gamma_1} &= u(r, s(r) + \varepsilon g(r)) \\ &= u(r, s(r)) + \frac{\partial u}{\partial z}(r, s(r))\cdot\varepsilon g(r) + O(\varepsilon^2) \\ &= u|_{\overline{\Gamma}_1} + \frac{\partial u}{\partial z}\Big|_{\overline{\Gamma}_1}\cdot\varepsilon g(r) + O(\varepsilon^2) \\ &= u_0|_{\overline{\Gamma}_1} + \varepsilon u_1|_{\overline{\Gamma}_1} + \frac{\partial u_0}{\partial z}\Big|_{\overline{\Gamma}_1}\cdot\varepsilon g(r) + O(\varepsilon^2) \\ &= f. \end{aligned}$$

Comparing the coefficients of the same power of ε, we obtain

$$u_0|_{\overline{\Gamma}_1} = f \quad \text{and} \quad u_1|_{\overline{\Gamma}_1} = -\frac{\partial u_0}{\partial z}\Big|_{\overline{\Gamma}_1}\cdot g(r).$$

It is easy to change the form of the boundary condition for Γ_2, and we obtain

$$\Big(k\frac{\partial u_0}{\partial n} + \alpha u_0\Big)\Big|_{\Gamma_2} = q \quad \text{and} \quad \Big(k\frac{\partial u_1}{\partial n} + \alpha u_1\Big)\Big|_{\Gamma_2} = 0.$$

With the same method, we can rewrite the Neumann condition on Γ_1

$$\Big(k\frac{\partial u}{\partial n} + \alpha u\Big)\Big|_{\Gamma_1} = \alpha\overline{u}$$

as

$$\Big(k\frac{\partial u_0}{\partial n}+\alpha u_0\Big)\Big|_{\overline{\Gamma}_1}=\alpha\overline{u}$$

and

$$\Big(k\frac{\partial u_1}{\partial n}+\alpha u_1\Big)\Big|_{\overline{\Gamma}_1}=\frac{s'(r)g'(r)}{1+s'^2(r)}\alpha(\overline{u}-u_0)\Big|_{\overline{\Gamma}_1}+\frac{kg'(r)}{\sqrt{1+s'^2(r)}}\cdot\frac{\partial u_0}{\partial r}\Big|_{\overline{\Gamma}_1}-\alpha\frac{\partial u_0}{\partial z}\cdot g(r)\Big|_{\overline{\Gamma}_1}.$$

With this perturbation method, we change problem (3.1) into two problems satisfied by u_0 and u_1, respectively:

$$\begin{cases}\triangle u_0=0, \quad \text{in } \Omega,\\ u_0|_{\overline{\Gamma}_1}=f,\\ \Big(k\dfrac{\partial u_0}{\partial n}+\alpha u_0\Big)\Big|_{\Gamma_2}=q,\end{cases}\tag{3.2}$$

$$\begin{cases}\triangle u_1=0, \quad \text{in } \Omega,\\ u_1|_{\overline{\Gamma}_1}=-\dfrac{\partial u_0}{\partial z}\cdot g(r)|_{\overline{\Gamma}_1},\\ \Big(k\dfrac{\partial u_1}{\partial n}+\alpha u_1\Big)\Big|_{\Gamma_2}=0.\end{cases}\tag{3.3}$$

They are both boundary value problems with fixed boundary. However, in (3.3) the boundary condition at the right hand depends on u_0.

Solve problems (3.2) and (3.3), and let $\overline{u}=u_0+\varepsilon u_1$ be the approximate solution for the original problem (3.1). Here we use the fundamental solution method (see [6, 14]) to get the numerical solution. The fundamental solution for axi-symmetric Laplace equation with boundary condition $Bu|_{\partial\Omega}=q$ is denoted by $E(X,X')$, which satisfies

$$\triangle E(X,X')=\delta(X,X').$$

We will give its specific expression in the next section. Select a set of control points, $X^j\notin\overline{\Omega}$ $(1\le j\le N)$, and take

$$u^N(X,\{C_j\},\{X^j\}):=\sum_{j=1}^{N}C_jE(X,X^j)$$

as an approximation of $u(X)$. In order to determine the coefficient vector C, we select a set of collocation points, $Y_i\in\partial\Omega$ $(1\le i\le M)$, so that the boundary condition $Bu^N(Y_i)=q(Y_i)$ is satisfied, i.e.,

$$\sum_{j=1}^{N}C_jBE(Y_i,X^j)=q(Y_i), \quad 1\le i\le M.$$

We denote it as

$$HC=Q,$$

where $H=(h_{ij}),\ h_{ij}=BE(Y_i,X^j)$.

In the case that the conductivity k is a piecewise constant, the solution will be a little different. The simplified case is that the domain is divided into two parts Ω_1 and Ω_2, correspondingly, the boundary is split into two segments Γ_1 and Γ_2, and the interface is denoted

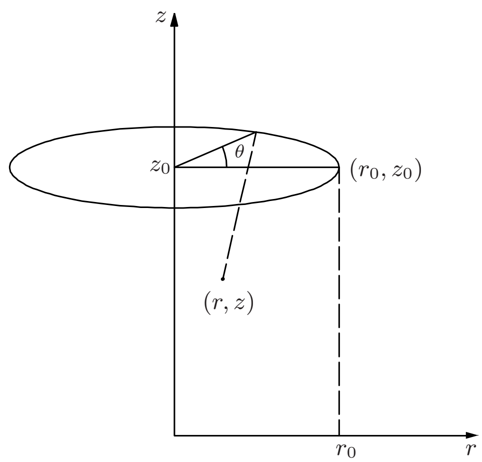


Figure 3 Axi-symmetric fundamental solution

by Γ'. The conductivity equals k_1 in Ω_1 and k_2 in Ω_2. The problem will be solved in two sub-domain respectively, and suppose u_1 is the solution in Ω_1 and u_2 is the solution in Ω_2. What's more, we should add two boundary conditions, so the problem becomes

$$\begin{cases} \triangle u_1 = 0, \quad \text{in } \Omega_1, \\ \triangle u_2 = 0, \quad \text{in } \Omega_2, \\ Bu_1|_{\Gamma_1} = g_1, \\ Bu_2|_{\Gamma_2} = g_2, \\ u_1|_{\Gamma'} = u_2|_{\Gamma'}, \\ k_1 \dfrac{\partial u_1}{\partial n}\Big|_{\Gamma'} = k_2 \dfrac{\partial u_2}{\partial n}\Big|_{\Gamma'}. \end{cases}$$

4 Axi-Symmetric Fundamental Solution

As we mentioned before, the domain of the problem is axi-symmetric. In order to use FSM, the axi-symmetric fundamental solution must be found. We can derive it from the 3-D fundamental solution.

As we know, the physical meaning of the 3-D fundamental solution,

$$E(X, X') = \frac{1}{4\pi|X - X'|},$$

is the electric potential at point $X = (x, y, z)$, generated by a point charge located at point $X' = (x', y', z')$. In the axi-symmetric case, the electric potential generated by a circle-distributing line charge (r_0, z_0) (see Figure 3) can be expressed as an integration,

$$E(r, z; r_0, z_0) = \frac{1}{4\pi}\int_0^{2\pi} \frac{1}{2\pi\sqrt{(r - r_0\cos\theta)^2 + (r_0\sin\theta)^2 + (z - z_0)^2}} d\theta = \frac{1}{4\pi}\int_0^{\pi} \frac{d\theta}{\pi\sqrt{G}}, \quad (4.1)$$

where $G = G(r, z; r_0, z_0, \theta) = r^2 + r_0^2 - 2rr_0\cos(\theta) + (z - z_0)^2$.

Substituting (4.1) into axi-symmetric Laplace equation

$$\Delta u = \frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} = 0$$

and do numerical integration, we can find that the result of $\delta E(r,z;r_0,z_0)$ is very approximate to 0. So we can believe that (4.1) is an expression of axi-symmetric fundamental solution of Laplace equation.

We calculate some numerical examples which have exact solutions to find that the results are very satisfying. The maximal relative error is about 6×10^{-6}. Even in the more complicated case, where k is a piecewise constant, the maximal relative error is about 3.7×10^{-3}.

Another form of axi-symmetric fundamental solution can be found in [6].

5 Inverse Problem

The unknown wear-line denoted by Γ_5 will be estimated by utilizing a finite number of measurements of temperature

$$T_l,\ l=1,\cdots,L$$

at given locations $\{P_l\}_{l=1}^L$ in Ω, in combination with solving the mathematical model of heat conduction governed by equations (2.1)–(2.7). The locations used in our studies are shown in Figure 2, where 9 thermocouples are used for measuring.

By using perturbation method, the unknown boundary Γ_5 can be represented by a given curve plus a small perturbation

$$\Gamma_5=\{r,z\mid z=s(r)+\varepsilon g(r)\}.$$

The inverse problem is commonly solved by minimizing the sum of the squared residual at the measurement points,

$$\min\sum_{l=1}^{L}(u_l-T_l)^2.$$

In practice, since T_l are measured, there will be measurement errors. Therefore, instead of minimizing the pure squared sum, we minimize

$$\Phi(Z)=\sum_{l=1}^{L}(u_l-T_l)^2+J(Z)=\sum_{l=1}^{L}[(u_0(P_l)+\varepsilon u_1(P_l,Z))-T_l]^2+J(Z) \tag{5.1}$$

with respect to the curve parameter vector of $g(r)$, $Z=(z_1,\cdots,z_m)$, which are selected along $g(r)$ and m is the number of nodes. We can get the shape of g by interpolation with the known value . The term J is a regulizer.

In calculations presented here J is a finite difference approximation to

$$\beta\int_0^R(g^2(r)+g'^2(r))dr, \tag{5.2}$$

where β is a constant (see [11, 13]). The integration is calculated by trapezoidal integration.

6 Numerical Results

Some test cases were created to test the algorithm expressed above for solving the inverse problem. For simplicity, the side wall Γ_3 is set to be vertical to the ground (see Figure 4).

In each test, the shape of Γ_5 and $\overline{\Gamma}_5$ are given, that is, the express of $s(r)$, and $g(r)$ are known.

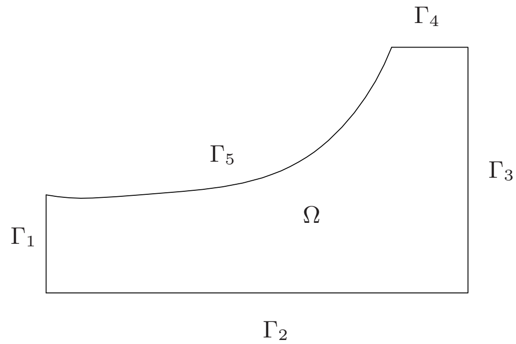


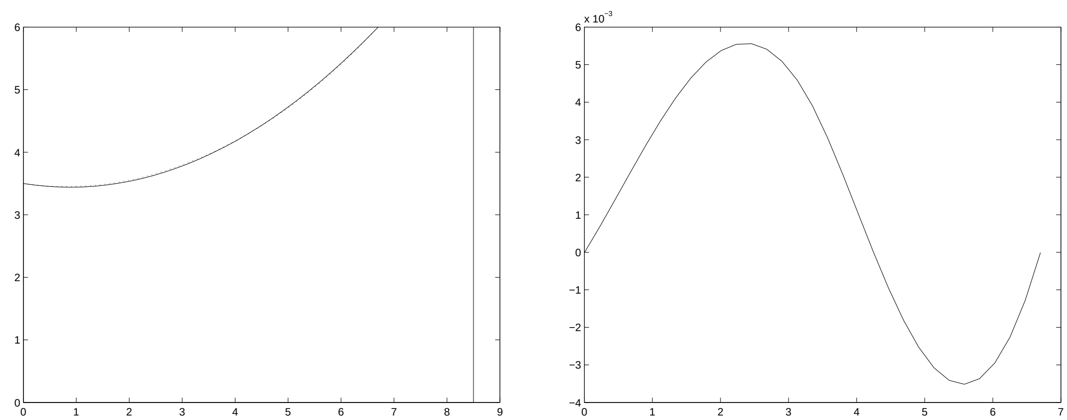
Figure 4 Domain of the test

Giving value of $k, \alpha_2, \alpha_3, u_2, u_3$ and solving direct problem (2.1)–(2.7), we obtain calculating temperature at the thermocouple locations. Then we add a random error vector to U and use it as the measured temperature T. The range of the random error is chosen to be 0.5%.

Given a starting value vector V_0, we solve the inverse problem by iteration and obtain the curve parameter vector V. Then we can calculate the curve by interpolation and compare it with the originally given curve $s(r)$ and $g(r)$.

Test 1
$$s(r) = 3.5 + 2.5 \times \left(\frac{r}{6.7}\right)^2, \quad g(r) = r^2 - 6.7r,$$
$$k = 10, \quad \alpha_2 = 30, \quad \alpha_3 = 70, \quad u_2 = 35, \quad u_3 = 33,$$
$$\varepsilon = 0.02, \quad \beta = 0.005.$$

The maximal relative error of the wear-line is about 5×10^{-3}, as shown in Figure 5.

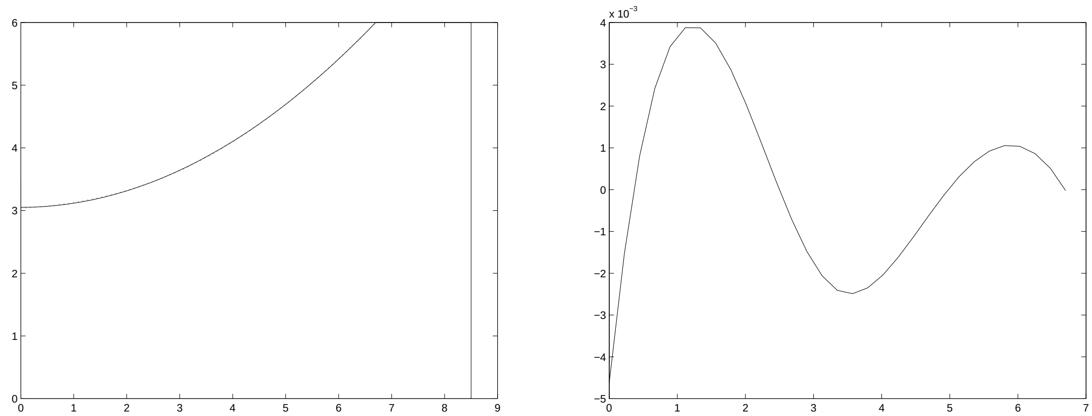


(a) boundary (b) relative error of boundary

Figure 5 Result of Test 1

Test 2
$$s(r) = 3.5 + 2.5 \times \left(\frac{r}{6.7}\right)^2, \quad g(r) = \frac{1}{2}r^2 - 22.445,$$
$$k = 10, \quad \alpha_2 = 30, \quad \alpha_3 = 70, \quad u_2 = 35, \quad u_3 = 33,$$
$$\varepsilon = 0.02, \quad \beta = 0.005.$$

The maximal relative error of the wear-line is about 4.6×10^{-3}, as shown in Figure 6.



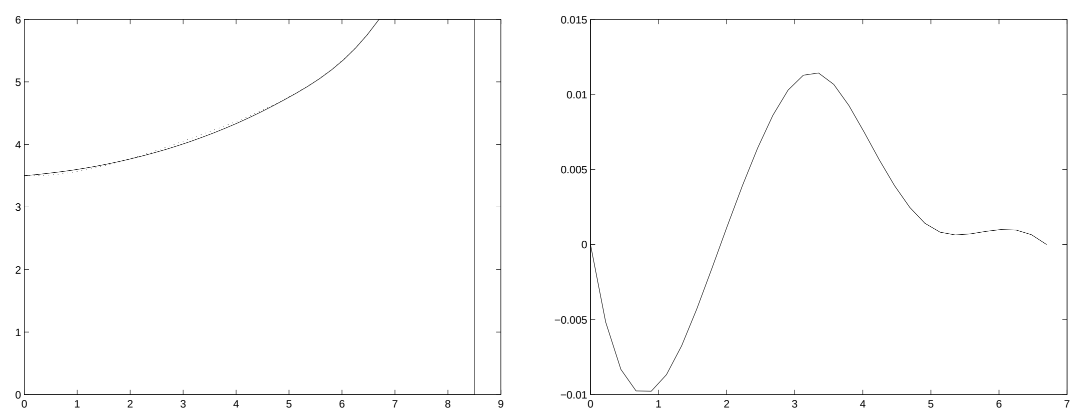
(a) boundary (b) relative error of boundary

Figure 6 Result of Test 2

Test 3 $\qquad s(r) = 2.5 + e^{\frac{r}{6.7}\ln(3.5)}, \quad g(r) = \begin{cases} 0.5832(r-5)^2 - 14.58, & 0 \le r \le 5, \\ 2(r-5)^2 - 14.58, & 5 \le r \le 6.7, \end{cases}$

$$k = 10, \quad \alpha_2 = 30, \quad \alpha_3 = 70, \quad u_2 = 35, \quad u_3 = 33,$$
$$\varepsilon = 0.02, \quad \beta = 0.005.$$

The maximal relative error of the wear-line is about 9.6×10^{-3}, as shown in Figure 7.



(a) boundary (b) relative error of boundary

Figure 7 Result of Test 3

After the tests, we present simulations of the wear-line using actual temperature measurements from a blast furnace. Unlike the testing cases, the actual wear-line Γ_5 can only be expressed by a parametric curve, as

$$\begin{cases} r = r(t) + \varepsilon h(t), \\ z = z(t) + \varepsilon l(t). \end{cases}$$

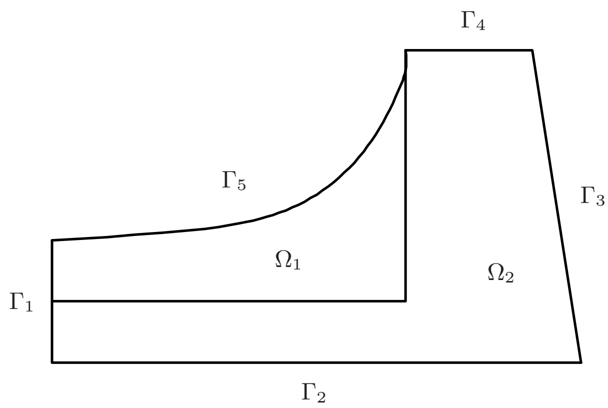
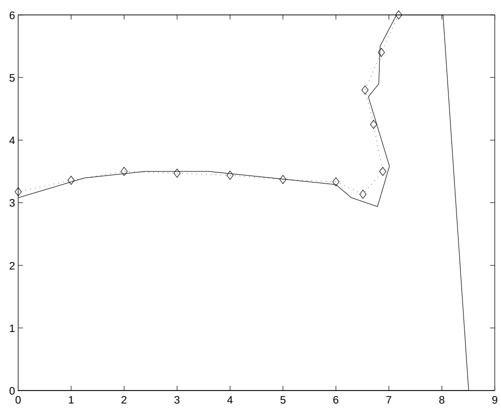
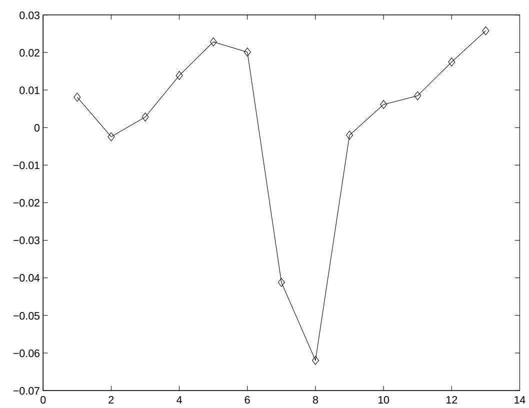


Figure 8 Domain of the blast furnace

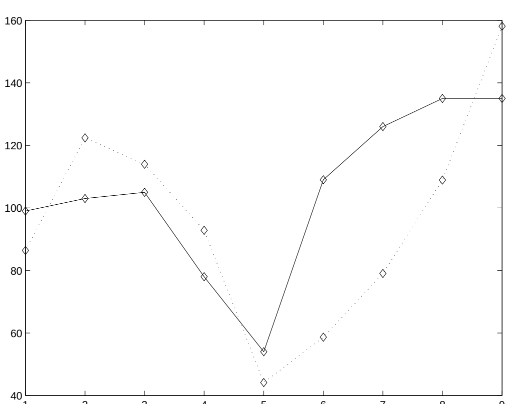


(a) calculated boundary

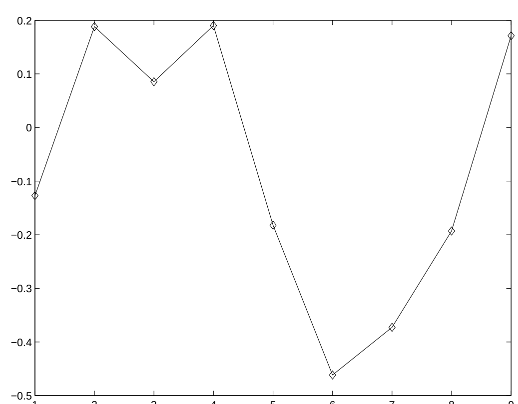


(b) relative error of the boundary

Figure 9 Result of boundary



(a) calculated temperature



(b) relative error of the temperature

Figure 10 Result of temperature

The boundary condition $u|_{\Gamma_5} = f$ converts to

$$u_0|_{\overline{\Gamma}_5} = f \quad \text{and} \quad u_1|_{\overline{\Gamma}_5} = -\Big(\frac{\partial u_0}{\partial r}\cdot h(t) + \frac{\partial u_0}{\partial z}\cdot l(t)\Big)\Big|_{\overline{\Gamma}_5}.$$

Because of the difference of material, the conductivity varies. In actual furnace, the domain Ω is divided into two parts, and $k_1 = 10,\ k_2 = 3.3$ (see Figure 8). The other parameters are $\alpha_2 = 30,\ \alpha_3 = 70,\ u_2 = 35,\ u_3 = 33$, and the temperature of the wear-line is 1450°C.

Solving inverse problem, we get the calculated shape of the wear-line. As shown in Figure 9, the difference between the calculated curve and the wear-line shape computed by FEM method without perturbation is quite acceptable.

We then calculate the temperature at the measuring points. Comparing with the measured temperature, the difference is partly caused by measurement error, so it is also acceptable (see Figure 10).

7 Comment and Conclusion

Here we try to give the rationality of the perturbation method theoretically. Remember that we expand the solution u about ε as

$$u = u_0 + \varepsilon u_1 + \cdots$$

and take the first two terms $\overline{u} = u_0 + \varepsilon u_1$, as an approximation of u. Now we let $v = u - \overline{u}$. Then we have

$$\frac{\partial v}{\partial n}\Big|_{\Gamma_1} = \frac{\partial v}{\partial n}\Big|_{\Gamma_4} = \Big(k\frac{\partial v}{\partial n} + \alpha_2 v\Big)\Big|_{\Gamma_2} = \Big(k\frac{\partial v}{\partial n} + \alpha_3 v\Big)\Big|_{\Gamma_3} = 0 \tag{7.1}$$

and

$$\begin{aligned} v|_{\Gamma_5} &= u|_{\Gamma_5} - (u_0|_{\Gamma_5} + \varepsilon u_1|_{\Gamma_5}) \\ &= \overline{u}_0 - \Big(u_0|_{\overline{\Gamma}_5} + \Big(\frac{\partial u_0}{\partial z}\varepsilon g(r)\Big)\Big|_{\overline{\Gamma}_5} + O(\varepsilon^2) + \varepsilon\Big(u_1|_{\overline{\Gamma}_5} + \Big(\frac{\partial u_1}{\partial z}\varepsilon g(r)\Big)\Big|_{\overline{\Gamma}_5} + O(\varepsilon^2)\Big)\Big) \\ &= \overline{u}_0 - \Big(\overline{u}_0 + \varepsilon\Big(\frac{\partial u_0}{\partial z} g(r)\Big)\Big|_{\overline{\Gamma}_5} + O(\varepsilon^2)\Big) - \varepsilon\Big(-\Big(\frac{\partial u_0}{\partial z} g(r)\Big)\Big|_{\overline{\Gamma}_5} + \varepsilon\Big(\frac{\partial u_1}{\partial z} g(r)\Big)|_{\overline{\Gamma}_5} + O(\varepsilon^2)\Big) \\ &= O(\varepsilon^2) - \varepsilon^2\Big(\frac{\partial u_1}{\partial z} g(r)\Big)\Big|_{\overline{\Gamma}_5} + O(\varepsilon^3) \\ &= O(\varepsilon^2). \end{aligned}$$

According to the extremum principle (see [12]), we can get from (7.1) that the extremum value of v can not be yielded on boundary Γ_1, Γ_4. If it is yielded on Γ_5, we can get that

$$|v| = O(\varepsilon^2).$$

Suppose that the maximal value of v is obtained on Γ_2 (or Γ_3), we can get from the extremum principle that

$$\frac{\partial v}{\partial n} > 0, \quad \text{on } \Gamma_2 \text{ (or } \Gamma_3).$$

So we have $v_{\max} < 0$ from (7.1). Therefore,

$$0 > v_{\max} \geq v|_{\Gamma_5} = O(\varepsilon^2), \quad |v| \leq O(\varepsilon^2).$$

In like manner, supposing the minimal value of v is obtained on Γ_2 (or Γ_3), we can get from the extremum principle that

$$\frac{\partial v}{\partial n} < 0, \quad \text{on } \Gamma_2 \text{ (or } \Gamma_3\text{)}.$$

So we have $v_{\min} > 0$ from (7.1). Therefore,

$$0 < v_{\min} \leq v|_{\Gamma_5} = O(\varepsilon^2), \quad |v| \leq O(\varepsilon^2).$$

Thus we get $|u - \overline{u}| = O(\varepsilon^2)$. The error between the true value and the approximation value is about the magnitude of ε^2, that is acceptable.

In this paper, we have discussed a method for determine a part of the boundary for a melting furnace. The main thought is to solve the problem with a fixed boundary while the original boundary is flexible. We use perturbation method for this purpose and prove its rationality in both numerical and theoretical ways. Another creation is that we give a possible expression of an axi-symmetric fundamental solution of Laplacian. We give some test cases to test the perturbation method and the fundamental solution and the results are satisfying. We also use this method on real furnace data, and the relative error is acceptable.

References

[1] Tan, Y. J. and Chen, X. X., Identifying corrosion boundary by perturbation method, Differential Equations & Asymptotic Theory in Mathematical Physics, Chen Hua and Roderick Wong (eds.), World Scientific, Singapore, 2004.

[2] Sorli, K. and Skaar, I. M., Monitoring the wear-line of a melting furnace, Inverse Problems in Engineering: Theory and Practice, June 13–18, Port Ludlow, WA, USA, 1999.

[3] Berntsson, F., Boundary identification for an elliptic equation, *Inverse Problems*, **18**, 2002, 1579–1592.

[4] Baker, D. P., Dulikravich G. S., Dennis, B. H. and Martin, T. J., Inverse determination of eroded smelter wall thickness variation using an elastic membrane concept, Proceeding of NHTC'03 ASME Summer Heat Transfer Conference, July 21–23, Las Vegas, Nevada, 2003.

[5] Yoshikawa, H. et al., Estimation of erosion line of refractory and solidification layer in blast furnace hearth, Proc. of 4th Conf. on Simulation Technology, Japan Society of Simulation Technology, 1984, 75–78.

[6] Bogomolny, A., Fundamental solutions method for elliptic boundary value problems, *SIAM J. Numer. Anal.*, **22**(4), 1985, 644–669.

[7] Brebbia, C. A., Progress in Boundary Element Methods, Pentech Press Limited, Denvon, USA, 1983.

[8] Acevedo, U. and Manuel, C., Mathematical Modelling and Control of the Blast Furnace Process, Purdue University, 1981.

[9] Hinch, J., Perturbation Methods, Cambridge University Press, Cambridge, 1991.

[10] Murdock, J. A., Perturbations: Theory and Methods, Corrected reprint of the 1991 original, Classics in Applied Mathematics, **27**, Society for Industrial and Applied Mathematics, Philadelphia, 1999.

[11] Engl, H. W., Hanke, M. and Neubauer, A., Regularization of Inverse Problems, Kluwer Academic Publishers, Dordrecht, 1996.

[12] Ockendon, J., Howison, S., Lacey, A. and Movchan, A., Applied Partial Differential Equations (revised edition), Oxford University Press, Oxford, 2003.

[13] Szczygiel, I. and Fic, A., Inverse convection-diffusion problem of estimating boundary veloctiy based on internal temperature measurements, *Inverse Problems in Engineering*, **10**(3), 2002, 271–291.

[14] Mathon, R. and Johnson, R. L., The approximate solution of elliptic boundary-value problems by fundamental solutions, *SIAM J. Numer. Anal.*, **14**(4), 1977, 638–650.