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On the Irrotational Approximation to the 2-Dimensional Isothermal Euler System*

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Abstract The author mainly studies the difference of the weak solutions generated by a wave front tracking algorithm to the steady Euler system and the isothermal Euler system. Under the hypothesis that the initial data are of sufficiently small total variation, it is proved that the difference between the solutions of the steady Euler system and the system of isothermal supersonic flow can be bounded by the cube of the total variation of the initial perturbation.

Keywords Irrotational approximation, Isothermal Euler system, Semigroup
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1 Introduction

The planar steady full Euler system for compressible fluids can be written as

$$\begin{cases}(\rho u)_x+(\rho v)_y=0,\\ (\rho u^2+p)_x+(\rho uv)_y=0,\\ (\rho uv)_x+(\rho v^2+p)_y=0,\\ (u(\frac{1}{2}\rho(u^2+v^2)+\rho e+p))_x+(v(\frac{1}{2}\rho(u^2+v^2)+\rho e+p))_y=0,\end{cases}\tag{1.1}$$

where ρ is the density of the fluid, (u,v) is the velocity, p is the pressure, and e is the internal energy. The second law of thermodynamics asserts that

$$T\mathrm{d}S=\mathrm{d}e+p\mathrm{d}v,\tag{1.2}$$

where $T>0$ is the temperature, and S is the entropy. This implies that

$$e_S(S,v)=T,\quad e_v(S,v)=-p.\tag{1.3}$$

For polytropic gas,

$$p=R\rho T,\quad e=c_v(T-T_0),\quad c_v=\frac{R}{\gamma-1},\tag{1.4}$$

where R and T_0 are constants. Then by scaling, we have

$$p=\exp\Big\{\frac{S\varepsilon}{R}\Big\}\rho^{\varepsilon+1},\quad e(\rho,S,\varepsilon)=\frac{1}{\varepsilon}\Big(\Big(\rho\exp\Big\{\frac{S}{R}\Big\}\Big)^{\varepsilon}-1\Big),\tag{1.5}$$

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where

$$\varepsilon = \gamma - 1 > 0, \tag{1.6}$$

and γ is the adiabatic index. It follows that

$$p = \rho(1 + e\varepsilon). \tag{1.7}$$

The sound speed is $c = \sqrt{(1+\varepsilon)(1+e\varepsilon)}$.

As $\varepsilon \to 0$, the internal energy is given by (see [1])

$$e_0(\rho, S) = \lim_{\varepsilon\to 0} e(\rho, S, \varepsilon) = \ln\rho + \frac{S}{R}, \tag{1.8}$$

and the pressure $p = \rho$. Correspondingly, the sound speed is $c \equiv 1$, and the non-isentropic Euler system (1.1) formally converges to the limiting system as follows:

$$\begin{cases} (\rho u)_x + (\rho v)_y = 0, \\ (\rho u^2 + p_0)_x + (\rho uv)_y = 0, \\ (\rho uv)_x + (\rho v^2 + p_0)_y = 0, \\ \Big(u\Big(\dfrac{1}{2}\rho(u^2+v^2) + \rho e_0 + p_0\Big)\Big)_x + \Big(v\Big(\dfrac{1}{2}\rho(u^2+v^2) + \rho e_0 + p_0\Big)\Big)_y = 0, \end{cases} \tag{1.9}$$

which is called an isothermal Euler system.

System (1.9) has a simplified approximation

$$\begin{cases} (\rho u)_x + (\rho v)_y = 0, \\ v_x - u_y = 0, \end{cases} \tag{1.10}$$

where the density ρ and the velocity (u, v) satisfy the following Bernoulli equation:

$$\frac{u^2+v^2}{2} + \ln\rho = B_0, \tag{1.11}$$

which gives $\rho = \exp\{B_0 - \frac{u^2+v^2}{2}\} := g(u, v)$, where B_0 is a constant. From $e = \ln\rho + \frac{S}{R}$, we know that $e = \ln g(u, v) + \frac{S_0}{R} := f(u, v)$ for isothermal gas.

The main purpose of this paper is to estimate the difference of the solutions to system (1.9) and system (1.10)–(1.11) in the supersonic region.

Let $U_0^{(0)} = (u_0, v_0)^{\mathrm{T}}$ be a constant state with $u_0^2 + v_0^2 < 2B_0$ and $u_0 > 1$, where the superscript T stands for the transpose. Denote $U_0^{(1)} = (g(u_0, v_0), u_0, v_0, f(u_0, v_0))^{\mathrm{T}}$, and then the main result is stated as follows.

Theorem 1.1 *Denote by $U_0 = (u_0(y), v_0(y))^{\mathrm{T}}$ a bounded measurable function with small bounded variation, such that $U_0(y) \to U_0^{(0)} (y \to \pm\infty)$. Let $U_1 = (\rho_1, u_1, v_1, e_1)^{\mathrm{T}}$ and $U_2 = (g(u_2, v_2), u_2, v_2, f(u_2, v_2))^{\mathrm{T}}$ be the respective solutions on $R_x^+ \times R_y$ to the isothermal Euler system* (1.9) *and system* (1.10)*, taking $U_0^{(1)} = (g(u_0, v_0), u_0, v_0, f(u_0, v_0))^{\mathrm{T}}$ as the initial data. Assume that $TV(U_0)$ is sufficiently small so that U_1 and U_2 are well defined for all $x > 0$, and $U_1(x, y)$ and $U_2(x, y)$ lie in the supersonic region for $x \geq 0$. Then, there exist constants $\delta > 0$ and $K > 0$, such that for all U_0 with $\|U_0 - U_0^{(0)}\|_{L^\infty(R^1)} + TV(U_0) < \delta$ and $U - U_0^{(0)} \in L^1$, and for all $x > 0$,*

$$\|U_1(x, \cdot) - U_2(x, \cdot)\|_{L^1(R^1)} \leq Kx(TV(U_0))^3. \tag{1.12}$$

Here $TV(U_0)$ stands for the total variation of U_0. The superscript T *stands for the transpose, and $\|\cdot\|_{L^1(R^1)}$ stands for the L^1-norm.*

Here and hereafter x is regarded as the time variable. The proof of Theorem 1.1 is given in Section 5. We follow the idea of Bressan [2–3] to compare the solutions in L^1. That is, the proof is based on the comparison of solutions to the isothermal Euler system (1.9) and system (1.10), and we use some L^1-stability estimates for the standard Riemann semigroup generated by the isothermal Euler system (1.9). The same ideas were used to treat the isentropic approximation for the full Euler system by Saint-Raymond [4] and the classical limit of the relativistic Euler equations by Bianchini and Colombo [5]. Zhang [6] compared the solutions to the isentropic Euler system and the system of irrotational supersonic flow for $\gamma > 1$. Liu [7] discussed the difference between the solutions to the full Euler system and the system of isentropic supersonic flow for $\gamma > 1$. Note that the isothermal case $\gamma = 1$ is also important in practice, and engineers often use the potential flow to simulate the solution, so it is also necessary to compare the solutions to the full isothermal Euler system and the system of irrotational supersonic flow for $\gamma = 1$.

The remaining is organized as follows. In Sections 2 and 3, we study some properties on the wave curves for the isothermal Euler system (1.9) and system (1.10). In Section 4, we compare the solution to the Riemann problems for these two systems. In Section 5, we first estimate the difference between the ε-approximate solutions for irrotational flow equations and the solution to the isothermal Euler system. Then, by taking the limit we prove the main result.

2 Wave Curves for the System of Irrotational Flow

The matrix form of system (1.10) is

$$\begin{pmatrix} u\rho_u + \rho & u\rho_v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_x + \begin{pmatrix} v\rho_u & v\rho_v + \rho \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_y = 0. \tag{2.1}$$

From (1.11), we know that $\rho_u = -u\rho$, $\rho_v = -v\rho$.

The corresponding two eigenvalues of (2.1) are

$$\lambda_j = \frac{uv + (-1)^j \sqrt{u^2 + v^2 - 1}}{u^2 - 1}, \quad j = 1, 4. \tag{2.2}$$

The corresponding right eigenvectors (2.1) are

$$\overrightarrow{r}_j = b_j \widetilde{r}_j, \quad j = 1, 4, \tag{2.3}$$

where

$$\widetilde{r}_j = (-\lambda_j, 1)^{\mathrm{T}}, \quad j = 1, 4 \tag{2.4}$$

and

$$b_j = (\nabla \lambda_j \cdot \widetilde{r}_j)^{-1}, \tag{2.5}$$

where if $u > 1$, then $\nabla \lambda_j \cdot \widetilde{r}_j \neq 0$. So the system (1.10) is strictly hyperbolic.

Due to [8] (see also [9–10]), there is a $\delta_1 > 0$, such that the following holds. For any constant state $U_l \in O_{\delta_1}(U_0^{(0)}) \subset R^2$, the wave curves through U_l for the system of irrotational flow can be parameterized by $\tau_j \to \Phi_j(\tau_j, U_l)$ with $\Phi_j \in C^2$ and

$$\begin{aligned} &\Phi_j(0, U_l) = U_l, \\ &\frac{\partial \Phi_j}{\partial \tau_j}(0, U_l) = \overrightarrow{r}_j(U_l), \\ &\frac{\partial^2 \Phi_j}{\partial \tau_j^2}(0, U_l) = \nabla \overrightarrow{r}_j(U_l) \cdot \overrightarrow{r}_j(U_l). \end{aligned}$$

Moreover, for $\tau_j \geq 0$ and $j = 1, 4$,

$$\frac{\mathrm{d}}{\mathrm{d}\tau_j}\Phi_j(\tau_j, U_l) = \overrightarrow{r}_j(\Phi_j(\tau_j, U_l)). \tag{2.6}$$

Define

$$D = \{(u, v)^{\mathrm{T}} \in O_{\delta_1}(U_0^{(0)}) \subset R^2 \mid u > 1\}.$$

Consider the following Riemann problem for system (1.10):

$$U|_{x=0} = \begin{cases} U_l = (u_l, v_l), & y < 0, \\ U_r = (u_r, v_r), & y > 0, \end{cases} \tag{2.7}$$

where $U_l, U_r \in D$ are constant states. From the above discussion, the set of right states U_r, which can be connected to the left states U_l by a j-simple wave must lie in the curve Φ_j parameterized by τ_j. Define

$$\Phi(\tau_1, \tau_4, U_l) = \Phi_4(\tau_4, \Phi_1(\tau_1, U_l)).$$

Due to Lax [8] (see also [9–12]), we have the following result.

Lemma 2.1 *There exists a constant $\delta_1 > 0$, such that for any $U_l, U_r \in D$, the Riemann problem* (2.7) *admits a unique solution, i.e.,*

$$U_r = \Phi(\tau_1, \tau_4, U_l). \tag{2.8}$$

Moreover, $\Phi|_{\tau_1=\tau_4=0} = U_l,\ \frac{\partial \Phi}{\partial \tau_j}\Big|_{\tau_1=\tau_4=0} = \overrightarrow{r}_j(U_l),\ j = 1, 4.$

Proof Differentiating (2.8) with respect to τ_j, we have

$$\det\Big(\frac{\partial \Phi}{\partial(\tau_1, \tau_4)}\Big)\Big|_{\tau_1=\tau_4=0} = \det(\overrightarrow{r}_1(U_l), \overrightarrow{r}_4(U_l)) \neq 0.$$

For sufficiently small δ_1, $U_l, U_r \in D$, system (2.8) has a unique solution $\tau_j = \tau_j(U_r, U_l),\ j = 1, 4$, by the implicit function theorem.

3 Wave Curves for the System of Isothermal Euler System

System (1.9) has four eigenvalues

$$\lambda_j^* = \frac{uv + (-1)^j\sqrt{u^2 + v^2 - 1}}{u^2 - 1}, \quad j = 1, 4, \tag{3.1}$$

$$\lambda_2^* = \lambda_3^* = \frac{v}{u}. \tag{3.2}$$

The corresponding right eigenvectors are

$$\begin{aligned} \overrightarrow{r}_j^* &= b_j^* \widetilde{r}_j^*, \quad j = 1, 4, \\ \overrightarrow{r}_2^* &= (0, u, v, 0)^{\mathrm{T}}, \\ \overrightarrow{r}_3^* &= (0, 0, 0, 1)^{\mathrm{T}}, \end{aligned}$$

where

$$\begin{aligned} \widetilde{r}_j^* &= (\rho(\lambda_j^* u - v), -\lambda_j^*, 1, \lambda_j^* u - v)^{\mathrm{T}}, \\ b_j^* &= (\nabla \lambda_j^* \cdot \widetilde{r}_j^*)^{-1}, \quad j = 1, 4, \end{aligned}$$

and if $u > 1$, then $\nabla\lambda_j^* \cdot \widetilde{r}_j^* \neq 0$ $(j = 1, 4)$ and $\nabla\lambda_j^* \cdot \widetilde{r}_j^* = 0$ $(j = 2, 3)$.

Due to [8], there is a $\delta_2 > 0$, such that the following hold. For any constant state $U_l^* \in O_{\delta_2}(U_0^{(1)}) \subset R^4$, the j-th wave curves through U_l^* for the isothermal Euler system can be parameterized by $\tau_j \to \Phi_j^*(\tau_j, U_l^*)$ with $\Phi_j^* \in C^2$ and

$$\begin{aligned}
&\Phi_j^*(0, U_l^*) = U_l^*,\\
&\frac{\partial \Phi_j^*}{\partial \tau_j}(0, U_l^*) = \overrightarrow{r}_j^*(U_l^*),\\
&\frac{\partial^2 \Phi_j^*}{\partial \tau_j^2}(0, U_l^*) = \nabla \overrightarrow{r}_j^*(U_l^*) \cdot \overrightarrow{r}_j^*(U_l^*).
\end{aligned}$$

Moreover,

$$\frac{\mathrm{d}}{\mathrm{d}\tau_j}\Phi_j^*(\tau_j, U_l^*) = \overrightarrow{r}_j^*(\Phi_j^*(\tau_j, U_l^*)), \quad \tau_j \geq 0, \ j = 1, 4 \tag{3.3}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}\tau_j}\Phi_j^*(\tau_j, U_l^*) = \overrightarrow{r}_j^*(\Phi_j^*(\tau_j, U_l^*)), \quad j = 2, 3.$$

Here the superscript $*$ stands for the isothermal Euler system.

Let $\Phi_j^*(\tau_j, U_l^*) = (\rho_j^*(\tau_j, U_l^*), u_j^*(\tau_j, U_l^*), v_j^*(\tau_j, U_l^*), e_j^*(\tau_j, U_l^*))$. Then define

$$D^* = \{(\rho, u, v, e)^{\mathrm{T}} \in O_{\delta_2}(U_0^{(1)}) \subset R^4 \mid u > 1\}.$$

Consider the following Riemann problem for system (1.9):

$$U^*|_{x=0} = \begin{cases} U_l^* = (\rho_l, u_l, v_l, e_l), & y < 0,\\ U_r^* = (\rho_r, u_r, v_r, e_r), & y > 0, \end{cases} \tag{3.4}$$

where $U_l^*, U_r^* \in D^*$ are constant states. From the above discussion, the set of right states U_r^* which can be connected to the left states U_l^* by a j-simple wave must lie in the curve Φ_j^* parameterized by τ_j. Define

$$\Phi^*(\tau_1, \tau_2, \tau_3, \tau_4, U_l^*) = \Phi_4^*(\tau_4, \Phi_3^*(\tau_3, \Phi_2^*(\tau_2, \Phi_1^*(\tau_1, U_l^*)))).$$

Due to Lax [8] (see also [9–12]), we have the following result.

Lemma 3.1 *There exists a constant $\delta_2 > 0$, such that for any $U_l^*, U_r^* \in D^*$, the Riemann problem* (3.4) *admits a unique solution, i.e.,*

$$U_r^* = \Phi^*(\tau_1, \tau_2, \tau_3, \tau_4, U_l^*). \tag{3.5}$$

Moreover, $\Phi^*|_{\tau_1=\tau_2=\tau_3=\tau_4=0} = U_l^*$, $\frac{\partial \Phi^*}{\partial \tau_j}|_{\tau_1=\tau_2=\tau_3=\tau_4=0} = \overrightarrow{r}_j^*(U_l^*)$, $j = 1, 2, 3, 4$.

Proof Differentiating (3.5) with respect to τ_j, we have

$$\det\Big(\frac{\partial \Phi^*}{\partial(\tau_1, \tau_2, \tau_3, \tau_4)}\Big)\Big|_{\tau_1=\tau_2=\tau_3=\tau_4=0} = \det(\overrightarrow{r}_1^*(U_l^*), \overrightarrow{r}_2^*(U_l^*), \overrightarrow{r}_3^*(U_l^*), \overrightarrow{r}_4^*(U_l^*)) \neq 0.$$

For sufficiently small δ_2, $U_l^*, U_r^* \in D^*$, the system (3.5) has a unique solution $\tau_j = \tau_j(U_r^*, U_l^*)$, $j = 1, 2, 3, 4$ by the implicit function theorem.

The solution to the Riemann problem (1.9) and (3.4) satisfies the following properties.

Lemma 3.2 *For $\tau_j \geq 0$ and $j = 1, 4$, it holds that*

$$e_j^* - \ln \rho_j^* = \text{constant}. \tag{3.6}$$

Proof From (3.3), we know that

$$\frac{\mathrm{d}\rho_j^*}{\mathrm{d}\tau_j}(\tau_j, U_l^*) = b_j^* \rho_j^* (\lambda_j^* u_j^* - v_j^*), \tag{3.7}$$

$$\frac{\mathrm{d}e_j^*}{\mathrm{d}\tau_j}(\tau_j, U_l^*) = b_j^* (\lambda_j^* u_j^* - v_j^*), \tag{3.8}$$

which implies

$$\frac{\mathrm{d}\rho_j^*}{\mathrm{d}\tau_j} - \rho_j^* \frac{\mathrm{d}e_j^*}{\mathrm{d}\tau_j} = 0. \tag{3.9}$$

Thus, we have $e_j^* - \ln \rho_j^* = \text{constant}$.

Lemma 3.3 *For $\tau_j \geq 0$ and $j = 1, 4$, it holds that*

$$\frac{(u_j^*)^2 + (v_j^*)^2}{2} + \ln \rho_j^* = B_0. \tag{3.10}$$

Proof From (3.3), we know that

$$\begin{aligned}
&\frac{\mathrm{d}\rho_j^*}{\mathrm{d}\tau_j}(\tau_j, U_l^*) = b_j^* \rho_j^* (\lambda_j^* u_j^* - v_j^*), \\
&\frac{\mathrm{d}u_j^*}{\mathrm{d}\tau_j}(\tau_j, U_l^*) = b_j^* (-\lambda_j^*), \\
&\frac{\mathrm{d}v_j^*}{\mathrm{d}\tau_j}(\tau_j, U_l^*) = b_j^*,
\end{aligned}$$

which implies

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\tau_j}\Big\{\frac{(u_j^*)^2 + (v_j^*)^2}{2} + \ln \rho_j^*\Big\} &= u_j^* \frac{\mathrm{d}u_j^*}{\mathrm{d}\tau_j} + v_j^* \frac{\mathrm{d}v_j^*}{\mathrm{d}\tau_j} + \frac{1}{\rho_j^*}\frac{\mathrm{d}\rho_j^*}{\mathrm{d}\tau_j} \\
&= b_j^*(-\lambda_j^* u_j^* + v_j^* + \lambda_j^* u_j^* - v_j^*) \\
&= 0,
\end{aligned}$$

yielding

$$\frac{(u_j^*)^2 + (v_j^*)^2}{2} + \ln \rho_j^* = B_0.$$

Moreover, we get the lemma as follows.

Lemma 3.4 *For $\tau_j \geq 0$ and $j = 1, 4$, it holds that*
(1) $\lambda_j^*(\Phi_j^*(\tau_j, U_l^*)) = \lambda_j(u_j^*(\tau_j, U_l^*), v_j^*(\tau_j, U_l^*))$,
(2) $b_j^*(\Phi_j^*(\tau_j, U_l^*)) = b_j(u_j^*(\tau_j, U_l^*), v_j^*(\tau_j, U_l^*))$.

Proof From (2.2) and (3.1), we can easily get statement (1). It suffices to prove statement (2).

$$
\begin{aligned}
&b_j^{*-1}(\Phi_j^*(\tau_j, U_l^*)) - b_j^{-1}(u_j^*(\tau_j, U_l^*), v_j^*(\tau_j, U_l^*))\\
&= (\nabla_{(\rho,u,v,e)}\lambda_j^* \cdot \overrightarrow{r}_j^* - \nabla_{(u,v)}\lambda_j \cdot \overrightarrow{r}_j)\Big|_{(\rho,u,v,e)=(\rho_j^*,u_j^*,v_j^*,e_j^*)}\\
&= \Big(\frac{\partial\lambda_j^*}{\rho}, \frac{\partial\lambda_j^*}{u}, \frac{\partial\lambda_j^*}{v}, \frac{\partial\lambda_j^*}{e}\Big)(\rho(\lambda_j^* u - v), -\lambda_j^*, 1, (\lambda_j^* u - v)^{\mathrm{T}})\Big|_{(\rho,u,v,e)=(\rho_j^*,u_j^*,v_j^*,e_j^*)}\\
&\quad - \Big(\frac{\partial\lambda_j}{u}, \frac{\partial\lambda_j}{v}\Big)(-\lambda_j, 1)^{\mathrm{T}}\Big|_{(u,v)=(u_j^*,v_j^*)}\\
&= \Big(0, \frac{\partial\lambda_j^*}{u}, \frac{\partial\lambda_j^*}{v}, 0\Big)(\rho(\lambda_j^* u - v), -\lambda_j^*, 1, (\lambda_j^* u - v)^{\mathrm{T}})\Big|_{(\rho,u,v,e)=(\rho_j^*,u_j^*,v_j^*,e_j^*)}\\
&\quad - \Big(\frac{\partial\lambda_j}{u}, \frac{\partial\lambda_j}{v}\Big)(-\lambda_j, 1)^{\mathrm{T}}|_{(u,v)=(u_j^*,v_j^*)}.
\end{aligned}
$$

In view of statement (1), we get

$$
\begin{aligned}
&b_j^{*-1}(\Phi_j^*(\tau_j, U_l^*)) - b_j^{-1}(u_j^*(\tau_j, U_l^*), v_j^*(\tau_j, U_l^*))\\
&= \Big(-\frac{\partial\lambda_j}{u}\lambda_j + \frac{\partial\lambda_j}{v}\Big)\Big|_{(u,v)=(u_j^*,v_j^*)} - \Big(-\frac{\partial\lambda_j}{u}\lambda_j + \frac{\partial\lambda_j}{v}\Big)\Big|_{(u,v)=(u_j^*,v_j^*)}\\
&= 0.
\end{aligned}
$$

The proof is complete.

Lemma 3.5 *For $\tau_j \geq 0$ and $j = 1, 4$, there hold that*

$$
\frac{\mathrm{d}}{\mathrm{d}\tau_j}\begin{pmatrix} u_j^* \\ v_j^* \end{pmatrix} = b_j(u_j^*, v_j^*)\begin{pmatrix} -\lambda_j \\ 1 \end{pmatrix}\Big|_{(u,v)=(u_j^*,v_j^*)} \tag{3.11}
$$

and

$$
\begin{pmatrix} u_j^*(0, U_l^*) \\ v_j^*(0, U_l^*) \end{pmatrix} = \begin{pmatrix} u_l \\ v_l \end{pmatrix}. \tag{3.12}
$$

Proof Lemma 3.4 implies that

$$
b_j^*(\Phi_j^*(\tau_j, U_l^*)) = b_j(u_j^*(\tau_j, U_l^*), v_j^*(\tau_j, U_l^*)) \tag{3.13}
$$

for $j = 1, 4$. Then, applying (3.13) and Lemma 3.4 to (3.3) gives the result.

The proof is complete.

4 Comparison of the Solutions to the Riemann Problems

Set

$$
D = \{(u, v)^{\mathrm{T}} \in O_{\delta_1}(U_0^{(0)}) \subset R^2 \mid u > 1\}
$$

and

$$
D^* = \{(\rho, u, v, e)^{\mathrm{T}} \in O_{\delta_2}(U_0^{(1)}) \subset R^4 \mid u > 1\}.
$$

Denote

$$
\Phi(\tau_1, \tau_4, U_l) = \Phi_4(\tau_4, \Phi_1(\tau_1, U_l))
$$

for $U_l \in D$,

$$\Phi^*(\tau_1, \tau_2, \tau_3, \tau_4, U_l^*) = \Phi_4^*(\tau_4, \Phi_3^*(\tau_3, \Phi_2^*(\tau_2, \Phi_1^*(\tau_1, U_l^*))))$$

for $U_l^* \in D^*$, and

$$H_j(\tau_j, U_l) := \begin{pmatrix} g(\phi_j(\tau_j, U_l)) \\ \phi_j(\tau_j, U_l) \\ f(\phi_j(\tau_j, U_l)) \end{pmatrix}.$$

Lemma 4.1 *For $j = 1, 4$, it holds that*

$$\phi_j^*(\tau_j, U_l^*) = H_j(\tau_j, U_l) \tag{4.1}$$

for $\tau_j \geq 0$,

$$\frac{\partial H_j}{\partial \tau_j}(0, U_l) = r_j^*(U_l^*) \tag{4.2}$$

and

$$\frac{\partial^2 H_j}{\partial \tau_j^2}(0, U_l) = \frac{\partial^2 \phi_j^*}{\partial \tau_j^2}(0, U_l^*), \tag{4.3}$$

where $U_l^* = (g(U_l), U_l, f(U_l))$.

Proof For $j = 1, 4$, by (3.11)–(3.12), we have

$$\begin{pmatrix} u_j^*(\tau_j, U_l^*) \\ v_j^*(\tau_j, U_l^*) \end{pmatrix} = \phi_j(\tau_j, U_l)$$

for $\tau_j \geq 0$. Then, by Lemma 3.3,

$$\rho_j^*(\tau_j, U_l^*) = g(\phi_j(\tau_j, U_l))$$

for $\tau_j \geq 0$. Moreover, by Lemma 3.2,

$$e_j^*(\tau_j, U_l^*) = f(\phi_j(\tau_j, U_l))$$

for $\tau_j \geq 0$. These lead to (4.1) for $\tau_j \geq 0$.

On the other hand, both ϕ_j^* and H_j are functions of C^2 class. Then, by (4.1), we have the results (4.2)–(4.3). The proof is complete.

Proposition 4.1 *Suppose that $U_l \in D$. For small α_k with $k = 1$ or 4, the equations*

$$\Phi^*(\beta_1, \beta_2, \beta_3, \beta_4, U_l^*) = \begin{pmatrix} g(\phi_k(\alpha_k, U_l)) \\ \phi_k(\alpha_k, U_l) \\ f(\phi_k(\alpha_k, U_l)) \end{pmatrix} = H(\alpha_k, U_l) \tag{4.4}$$

have a unique solution $(\beta_1, \beta_2, \beta_3, \beta_4)$. Moreover,

$$\beta_k = \alpha_k \delta_{jk} + O(1)|\alpha_k^-|^3 \tag{4.5}$$

and

$$\beta_j = O(1)|\alpha_k^-|^3, \quad j \neq k. \tag{4.6}$$

Here $U_l^* = (g(U_l), U_l, f(U_l))$ and $\alpha^- = \min\{\alpha, 0\}$. The bound of $O(1)$ is independent of α_k and U_l.

Proof Since

$$\det\Big(\frac{\partial\Phi^*}{\partial(\beta_1,\beta_2,\beta_3,\beta_4)}\Big)\Big|_{\beta_i=0}\neq 0, \tag{4.7}$$

then the implicit function theorem implies the existence of the solution $(\beta_1,\beta_2,\beta_3,\beta_4)$ to (4.4) with $\beta_j=\beta_j(\alpha_k,U_l)\in C^2$. Moreover, $\beta_j|_{\alpha_k=0}=0$.

Next we go to get the Taylor expansion for β_j. Differentiating (4.4) with respect to α_k and letting $\alpha_k=0$, by Lemma 4.1, we have

$$\sum_{j=1}^{4}\frac{\partial\beta_j}{\partial\alpha_k}\Big|_{\alpha_k=0}\overrightarrow{r}_j^*(U_l^*)=\overrightarrow{r}_k^*(U_l^*) \tag{4.8}$$

for $k=1,4$, which gives the coefficients in the first order terms,

$$\frac{\partial\beta_j}{\partial\alpha_k}\Big|_{\alpha_k=0}=\delta_{jk}, \tag{4.9}$$

where $\delta_{jk}=1$ for $j=k$ and $\delta_{jk}=0$ for $j\neq k$.

To get the terms of the second order in the Taylor expansion, we differentiate the equations (4.4) with respect to α_k again and let $\alpha_k=0$. Then together with Lemma 4.1 and (4.9), we have

$$\sum_{j=1}^{4}\frac{\partial^2\beta_j}{\partial\alpha_k^2}\Big|_{\alpha_k=0}\overrightarrow{r}_j^*(U_l^*)+\frac{\partial^2\Phi^*}{\partial\beta_k^2}(0,U_l^*)=\frac{\partial^2 H_k}{\partial\alpha_k^2}(0,U_l) \tag{4.10}$$

for $k=1$ or $k=4$. Therefore, applying Lemma 4.1 to (4.10) gives

$$\frac{\partial^2\beta_j}{\partial\alpha_k^2}\Big|_{\alpha_k=0}=0. \tag{4.11}$$

Therefore, when $\alpha_k<0$, combining (4.9) with (4.10), we can derive the result. Moreover, when $\alpha_k\geq 0$, due to Lemma 4.1 and the uniqueness of the solution $(\beta_1,\beta_2,\beta_3,\beta_4)$ from the implicit function theorem, we have $\beta_k=\alpha_k$ and $\beta_j=0$ $(j\neq k)$. The proof is complete.

Proposition 4.2 *Suppose that* $(u_l,v_l),(u_r,v_r)\in D$ *with*

$$\begin{pmatrix}u_r\\ v_r\end{pmatrix}=\phi(\alpha_1,\alpha_4,(u_l,v_l)),$$

and that

$$\begin{pmatrix}\rho_r\\ u_r\\ v_r\\ e_r\end{pmatrix}=\phi^*(\beta_1,\beta_2,\beta_3,\beta_4,(\rho_l,u_l,v_l,e_l))$$

with $\rho_i=g(u_i,v_i)$, $e_i=f(u_i,v_i)$ *for* $i=l,r$. *Then*

$$\beta_j=\alpha_j+O(1)\{|\alpha_1^-|+|\alpha_4^-|\}^3,\quad j=1,4 \tag{4.12}$$

and

$$\beta_k=O(1)\{|\alpha_1^-|+|\alpha_4^-|\}^3,\quad k=2,3, \tag{4.13}$$

where $\alpha^-=\min\{\alpha,0\}$. *The bound of* $O(1)$ *is independent of* α_1,α_4 *and* (u_l,v_l).

Proof It suffices to solve the following equations for any given α_1, α_4 and $(u_l, v_l) \in D$:

$$\phi^*(\beta_1, \beta_2, \beta_3, \beta_4, \rho_l, u_l, v_l, e_l) = \begin{pmatrix} g(\phi(\alpha_1, \alpha_4, u_l, v_l)) \\ \phi(\alpha_1, \alpha_4, u_l, v_l) \\ f(\phi(\alpha_1, \alpha_4, u_l, v_l)) \end{pmatrix}.$$

In the same way as in the proof of Proposition 4.1, we can find C^2 functions $\beta_k = \beta_k(\alpha_1, \alpha_4, u_l, v_l)$, such that $(\beta_1, \beta_2, \beta_3, \beta_4)$ is the unique solution to the above system.

To derive the estimates on β_k, we let

$$(u_m, v_m)^{\mathrm{T}} = \Phi(\alpha_1, 0, u_l, v_l), \quad \rho_m = g(u_m, v_m), \quad e_m = f(u_m, v_m) \tag{4.14}$$

and consider the following equations:

$$\begin{pmatrix} \rho_m \\ u_m \\ v_m \\ e_m \end{pmatrix} = \phi^*(\beta_1', \beta_2', \beta_3', \beta_4', \rho_l, u_l, v_l, e_l),$$
$$\begin{pmatrix} \rho_r \\ u_r \\ v_r \\ e_r \end{pmatrix} = \phi^*(\beta_1'', \beta_2'', \beta_3'', \beta_4'', \rho_m, u_m, v_m, e_m).$$

By Proposition 4.1, we have

$$\beta_k' = \alpha_1 \delta_{k1} + O(1)|\alpha_1^-|^3, \tag{4.15}$$
$$\beta_k'' = \alpha_4 \delta_{k4} + O(1)|\alpha_4^-|^3, \tag{4.16}$$

where δ_{ij} is a kronecker symbol. Therefore, with the Glimm interaction estimates (see [3, 9, 13–16]), (4.15)–(4.16) give the estimates on $\beta_k, k = 1, 2, 3, 4$. The proof is complete.

5 Proof of the Main Result

Let $\{U_2^\varepsilon(x, y)\}_{\varepsilon>0}$ be a sequence of approximate solutions to system (1.10) constructed by a wave-front tracking algorithm (see [3, 18]), such that

$$\|U_2^\varepsilon(0, \cdot) - U_0(y)\|_{L^1} < \varepsilon.$$

U_2^ε is called an ε-approximate solution to system (1.10) and is a piecewise constant function in $x > 0$ with a finite number of wave fronts, which consist of shocks, rarefaction-fronts and non-physical-fronts. The small parameter ε controls three types of errors in U_2^ε as follows:

(1) The error in the speeds of shock and rarefaction fronts.

(2) The maximum strength of rarefaction fronts.

(3) The total strength of all non-physical waves.

As $\varepsilon \to 0+$, U_2^ε tends in L^1_{loc} to the entropy weak solution U_2 to (1.10). Due to [2, 18], the limit is unique. Therefore, the solution U_2 to system (1.10) is given by $U_2 = (g(u, v), u, v, f(u, v))^{\mathrm{T}}$.

To prove the main result, we need to study these approximate solutions. Due to [3], such approximate solutions have the following properties.

Lemma 5.1 *There exists a constant $M > 0$ independent of ε and U_ε, such that*

$$\|U_2^\varepsilon(x_1, \cdot) - U_2^\varepsilon(x_2, \cdot)\|_{L^1(R^1)} \leq M|x_1 - x_2|, \quad x_1, x_2 \in [0, +\infty). \tag{5.1}$$

By Kong-Yang's result (see [17]), we can obtain the standard Riemann semigroup S_x generated by system (1.9). Suppose that the initial data $\overline{U}$ for system (1.9) satisfies $\overline{U} = \overline{U}_0^{(0)}$ $(y \to \pm\infty)$ and $\overline{U} - \overline{U}_0^{(0)} \in L^1$. Then the ε-approximate solution $\overline{U}_2^{\varepsilon}$ to system (1.9) with an initial data $\overline{U}$ converges to a limiting function $U(x, y)$. Such a mapping $(\overline{U}, x) \to U(x, \cdot) \doteq S_x\overline{U}$ generates a standard Riemann semigroup. Furthermore, there exists a positive constant L, such that for any initial data $\overline{U}, \overline{V}$ and $s, t > 0$, we have

$$S_0\overline{U} = \overline{U}, \tag{5.2}$$

$$S_s(S_t\overline{U}) = S_{s+t}\overline{U}, \tag{5.3}$$

$$\|S_t\overline{U} - S_s\overline{V}\| \leq L(\|\overline{U} - \overline{V}\ \|_{L^1} + |\ t - s\ |). \tag{5.4}$$

Here, $\overline{U}_0^{(0)} = (g(u_0, v_0), u_0, v_0, f(u_0, v_0))$.

Corollary 5.1 *Suppose $U_l, U_r \in D$, $|\lambda| \leq \widehat{\lambda}$ and $x > 0$, where $\widehat{\lambda}$ is a fixed constant. Define $W = W(x, y)$ as the self-similar solution to system* (1.9) *with the Riemann initial data*

$$W(0, y) = \begin{cases} U_l^* = (\rho_l = g(u_l, v_l), u_l, v_l, f(u_l, v_l)), & y < 0, \\ U_r^* = (\rho_r = g(u_r, v_r), u_r, v_r, f(u_r, v_r)), & y > 0. \end{cases} \tag{5.5}$$

Here $U_l = (u_l, v_l)^{\mathrm{T}}$ and $U_r = (u_r, v_r)^{\mathrm{T}}$. Consider the following function:

$$V(x, y) = \begin{cases} U_l^*, & y < \lambda x, \\ U_r^*, & y \geq \lambda x. \end{cases} \tag{5.6}$$

It holds that

(i) *Generally, we have*

$$\frac{1}{x}\|V(x, \cdot) - W(x, \cdot)\ \|_{L^1(R^1)} = O(1)|U_r - U_l|. \tag{5.7}$$

(ii) *If $U_r = \phi_j(\sigma_j, U_l)$ for $\sigma_j > 0$, $j = 1$ or 4 and $\lambda = \lambda_j(U_r)$, i.e., U_l and U_r can be connected by a j-simple wave for system* (1.10)*, then we have*

$$\frac{1}{x}\|V(x, \cdot) - W(x, \cdot)\ \|_{L^1(R^1)} = O(1)\sigma_j^2. \tag{5.8}$$

Here ϕ_j is the j-simple wave curve, λ_j is the j-eigenvalue, and σ_j is the strength of the j-simple wave.

(iii) *If U_l and U_r can be connected by a j-shock wave, i.e., $U_r = \phi_j(\sigma_j, U_l)$ for $\sigma_j > 0, j = 1$ or 4 and $\lambda = s_j(U_r, U_l)$, where $s_j(U_r, U_l)$ is the shock wave speed for system* (1.10)*, then we have*

$$\frac{1}{x}\|V(x, \cdot) - W(x, \cdot)\ \|_{L^1(R^1)} = O(1)|U_r - U_l|^3. \tag{5.9}$$

Proof (i) (5.7) can be deduced as follows:

$$\begin{aligned} \frac{1}{x}\|V(x, \cdot) - W(x, \cdot)\ \|_{L^1(R^1)} &= \frac{1}{x}\int_{-\infty}^{\infty} |V(x, y) - W(x, y)|\mathrm{d}y \\ &= \frac{1}{x}\int_{-x\widehat{\lambda}}^{x\widehat{\lambda}} |V(x, y) - W(x, y)|\mathrm{d}y \\ &= O(1)|U_r - U_l|. \end{aligned}$$

(ii) Here we only prove the case of $k = 1$, and the result for $k = 4$ can be deduced similarly. If $\sigma_1 > 0$, the Riemann solution to system (1.9) can be given as follows:

$$\phi^*(\beta_1, \beta_2, \beta_3, \beta_4, U_l^*) = \begin{pmatrix} g(\phi_1(\sigma_1, U_l)) \\ \phi_1(\sigma_1, U_l) \\ f(\phi_1(\sigma_1, U_l)) \end{pmatrix}.$$

By Proposition 4.1, the above equation has a unique solution for $\sigma_1 > 0$

$$\beta_1 = \sigma_1, \quad \beta_k = 0, \quad k = 2, 3, 4. \tag{5.10}$$

Thus we have

$$\begin{aligned} \frac{1}{x}\|V(x,\cdot) - W(x,\cdot)\,\|_{L^1(R^1)} &= \frac{1}{x}\int_{-\infty}^{\infty} |V(x,y) - W(x,y)|\mathrm{d}y \\ &= \frac{1}{x}\int_{x\lambda_1^*(U_l^*)}^{x\lambda_1^*(U_r^*)} |V(x,y) - W(x,y)|\mathrm{d}y \\ &= O(1)|U_r^* - U_l^*|(\lambda_1^*(U_r^*) - \lambda_1^*(U_l^*)) \\ &= O(1)\sigma_1^2. \end{aligned}$$

(iii) Similarly, by Proposition 4.1,

$$\phi^*(\beta_1, \beta_2, \beta_3, \beta_4, U_l^*) = \begin{pmatrix} g(\phi_1(\sigma_1, U_l)) \\ \phi_1(\sigma_1, U_l) \\ f(\phi_1(\sigma_1, U_l)) \end{pmatrix}$$

has a unique solution

$$\beta_1 = \sigma_1 + O(1)|\sigma_1|^3, \tag{5.11}$$

$$\beta_k = O(1)|\sigma_1|^3, \quad k = 2, 3, 4. \tag{5.12}$$

Suppose that W_m, W_m' are the intermediate states of the Riemann problem. Then

$$\begin{aligned} |W_m' - U_r| &= O(1)|\beta_4| = O(1)|U_r - U_l|^3, \\ |W_m - U_r| &\le |W_m - W_m'| + |W_m' - U_r| \\ &= O(1)(|\beta_2| + |\beta_3| + |\beta_4|) \\ &= O(1)|U_r - U_l|^3. \end{aligned}$$

Since $\beta_1 < 0$, assume that s_1^* is the corresponding shock speed. From

$$\begin{aligned} &s_1(\sigma_1) = \lambda_1(U_l) + \frac{1}{2}\sigma_1 + O(1)\sigma_1^2, \\ &s_1^*(\beta_1) = \lambda_1^*(U_l^*) + \frac{1}{2}\beta_1 + O(1)\sigma_1^2 = \lambda_1(U_l) + \frac{1}{2}\sigma_1 + O(1)\sigma_1^2, \end{aligned}$$

we know that

$$s_1^* - s_1 = O(1)|\sigma_1|^2 = O(1)|U_r - U_l|^2.$$

Define

$$\begin{aligned} &q_M^1 = \max(s_1, s_1^*), \quad q_m^1 = \min(s_1, s_1^*), \\ &q_M^4 = \max(\lambda_4(U_r), \lambda_4(U_m)), \quad q_m^4 = \min(\lambda_4(U_r), \lambda_4(U_m)). \end{aligned}$$

Then we have

$$\begin{aligned}\frac{1}{x}\|V(x,\cdot)-W(x,\cdot)\ \|_{L^1(R^1)}&=\frac{1}{x}\int_{-\infty}^{\infty}|V(x,y)-W(x,y)|\mathrm{d}y\\&=\frac{1}{x}\Big(\int_{q_m^1x}^{q_M^1x}+\int_{q_M^1x}^{q_M^4x}\Big)|V(x,y)-W(x,y)|\mathrm{d}y\\&=O(1)|U_r-U_l|\cdot|s_1-s_1^*|+O(1)|U_r-U_l|^3\\&=O(1)|U_r-U_l|^3.\end{aligned}$$

Proof of Theorem 1.1 Let S be the standard Riemann semigroup generated by system (1.9). By Lemma 5.1, for any $\varepsilon>0$, there exists an ε-approximate solution $U_2^\varepsilon(x,y)=(u_2^\varepsilon(x,y),v_2^\varepsilon(x,y))$ to system (1.10). Define

$$U_2^{\varepsilon^*}(x,y)=(g(u_2^\varepsilon(x,y),v_2^\varepsilon(x,y)),u_2^\varepsilon(x,y),v_2^\varepsilon(x,y),f(u_2^\varepsilon(x,y),v_2^\varepsilon(x,y))). \tag{5.13}$$

Firstly, we estimate the error between $U_2^\varepsilon(x,y)$ and the weak solution $x\to S_xU_0^*$. In view of (5.1)–(5.4) and by [3, Theorem 2.9], we have the following error formula:

$$\|U_2^{\varepsilon^*}(x,\cdot)-S_xU_0^*\|_{L^1(R^1)}\le L\int_0^x\Big(\lim_{h\to0+}\inf\frac{\|U_2^{\varepsilon^*}(\tau+h)-S_hU_2^{\varepsilon^*}(\tau)\|_{L^1(R^1)}}{h}\Big)\mathrm{d}\tau. \tag{5.14}$$

Suppose that at time $\tau\in[0,x]$, no interaction takes place, and $U_2^{\varepsilon^*}$ has jumps at points $y_1<y_2<\cdots<y_N$. Denote $S=\{\alpha\mid 1\le\alpha\le N,\ U_2^{\varepsilon^*}(\tau,y_\alpha-)$ and $U_2^{\varepsilon^*}(\tau,y_\alpha+)$ can be connected by shock fronts or contact discontinuities$\}$, $R=\{\alpha\mid 1\le\alpha\le N,\ U_2^{\varepsilon^*}(\tau,y_\alpha-)$ and $U_2^{\varepsilon^*}(\tau,y_\alpha+)$ can be connected by rarefaction fronts$\}$ and $NP=\{\alpha\mid 1\le\alpha\le N,U_2^{\varepsilon^*}(\tau,y_\alpha-)$ and $U_2^{\varepsilon^*}(\tau,y_\alpha+)$ can be connected by nonphysical fronts$\}$.

For any fixed α, we define W_α as the Riemann solution to system (1.9) with the initial data $U_l=U_2^{\varepsilon^*}(\tau,y_\alpha-)$ and $U_R=U_2^{\varepsilon^*}(\tau,y_\alpha+)$. For any small h, the mapping $h\to S_hU_2^{\varepsilon^*}$ is a piecewise constant function with a finite number of fronts. Then by Lemma 5.1 and (5.7)–(5.9), we have

$$\begin{aligned}&\lim_{h\to0+}\frac{\|U_2^{\varepsilon^*}(\tau+h)-S_hU_2^{\varepsilon^*}(\tau)\|_{L^1(R^1)}}{h}\\&=\sum_{\alpha\in S\cup R\cup NP}\Big(\lim_{h\to0+}\frac{1}{h}\int_{y_\alpha-\rho}^{y_\alpha+\rho}|U_2^{\varepsilon^*}(\tau+h,y)-W_\alpha(h,y-y_\alpha)|\mathrm{d}y\Big)\\&=\sum_{\alpha\in S}O(1)(\varepsilon|U_2^{\varepsilon^*}(\tau,y_\alpha+)-U_2^{\varepsilon^*}(\tau,y_\alpha-)|+|U_2^{\varepsilon^*}(\tau,y_\alpha+)-U_2^{\varepsilon^*}(\tau,y_\alpha-)|^3)\\&\quad+\sum_{\alpha\in R}O(1)|\sigma_\alpha|(|\sigma_\alpha|+\varepsilon)+\sum_{\alpha\in NP}O(1)|U_2^{\varepsilon^*}(\tau,y_\alpha+)-U_2^{\varepsilon^*}(\tau,y_\alpha-)|,\end{aligned} \tag{5.15}$$

where σ_α is the strength of the rarefaction wave, and ρ is a suitable small positive constant. By the properties of the ε-approximation solution, the strengthes of the rarefaction wave and nonphysical fronts are less than ε. So the terms in (5.15) can be controlled by $O(1)\cdot\varepsilon$. For any $\tau\in[0,x]$, we have

$$\begin{aligned}&\lim_{h\to0+}\frac{\|U_2^{\varepsilon^*}(\tau+h)-S_hU_2^{\varepsilon^*}(\tau)\|_{L^1(R^1)}}{h}\\&=O(1)(\varepsilon+\varepsilon TV(U_2^{\varepsilon^*}(\tau,\cdot))+TV(U_2^{\varepsilon^*}(\tau,\cdot))^3)\\&=O(1)(\varepsilon+\varepsilon TV(U_2^{\varepsilon^*}(0,\cdot))+TV(U_2^{\varepsilon^*}(0,\cdot))^3)\\&=O(1)(\varepsilon+TV(U_0))^3).\end{aligned}$$

Inserting the above equalities into (5.14), we get

$$\|U_2^{\varepsilon^*}(x,\cdot) - S_x U_0^*\|_{L^1(R^1)} \le O(1)(\varepsilon + TV(U_0))^3). \tag{5.16}$$

U_2^{ε} converges to the weak solution to (1.10) and $U_2^{\varepsilon^*}$ converges to U_1 uniformly. So letting $\varepsilon \to 0$ in (5.16), we complete the proof of Theorem 1.1.

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References

[1] Chen, G. Q., Christoforou, C. and Zhang, Y. Q., Continuous dependence of entropy solutions to the Euler equations on the adiabatic exponent and Mach number, *Arch. Ration. Mech. Anal.*, **189**(1), 2008, 97–130.

[2] Bressan, A., The unique limit of the Glimm scheme, *Arch. Rational Mech. Anal.*, **130**, 1995, 105–230.

[3] Bressan, A., Hyperbolic Systems of Conservation Laws, The One-Dimensional Cauchy Problem, Oxford Lecture Series in Mathematics and Its Applications, Vol. 20, Oxford University Press, Oxford, 2000.

[4] Saint-Raymond, L., Isentropic approximation of the compressible Euler system in one space dimension, *Arch. Rational Mech. Anal.*, **155**, 2000, 171–199.

[5] Bianchini, S. and Colombo, R. M., On the stability of the standard Riemann semigroup, *Proc. American Math. Soc.*, **130**, 2002, 1961–1973.

[6] Zhang, Y. Q., On the irrotational approximation to steady supersonic flow, *Z. Angew. Math. Phys.*, **58**(2), 2007, 209–223.

[7] Liu, C. and Zhang, Y. Q., Isentropic approximation of the steady Euler system in two space dimensions, *Commun. Pure Appl. Anal.*, **7**(2), 2008, 277–291.

[8] Lax, P. D., Hyperbolic systems of conservation laws. II, *Comm. Pure Appl. Math.*, **10**, 1957, 537–566.

[9] Smoller, J., Shock Waves and Reaction-Diffusion Equations, Springer-Verlag, New York, 1983.

[10] Zhang, Y. Q., Steady supersonic flow past an almost straight wedge with large vertex angle, *J. Diff. Eq.*, **192**, 2003, 1–46.

[11] Chen, G. Q., Zhang, Y. Q. and Zhu, D. W., Existence and stability of supersonic Euler flows past Lipschitz wedges, *Arch. Rational Mech. Anal.*, **181**, 2006, 261–310.

[12] Zhang, Y. Q., Global existence of steady supersonic potential flow past a curved wedge with a piecewise smooth boundary, *SIAM J. Math. Anal.*, **31**, 1999, 166–183.

[13] Corli, A. and Sable-Tougeron, M., Stability of contact discontinuities under perturbations of bounded variation, *Rend. Sem. Mat. Univ. Padova*, **97**, 1997, 35–60.

[14] Dafermos, C. M., Hyperbolic Conservation Laws in Continuum Physics, Springer-Verlag, Berlin, 2000.

[15] Glimm, J., Solutions in the large for nonlinear hyperbolic systems of equations, *Comm. Pure Appl. Math.*, **18**, 1965, 95–105.

[16] Serre, D., Systems of Conservation Laws, Cambridge University Press, Cambridge, 1999.

[17] Kong, D. and Yang, T., A note on well-posedness theory for hyperbolic conservation laws, *Applied Mathematics Letters*, **16**, 2003, 143–146.

[18] Bressan, A., Liu, T. P. and Yang, T., L^1 stability estimates for $n \times n$ conservation laws, *Arch. Rational Mech. Anal.*, **149**, 1999, 1–22.