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Fujita-Liouville Type Theorem for Coupled Fourth-Order Parabolic Inequalities*

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Abstract This paper deals with a coupled system of fourth-order parabolic inequalities $|u|_t \geq -\Delta^2 u + |v|^q$, $|v|_t \geq -\Delta^2 v + |u|^p$ in $\mathbb{S} = \mathbb{R}^n \times \mathbb{R}^+$ with $p, q > 1$, $n \geq 1$. A Fujita-Liouville type theorem is established that the inequality system does not admit nontrivial nonnegative global solutions on $\mathbb{S}$ whenever $\frac{n}{4} \leq \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$. Since the general maximum-comparison principle does not hold for the fourth-order problem, the authors use the test function method to get the global non-existence of nontrivial solutions.

Keywords Fujita exponent, Liouville type theorem, Higher-order parabolic inequalities, Test function method
2000 MR Subject Classification 35B33, 35K30, 35K55

1 Introduction

This paper deals with the Cauchy problem for the fourth-order semilinear parabolic inequalities

$$\begin{cases} |u|_t \geq -\Delta^2 u + |v|^q, \\ |v|_t \geq -\Delta^2 v + |u|^p, \end{cases} \quad (x,t) \in \mathbb{S} = \mathbb{R}^n \times \mathbb{R}^+, \tag{1.1}$$

where $p, q > 1$, $n \geq 1$. Higher-order nonlinear parabolic equations appear in numerous applications such as thin film theory, flame propagation, bi-stable phase transition and higher-order diffusion (see [1]). Refer to [2–5] and the references therein for studies of higher-order heat equations.

It is well-known that the Cauchy problem of heat equation with source

$$u_t = \Delta u + |u|^{p-1} u, \quad (x,t) \in \mathbb{S}$$

does not admit nontrivial nonnegative global solutions whenever $p \in (1, 1 + \frac{2}{n}]$, by Fujita [6], Hayakawa [7], Weissler [8], etc.

The Cauchy problem for the higher-order semilinear diffusion equation

$$\begin{cases} u_t = -(-\Delta)^m u + |u|^p, & (x,t) \in \mathbb{S}, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^n \end{cases} \tag{1.2}$$

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with $m, p > 1$ was considered by Galaktionov and Pohozaev [2]. Under $u_0 \in L^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$, the Fujita exponent of (1.2) was determined as $p_c = 1 + \frac{2m}{n}$ in [2]. The global nonexistence result for the corresponding inequality problem with $p \in (1, p_c]$ was obtained as well under $u_0 \in L^1_{\text{loc}}(\mathbb{R}^n)$, $\int_{\mathbb{R}^n} u_0(x)\mathrm{d}x \geq 0$, without considering the initial traces (see [9]).

In this paper, we study the nonexistence of nontrivial global solutions for the coupled fourth-order inequalities (1.1). We will establish a Fujita-Liouville type theorem that if $\frac{n}{4} \leq \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$, then (1.1) does not admit nontrivial solutions in $\mathbb{S}$ without considering their traces on the hyperplane $t = 0$. Notice that the general maximum-comparison principle does not hold for the fourth-order problem. The technique used in this paper is the "test function method" (refer to, e.g., [3, 9–11]). As a simple consequence, the inequalities

$$\begin{cases} u_t \geq -\Delta^2 u + |v|^{q-1}v, \\ v_t \geq -\Delta^2 v + |u|^{p-1}u, \end{cases} \quad (x,t) \in \mathbb{S} \tag{1.3}$$

have no nontrivial solutions if $\frac{n}{4} \leq \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$ without taking their traces on the hyperplane $t = 0$ into account.

To state the main result of the paper, we need the following definitions.

Definition 1.1 *A pair of functions (u, v) with $u \in L^p_{\text{loc}}(\mathbb{S})$ and $v \in L^q_{\text{loc}}(\mathbb{S})$ is called a solution to (1.1) if*

$$\int_{\mathbb{S}} (-|u|\phi_t + u\Delta^2\phi)\mathrm{d}x\mathrm{d}t \geq \int_{\mathbb{S}} |v|^q \phi \mathrm{d}x\mathrm{d}t, \tag{1.4}$$

$$\int_{\mathbb{S}} (-|v|\phi_t + v\Delta^2\phi)\mathrm{d}x\mathrm{d}t \geq \int_{\mathbb{S}} |u|^p \phi \mathrm{d}x\mathrm{d}t \tag{1.5}$$

for any positive function $\phi \in C_0^\infty(\mathbb{S})$.

Definition 1.2 *A pair of functions (u, v) with $u \in L^p_{\text{loc}}(\mathbb{S})$ and $v \in L^q_{\text{loc}}(\mathbb{S})$ is called a solution to (1.3) if*

$$\int_{\mathbb{S}} (-u\phi_t + u\Delta^2\phi)\mathrm{d}x\mathrm{d}t \geq \int_{\mathbb{S}} |v|^{q-1} v\phi \mathrm{d}x\mathrm{d}t, \tag{1.6}$$

$$\int_{\mathbb{S}} (-v\phi_t + v\Delta^2\phi)\mathrm{d}x\mathrm{d}t \geq \int_{\mathbb{S}} |u|^{p-1} u\phi \mathrm{d}x\mathrm{d}t \tag{1.7}$$

for any positive function $\phi \in C_0^\infty(\mathbb{S})$.

Definition 1.3 *A pair of functions (u, v) with $u \in L^p_{\text{loc}}(\mathbb{S})$ and $v \in L^q_{\text{loc}}(\mathbb{S})$ is said to be bounded below by a positive constant if there exists a constant $C > 0$ such that $u, v \geq C$ a.e. on $\mathbb{S}$.*

Theorem 1.1 *Let (u, v) be a nonnegative solution to (1.1) on $\mathbb{S}$. Then $u = v = 0$ a.e. on $\mathbb{S}$ if $\frac{n}{4} \leq \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$.*

Since the nonnegative solutions to (1.3) do satisfy (1.1), Theorem 1.1 yields the following corollary.

Corollary 1.1 *Let (u, v) be a nonnegative solution to (1.3) on $\mathbb{S}$. Then $u = v = 0$ a.e. on $\mathbb{S}$ if $\frac{n}{4} \leq \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$.*

Moreover, for the problem (1.3), there is the non-existence result of the solutions bounded below.

Theorem 1.2 *Let $p, q > 1$. Then there are no solutions to* (1.3) *on $\mathbb{S}$ bounded below by a positive constant.*

Remark 1.1 If $p = q$ in (1.1), the coupled system (1.1) becomes the scalar inequality problem corresponding to (1.2) with $m = 2$, for which Theorem 1.1 says that the inequality problem does not admit global solutions when $\frac{n}{4} \le \frac{1}{p-1}$, or equivalently, $1 < p \le p_c = 1 + \frac{4}{n}$. This agrees with that in [9].

2 Proof of Theorem 1.1

We will prove the Fujita-Liouville type theorem in this section by using the test function method, inspired by [10].

Proof of Theorem 1.1 Let (u, v) be a solution to (1.1), $p, q > 1$. For $0 < \tau < +\infty$, $0 < r < R < +\infty$, set $\eta \in C^\infty$ with $\eta' \ge 0$ and

$$\begin{cases} \eta(t) = 1, & \text{if } t \in [2\tau, +\infty), \\ \eta(t) = 0, & \text{if } t \in [0, \tau]. \end{cases} \tag{2.1}$$

Let $\xi \in C^\infty(\mathbb{S})$ satisfy

$$\begin{cases} \xi(x,t) = 1, & \text{if } (x,t) \in \overline{P(r)}, \\ \xi(x,t) = 0, & \text{if } (x,t) \in \mathbb{S} \setminus \overline{P(R)}, \end{cases} \tag{2.2}$$

where $P(r) := \{(x,t) \in \mathbb{S} : |x|^4 + t < r\}$. Take $\phi(x,t) = \xi^s(x,t)\eta^2(t)$ as a test function of (1.4), (1.5), with $s > 4$ to be determined.

Substituting $\phi(x,t)$ into (1.4), we have

$$\begin{aligned} &- s\int_{P(R)} |u|\xi_t\xi^{s-1}\eta^2 \mathrm{d}x\mathrm{d}t + \int_{P(R)} u\Delta^2\xi^s\eta^2 \mathrm{d}x\mathrm{d}t \\ \ge &\int_{P(R)} |v|^q\xi^s\eta^2 \mathrm{d}x\mathrm{d}t + 2\int_{P(R)} |u|\xi^s\eta\eta' \mathrm{d}x\mathrm{d}t. \end{aligned} \tag{2.3}$$

Since $\eta' \ge 0$ for all $t > 0$, the second integral in the right-hand side of (2.3) is nonnegative, and consequently,

$$s\int_{P(R)} |u||\xi_t|\xi^{s-1}\eta^2 \mathrm{d}x\mathrm{d}t + \int_{P(R)} |u||\Delta^2\xi^s|\eta^2 \mathrm{d}x\mathrm{d}t \ge \int_{P(R)} |v|^q\xi^s\eta^2 \mathrm{d}x\mathrm{d}t. \tag{2.4}$$

Due to

$$\begin{aligned} \Delta^2\xi^s = &\, s(s-1)(s-2)(s-3)\xi^{s-4}|\nabla_x\xi|^4 + 6s(s-1)(s-2)\xi^{s-3}|\nabla_x\xi|^2\Delta\xi \\ &+ 3s(s-1)\xi^{s-2}|\Delta\xi|^2 + 4s(s-1)\xi^{s-2}\nabla_x\xi \cdot \nabla_x(\Delta\xi) + s\xi^{s-1}\Delta^2\xi, \end{aligned}$$

the inequality (2.4) implies

$$\begin{aligned}
&s\int_{P(R)}|u||\xi_t|\xi^{s-1}\eta^2\mathrm{d}x\mathrm{d}t+s(s-1)(s-2)(s-3)\int_{P(R)}|u||\nabla_x\xi|^4\xi^{s-4}\eta^2\mathrm{d}x\mathrm{d}t\\
&+6s(s-1)(s-2)\int_{P(R)}|u||\nabla_x\xi|^2|\Delta\xi|\xi^{s-3}\eta^2\mathrm{d}x\mathrm{d}t\\
&+3s(s-1)\int_{P(R)}|u||\Delta\xi|^2\xi^{s-2}\eta^2\mathrm{d}x\mathrm{d}t\\
&+4s(s-1)\int_{P(R)}|u||\nabla_x\xi||\nabla_x(\Delta\xi)|\xi^{s-2}\eta^2\mathrm{d}x\mathrm{d}t+s\int_{P(R)}|u||\Delta^2\xi|\xi^{s-1}\eta^2\mathrm{d}x\mathrm{d}t\\
\geq&\int_{P(R)}|v|^q\xi^s\eta^2\mathrm{d}x\mathrm{d}t. \qquad (2.5)
\end{aligned}$$

By Hölder inequality, we deduce from (2.5) that

$$\begin{aligned}
&\Big(\int_{P(R)\setminus P(r)}|u|^p\xi^s\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{1}{p}}\Big\{\Big(\int_{P(R)}|\xi_t|^{\frac{p}{p-1}}\xi^{s-\frac{p}{p-1}}\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{p-1}{p}}\\
&+\Big(\int_{P(R)}|\nabla_x\xi|^{\frac{4p}{p-1}}\xi^{s-\frac{4p}{p-1}}\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{p-1}{p}}+\Big(\int_{P(R)}(|\nabla_x\xi|^2|\Delta\xi|)^{\frac{p}{p-1}}\xi^{s-\frac{3p}{p-1}}\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{p-1}{p}}\\
&+\Big(\int_{P(R)}|\Delta\xi|^{\frac{2p}{p-1}}\xi^{s-\frac{2p}{p-1}}\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{p-1}{p}}+\Big(\int_{P(R)}(|\nabla_x\xi||\nabla_x(\Delta\xi)|)^{\frac{p}{p-1}}\xi^{s-\frac{2p}{p-1}}\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{p-1}{p}}\\
&+\Big(\int_{P(R)}|\Delta^2\xi|^{\frac{p}{p-1}}\xi^{s-\frac{p}{p-1}}\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{p-1}{p}}\Big\}\\
\geq\ & c_1\int_{P(R)}|v|^q\xi^s\eta^2\mathrm{d}x\mathrm{d}t, \qquad (2.6)
\end{aligned}$$

where and in the sequel of the paper $c_i=c_i(n,p,s)$ $(i=1,\cdots,9)$ represent positive constants independent of τ,r,R. Set $\xi(x,t)=\psi(\frac{|x|^4+t}{R})$ in (2.6), where $\psi:\ [0,+\infty)\to[0,1]$ is a smooth function satisfying

$$\begin{cases}\psi(y)=1, & \text{if } y\in\Big[0,\dfrac{1}{2}\Big],\\ \psi(y)=0, & \text{if } y\in[1,+\infty).\end{cases} \qquad (2.7)$$

Direct computations show the following inequalities:

$$\begin{array}{lll}\mathrm{mes}\,|P(R)|\leq c_2R^{\frac{n+4}{4}}, & |\xi_t|\leq c_2R^{-1}, & |\Delta\xi|^2\leq c_2R^{-1},\\ |\nabla_x\xi|^4\leq c_2R^{-1}, & |\nabla_x(\Delta\xi)|\leq c_2R^{-\frac{3}{4}}, & |\Delta^2\xi|\leq c_2R^{-1}\end{array}$$

for arbitrary $R=2r>0$ and $(x,t)\in\mathbb{S}$. Since $s>\frac{4p}{p-1}$, it follows from (2.6) that

$$\Big(\int_{P(R)\setminus P(r)}|u|^p\xi^s\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{1}{p}}(R^{\frac{n+4}{4}-\frac{p}{p-1}})^{\frac{p-1}{p}}\geq c_3\int_{P(R)}|v|^q\xi^s\eta^2\mathrm{d}x\mathrm{d}t. \qquad (2.8)$$

In a similar way,

$$\Big(\int_{P(R)\setminus P(r)}|v|^q\xi^s\eta^2\mathrm{d}x\mathrm{d}t\Big)^{\frac{1}{q}}(R^{\frac{n+4}{4}-\frac{q}{q-1}})^{\frac{q-1}{q}}\geq c_3\int_{P(R)}|u|^p\xi^s\eta^2\mathrm{d}x\mathrm{d}t. \qquad (2.9)$$

Combining (2.8) and (2.9), we have

$$\begin{aligned} c_4 \int_{P(R)} |v|^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t &\le \Big(\int_{P(R)\setminus P(r)} |v|^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \Big)^{\frac{1}{pq}} (R^{\frac{n+4}{4}-\frac{q}{q-1}})^{\frac{q-1}{pq}} (R^{\frac{n+4}{4}-\frac{p}{p-1}})^{\frac{p-1}{p}} \\ &= \Big(\int_{P(R)\setminus P(r)} |v|^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \Big)^{\frac{1}{pq}} R^{\frac{n(pq-1)-4(1+q)}{4pq}}, \end{aligned} \tag{2.10}$$

and also

$$c_4 \int_{P(R)} |u|^p \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \le \Big(\int_{P(R)\setminus P(r)} |u|^p \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \Big)^{\frac{1}{pq}} R^{\frac{n(pq-1)-4(1+p)}{4pq}}. \tag{2.11}$$

It concludes from (2.10) and (2.11) that $\int_{\mathbb{S}} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t = \int_{\mathbb{S}} |v|^q \eta^2 \mathrm{d}x\mathrm{d}t = 0$ whenever $\frac{n}{4} < \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$.

If $\frac{n}{4} = \max(\frac{p+1}{pq-1}, \frac{q+1}{pq-1})$, e.g., $\frac{n}{4} = \frac{p+1}{pq-1}$, then $\int_{\mathbb{S}} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t \le c_5$, which implies

$$\int_{P(2r_k)\setminus P(r_k)} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t \to 0 \tag{2.12}$$

for any sequence $r_k \to +\infty$ with $R = 2r_k,\ r = r_k$.

On the other hand, we have by (2.11) that

$$\Big(\int_{P(R)\setminus P(r)} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t \Big)^{\frac{1}{pq}} R^{\frac{n(pq-1)-4(1+p)}{4pq}} \ge c_4 \int_{P(r)} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t. \tag{2.13}$$

Combining (2.12) and (2.13) with $\frac{n}{4} = \frac{p+1}{pq-1}$, we obtain $\lim\limits_{r_k\to+\infty} \int_{P(r_k)} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t = 0$, and hence $\int_{\mathbb{S}} |u|^p \eta^2 \mathrm{d}x\mathrm{d}t = 0$. Due to $\eta(t) = 1$ for $t \in [2\tau, +\infty)$ and $\tau > 0$ by (2.1), we get $u = 0$ a.e. on $\mathbb{S}$, and consequently, $v = 0$ a.e. on $\mathbb{S}$ by (2.8).

3 Proof of Theorem 1.2

We continue proving the non-existence of solutions bounded below by positive constants to the coupled inequality system (1.3).

Proof of Theorem 1.2 Suppose for contradiction that system (1.3) admits a solution (u, v) with $u, v \ge C > 0$ on $\mathbb{S}$. As in the proof of Theorem 1.1, for $0 < \tau < +\infty$, $0 < r < R < +\infty$, take $\eta \in C^\infty$ satisfying (2.1) with $\eta' \ge 0$, and $\xi \in C^\infty(\mathbb{S})$ defined by (2.2). Set $\phi(x,t) = \xi^s(x,t)\eta^2(t)$ as a test function of (1.6), with $s \ge \frac{4p}{p-1}$. We obtain

$$\begin{aligned} &- s \int_{P(R)} u \xi_t \xi^{s-1} \eta^2 \mathrm{d}x\mathrm{d}t + \int_{P(R)} u \Delta^2 \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \\ \ge& \int_{P(R)} v^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t + 2 \int_{P(R)} u \xi^s \eta \eta' \mathrm{d}x\mathrm{d}t. \end{aligned} \tag{3.1}$$

Since $\eta' \ge 0$ for $t > 0$, the second integral of right-hand side of (3.1) is nonnegative. So

$$s \int_{P(R)} u |\xi_t| \xi^{s-1} \eta^2 \mathrm{d}x\mathrm{d}t + \int_{P(R)} u |\Delta^2 \xi^s| \eta^2 \mathrm{d}x\mathrm{d}t \ge \int_{P(R)} v^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t.$$

Repeating the same procedures as (2.4)–(2.11) in the proof of Theorem 1.1, we can get

$$c_4 \int_{P(R)} v^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \le \Big(\int_{P(R)\setminus P(r)} v^q \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \Big)^{\frac{1}{pq}} R^{\frac{n(pq-1)-4(1+q)}{4pq}}, \tag{3.2}$$

$$c_4 \int_{P(R)} u^p \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \le \Big(\int_{P(R)\setminus P(r)} u^p \xi^s \eta^2 \mathrm{d}x\mathrm{d}t \Big)^{\frac{1}{pq}} R^{\frac{n(pq-1)-4(1+p)}{4pq}}. \tag{3.3}$$

It follows from (3.2) and (3.3) that

$$\int_{P(\frac{R}{2})} v^q \eta^2 \mathrm{d}x\mathrm{d}t \le c_6 R^{\frac{n(pq-1)-4(1+q)}{4(pq-1)}}, \quad \int_{P(\frac{R}{2})} u^p \eta^2 \mathrm{d}x\mathrm{d}t \le c_6 R^{\frac{n(pq-1)-4(1+p)}{4(pq-1)}}.$$

Passing to the limit as $\tau \to 0$, we obtain

$$\int_{P(\frac{R}{2})} v^q \mathrm{d}x\mathrm{d}t \le c_7 R^{\frac{n}{4}-\frac{1+q}{pq-1}}, \quad \int_{P(\frac{R}{2})} u^p \mathrm{d}x\mathrm{d}t \le c_7 R^{\frac{n}{4}-\frac{1+p}{pq-1}}. \tag{3.4}$$

Since $u, v \ge C$ and $\operatorname{mes}|P(\frac{R}{2})| \ge c_8 R^{\frac{n+4}{4}}$, we learn from (3.4) that $c_9 R^{\frac{n}{4}-\frac{1+q}{pq-1}} \ge R^{\frac{n+4}{4}}$, $c_9 R^{\frac{n}{4}-\frac{1+p}{pq-1}} \ge R^{\frac{n+4}{4}}$ hold with $p, q > 1$ and $R > 0$. This yields a contradiction for R large enough.

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