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Closed Geodesics and Volume Growth of Open Manifolds with Sectional Curvature Bounded from Below*

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Abstract In this paper, the relationship between the existence of closed geodesics and the volume growth of complete noncompact Riemannian manifolds is studied. First the authors prove a diffeomorphic result of such an n-manifold with nonnegative sectional curvature, which improves Marenich-Toponogov's theorem. As an application, a rigidity theorem is obtained for nonnegatively curved open manifold which contains a closed geodesic. Next the authors prove a theorem about the nonexistence of closed geodesics for Riemannian manifolds with sectional curvature bounded from below by a negative constant.

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1 Introduction

Let (M, g) be an n-dimensional complete noncompact Riemannian manifold with sectional curvature satisfying $K_M \geq c$, where $c \leq 0$ is a constant. Denote by $\alpha_n(r, c)$ the volume of a geodesic ball of radius r in an n-dimensional space form of constant curvature c. The relative volume comparison theorem (see [1]) implies that the function

$$r \to \frac{\mathrm{Vol}[B(p, r)]}{\alpha_n(r, c)}$$

is monotone decreasing, where $B(p, r)$ is the open metric ball with center p and radius r in M. It is well known that

$$\alpha_n(r, c) = \omega_{n-1} \int_0^r S_c(t)\mathrm{d}t,$$

where

$$S_c(t) = \begin{cases} t^{n-1}, & c = 0, \\ \Big(\dfrac{\sinh(\sqrt{-c}t)}{\sqrt{-c}}\Big)^{n-1}, & c < 0, \end{cases}$$

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and ω_m is the volume of $\mathbf{S}^m(1)$.

For any $p \in M$, we set

$$\nu_c(p) = \lim_{r\to\infty} \frac{\mathrm{Vol}[B(p,r)]}{\alpha_n(r,c)},$$

and define

$$\nu_c(M) = \inf_{p\in M} \nu_c(p).$$

One always has

$$\frac{\mathrm{Vol}[B(p,r)]}{\alpha_n(r,c)} \geq \nu_c(p) \geq \nu_c(M), \quad \forall r > 0,\ \forall p \in M.$$

Notice that $0 \leq \nu_c(M) \leq 1$, and M is isometric to an n-dimensional space form of constant curvature c if and only if $\nu_c(M) = 1$. Moreover, for $c = 0$, $\nu_0(p)$ is independent of the choice of the base point p, i.e., $\nu_0(M) = \nu_0(p)$.

Riemannian manifolds with large volume growth, i.e., $\nu_c(M) > 0$, have been studied extensively in the last two decades, see for examples [3, 6, 9–10] and the references therein. In this paper, we shall study the relationship between the existence of closed geodesics and volume growth.

It is well known that any compact Riemannian manifold contains at least one closed geodesic (see [5]), but this is not true for an open Riemannian manifold, since there is the following theorem (see [4, 9]).

Theorem 1.1 *If N is a closed minimal k-submanifold of a nonnegatively curved n-manifold M, then*

$$\mathrm{Vol}[B(N,r)] \leq \mathrm{Vol}(N)\alpha_{n-k}(r,0),$$

where $B(N,r) = \{x \in M : d(x,N) < r\}$ and $\alpha_{n-k}(r,0)$ denotes the volume of the r-ball in the Euclidean space $\mathbb{R}^{n-k}$.

As an application of Theorem 1.1, we shall prove the following result which improves Marenich and Toponogov's theorem (see [7, 9]).

Theorem 1.2 *Let M be an n-dimensional complete noncompact Riemannian manifold with sectional curvature $K_M \geq 0$. If*

$$\limsup_{r\to\infty} \frac{\mathrm{Vol}[B(p,r)]}{r^s} > 0$$

for some $p \in M$ and $s > n-1$, then M is diffeomorphic to $\mathbb{R}^n$.

Note that if $s = n$, then the above theorem is just Marenich and Toponogov's theorem.

By Theorem 1.2, it is natural to consider $\frac{\mathrm{Vol}[B(p,r)]}{\omega_{n-1}r^{n-1}}$. Thus we set

$$\mu(p) = \limsup_{r\to\infty} \frac{\mathrm{Vol}[B(p,r)]}{\omega_{n-1}r^{n-1}}$$

and define

$$\mu(M) = \inf_{p\in M} \mu(p).$$

Note that, the limit $\mu(p)$ may be infinity.

Using Wu's method in [9], we shall obtain the following rigidity theorem.

Theorem 1.3 *Let M be an n-dimensional complete noncompact Riemannian manifold with sectional curvature $K_M \geq 0$. If M contains a closed geodesic σ with length $L(\sigma)$, then $\mu(M) \leq L(\sigma)$, and the equality holds if and only if M is isometric to $\mathbf{S}^1 \times \mathbb{R}^{n-1}$ with flat metric.*

Next we discuss a Riemannian manifold with sectional curvature $K_M \geq -\kappa^2$ and $\nu_{-\kappa^2}(M) > 0$, where $\kappa > 0$. For the topology of this kind of manifolds, the reader can refer to [10] for more details. Here we prove the following theorem about the nonexistence of closed geodesics for this kind of manifolds.

Theorem 1.4 *Given $\nu \in (0,1)$ and $\kappa > 0$, let M be a complete Riemannian n (≥ 2)-manifold with $K_M \geq -\kappa^2$, and $\nu_{-\kappa^2}(M) > \nu$. Assume that $\theta_0 = \theta_0(\nu, n) \in \left(0, \frac{\pi}{2}\right)$ is the solution to*

$$2\int_0^{\theta_0} \sin^{n-2} t\mathrm{d}t = (1-\nu)\int_0^{\pi} \sin^{n-2} t\mathrm{d}t.$$

If $r_0 = \frac{1}{\kappa}\tanh^{-1}(\cos\theta_0)$, then M does not contain any closed geodesic γ with length $L(\gamma) < 2r_0$.

In Section 2, we shall recall some fundamental facts and several important lemmas. The main results are proved in Section 3.

2 Lemmas

In this section, we recall some fundamental facts for later use. Throughout this paper, all geodesics are assumed to have a unit speed.

By the first variation formula of arc length, it is easy to get the following lemma.

Lemma 2.1 *Let N be a smooth compact submanifold of a complete Riemannian manifold M. Assume that $q \in M$, $q \overline{\in} N$, and that $\gamma : [0, a] \to M$ is a minimal geodesic from N to q. Then $\langle \gamma'(0), \mathbf{v} \rangle = 0$ for any $\mathbf{v} \in T_{\gamma(0)}N$.*

Lemma 2.2 *Let M be an n-dimensional complete noncompact Riemannian manifold. Given $s > 0$, if $\lim\limits_{r\to\infty} \frac{\mathrm{Vol}[B(p,r)]}{r^s} = 0$ for some $p \in M$, then for any $q \in M$,*

$$\lim_{r\to\infty} \frac{\mathrm{Vol}[B(q,r)]}{r^s} = 0.$$

Proof Let $d = d(p,q)$ be the distance between p and q. For any $r > d$, one checks that

$$B(p, r-d) \subset B(q,r) \subset B(p, r+d).$$

Then we have

$$\frac{\mathrm{Vol}[B(p,r-d)]}{(r-d)^s} \cdot \frac{(r-d)^s}{r^s} \leq \frac{\mathrm{Vol}[B(q,r)]}{r^s} \leq \frac{\mathrm{Vol}[B(p,r+d)]}{(r+d)^s} \cdot \frac{(r+d)^s}{r^s}.$$

Letting $r \to \infty$, we get the conclusion.

Lemma 2.3 (cf. [2]) *Let M be a complete Riemannian manifold with $K_M \geq c$. Denote by $M^2(c)$ the complete simply connected surface of constant curvature c. Given $l_1, l_2 > 0$, let $\gamma_1 : [0, l_1] \to M$, $\gamma_2 : [0, l_2] \to M$ be two geodesic segments in M such that $\gamma_1(l_1) = \gamma_2(0)$ and $\angle(-\gamma_1'(l_1), \gamma_2'(0)) = \alpha$. We call such a configuration a hinge and denote it by $(\gamma_1, \gamma_2, \alpha)$. Let*

$\widetilde{\gamma}_1, \widetilde{\gamma}_2 \subset M^2(c)$ be two geodesic segments such that $\widetilde{\gamma}_1(l_1) = \widetilde{\gamma}_2(0)$, $L(\gamma_i) = L(\widetilde{\gamma}_i) = l_i$ $(i = 1, 2)$, and $\angle(-\widetilde{\gamma}_1'(l_1), \widetilde{\gamma}_2'(0)) = \alpha$.

Let γ_1 be minimal, and if $c > 0$, $L(\gamma_2) \leq \frac{\pi}{\sqrt{c}}$, then the following holds

$$d(\gamma_1(0), \gamma_2(l_2)) \leq d_c(\widetilde{\gamma}_1(0), \widetilde{\gamma}_2(l_2)),$$

where d_c denotes the distance function in $M^2(c)$.

3 Proofs of the Main Results

Proof of Theorem 1.2 Here we use a similar method as that of Theorem 3.1 in [8]. First by Theorem 1.1, if a nonnegatively curved open Riemannian n-manifold M contains a closed geodesic σ, then

$$\lim_{r\to\infty} \frac{\mathrm{Vol}[B(p,r)]}{r^s} = 0$$

for any $p \in \sigma$ and $s > n-1$. Now by Lemma 2.2, the above limit holds for any $p \in M$.

If M is not diffeomorphic to $\mathbb{R}^n$, by Cheeger-Gromoll's soul theorem, the soul of M is not a point. Then the soul must contain a closed geodesic σ (since any compact Riemannian manifold contains at least one closed geodesic). Because the soul is a totally geodesic submanifold, we have that σ is also a closed geodesic of M, which is a contradiction to the assumption. This finishes the proof of Theorem 1.2.

Before proving Theorem 1.3, let $\sigma : \mathbf{S}^1 \to M$, $u \to \sigma(u)$ be a closed geodesic of M. The normal space of $\sigma(u)$ in M is given by

$$N_{\sigma(u)}M = \{\xi \in T_{\sigma(u)}M \mid \langle \xi, \sigma'(u) \rangle = 0\},$$

where $\langle X, Y \rangle = g(X, Y)$ is the inner product of vectors X and Y, and g is the Riemannian metric of M. The corresponding normal bundle is

$$N_\sigma M = \bigcup_{u \in \mathbf{S}^1} N_{\sigma(u)}M.$$

We consider the following map

$$F : N_\sigma M \to M, \quad (\sigma(u), \xi) \to \exp_{\sigma(u)} \xi.$$

It is easy to show that, when $\|\xi\| = \sqrt{\langle \xi, \xi \rangle}$ is sufficiently small, the tangent map $F_*|_{(\sigma(u),\xi)}$ is a linear map of full rank.

The closed geodesic σ is of course a smooth compact submanifold of M, so by Lemma 2.1, we know that F is a surjective map.

Proof of Theorem 1.3 Assume that $\sigma = \sigma(u)$, and $u \in [0, L(\sigma)]$ is the unit speed closed geodesic. Let

$$D_r\sigma = \{(\sigma(u), \xi) \in N_\sigma M \mid u \in \mathbf{S}^1,\ \|\xi\| < r\},$$

and we have

$$F(D_r\sigma) = \{q \in M \mid d(\sigma, q) < r\},$$

where $d(\sigma, q)$ is the distance from q to σ. For any $u_0 \in u$, we have $B(\sigma(u_0), r) \subset F(D_r\sigma)$, and then

$$\mathrm{Vol}[B(\sigma(u_0), r)] \leq \mathrm{Vol}[F(D_r\sigma)].$$

Let $E(u,t) = \exp_{\sigma(u)}(t\xi(u))$, where $\xi(u)$ is a parallel vector field along σ, and $\xi(u) \in \mathbf{S}^{n-2}(1) \subset N_{\sigma(u)}M$. Set $\gamma(t) = E(u_0, t) = \exp_{\sigma(u_0)}(t\xi(u_0))$ and let $\{e_1, e_2, \cdots, e_{n-2}\}$ be an orthonormal basis of $T_{\xi(u_0)}\mathbf{S}^{n-2}(1)$.

We consider the following $n-1$ Jacobi fields $\{J_1(t), J_2(t), \cdots, J_{n-1}(t)\}$ along $\gamma(t)$

$$J_i(t) = (\exp_{\sigma(u_0)})_{*t\xi(u_0)}(te_i), \quad 1 \leq i \leq n-2$$

and

$$J_{n-1}(t) = E_*\Big(\frac{\partial}{\partial u}\Big)\Big|_{u=u_0}.$$

Then we derive

$$J_i(0) = 0, \quad J_i'(0) = e_i, \quad \langle J_i'(0), \xi(u_0)\rangle = 0, \quad \|J_i'(0)\| = 1, \quad 1 \leq i \leq n-2 \tag{3.1}$$

and

$$J_{n-1}(0) = \sigma'(u_0), \quad J_{n-1}'(0) = 0, \quad \langle J_{n-1}(0), \xi(u_0)\rangle = 0, \quad \|J_{n-1}(0)\| = 1. \tag{3.2}$$

Now

$$\mathrm{Vol}[F(D_r\sigma)] = \int_{\sigma(u_0)\in\sigma}\int_{\xi(u_0)\in\mathbf{S}^{n-2}(1)}\int_0^{\min\{c(\xi(u_0)),r\}}\sqrt{\det(F_*(t))}\mathrm{d}t\mathrm{d}\xi(u_0)\mathrm{d}u_0, \tag{3.3}$$

where

$$F_*(t) = F_*|_{(\sigma(u_0),t\xi(u_0))} = (g_{ij}(t))_{(n-1)\times(n-1)}, \quad g_{ij}(t) = \langle J_i(t), J_j(t)\rangle,$$

and $c(\xi(u_0))$ denotes the distance to the cut points of $\sigma(u_0)$ along the geodesic

$$\gamma(t) = \exp_{\sigma(u_0)}(t\xi(u_0)).$$

Notice that

$$t \leq \min\{c(\xi(u_0)), r\} \leq c(\xi(u_0)),$$

so by (3.1)–(3.2) and the Rauch comparison theorem, we get

$$\|J_i(t)\| \leq t \quad \text{for } 1 \leq i \leq n-2 \ \text{ and } \ \|J_{n-1}(t)\| \leq 1. \tag{3.4}$$

Since $F_*(t)$ is a positive definite symmetric matrix, we then have

$$\det(F_*(t)) \leq \prod_{1\leq i\leq n-1} g_{ii}(t).$$

From (3.4) we obtain

$$\sqrt{\det(F_*(t))} \leq t^{n-2},$$

and by (3.3) we know

$$\begin{aligned}\mathrm{Vol}[B(\sigma(u_0), r)] &\leq \mathrm{Vol}[F(D_r\sigma)] \\ &\leq L(\sigma)\mathrm{Vol}[\mathbf{S}^{n-2}(1)]\int_0^r t^{n-2}\mathrm{d}t \\ &= L(\sigma)\omega_{n-1}r^{n-1}.\end{aligned} \tag{3.5}$$

It is clear by (3.5) that $\mu(\sigma(u_0)) \le L(\sigma)$, so $\mu(M) \le L(\sigma)$.

If $\mu(M) = L(\sigma)$, then

$$g_{ij}(t) = \langle J_i(t), J_j(t)\rangle = t^2\delta_{ij} \quad \text{for } 1 \le i \le n-2$$

and

$$\langle J_{n-1}(t), J_{n-1}(t)\rangle = 1, \quad g_{i(n-1)}(t) = \langle J_i(t), J_{n-1}(t)\rangle = 0 \quad \text{for } 1 \le i \le n-2.$$

By the Gaussian lemma, we have

$$\langle J_i(t), \gamma'(t)\rangle = 0 \quad \text{for } 1 \le i \le n-1.$$

Therefore the metric on M is of the following form

$$g = \mathrm{d}u^2 + \mathrm{d}t^2 + t^2\mathrm{d}\xi^2, \quad u \in [0, L(\sigma)],\ t \ge 0,\ \xi \in \mathbf{S}^{n-2}(1),$$

which implies that M is isometric to $\mathbf{S}^1 \times \mathbb{R}^{n-1}$ with flat metric. This completes the proof of Theorem 1.3.

We use a similar method as that of Theorem 2 in [3] to prove Theorem 1.4.

Proof of Theorem 1.4 Assume that there is a normal closed geodesic γ with length $L(\gamma) = 2r_1 < 2r_0$ on M, and let $p = \gamma(0)$ and $q = \gamma(r_1)$. Denote by γ_1 the part of γ from p to q with $\gamma_1'(0) = \gamma'(0)$, and γ_2 the part of γ from p to q with $\gamma_2'(0) = -\gamma'(0)$, and then $L(\gamma_1) = L(\gamma_2) = r_1 < r_0$. Let $\Gamma = \{\gamma'(0), -\gamma'(0)\}$ be the set of two unit vectors in T_pM. For any $\theta \in \left[0, \frac{\pi}{2}\right]$, let

$$\Gamma(\theta) = \{u \in S_pM \mid \angle(u, \Gamma) \le \theta\}.$$

So

$$\mathrm{Vol}[\Gamma(\theta_0)] = 2V(\theta_0) = (1-\nu)\omega_{n-1},$$

where $V(\theta_0)$ denotes the volume of a geodesic ball of radius θ_0 in an $(n-1)$-unit sphere and ω_m is again the volume of $\mathbf{S}^m(1)$.

For each $u \in S_pM$, let $c(u)$ denote the distance to the cut points of p along the geodesic $\exp_p(tu)$. We claim that for any $u \in \Gamma(\theta_0)$,

$$c(u) \le R_0 := \frac{1}{\kappa}\tanh^{-1}\Big(\frac{\tanh\kappa r_1}{\cos\theta_0}\Big). \tag{3.6}$$

In fact, let $u \in \Gamma(\theta_0)$ and set $r = c(u)$. Then $\sigma_1(t) = \exp_p(tu)$, and $t \in [0, r]$ is a minimal geodesic. Without loss of generality, we can take $\gamma_1'(0) = \gamma'(0) \in \Gamma$ with $\beta = \angle(u, \gamma_1'(0)) \le \theta_0$. Let $z = \exp_p(ru)$ and $t = d(q, z)$. Lemma 2.3 applies to the hinge $(\sigma_1, \gamma_1, \beta)$ to give

$$\begin{aligned}\cosh\kappa t &\le \cosh\kappa r_1\cosh\kappa r - \sinh\kappa r_1\sinh\kappa r\cos\beta\\ &\le \cosh\kappa r_1\cosh\kappa r - \sinh\kappa r_1\sinh\kappa r\cos\theta_0.\end{aligned}$$

Take a minimal geodesic σ joining q with z. Since $\gamma_1'(r_1) = -\gamma_2'(r_1)$ and

$$\angle(\gamma_1'(r_1), \sigma'(0)) + \angle(\gamma_2'(r_1), \sigma'(0)) = \pi,$$

we can assume that $\beta_1 = \angle(\gamma_1'(r_1), \sigma'(0)) \leq \frac{\pi}{2}$. Lemma 2.3 applies to the hinge $(\sigma, \gamma_1, \beta_1)$ to give

$$\begin{aligned}\cosh \kappa r &\leq \cosh \kappa r_1 \cosh \kappa t - \sinh \kappa r_1 \sinh \kappa t \cos \beta_1 \\ &\leq \cosh \kappa r_1 \cosh \kappa t.\end{aligned}$$

Thus,

$$\cosh \kappa r \leq \cosh^2 \kappa r_1 \cosh \kappa r - \cosh \kappa r_1 \sinh \kappa r_1 \sinh \kappa r \cos \theta_0.$$

Simplifying the above inequality, we get

$$\tanh \kappa r \cos \theta_0 \leq \tanh \kappa r_1.$$

This proves our claim.

Let

$$\mathrm{d}V(\exp_p(t\xi)) = \sqrt{g(t;\xi)}\mathrm{d}t\mathrm{d}\mu_p(\xi)$$

be the volume form in the geodesic spherical coordinates around p, where $\mathrm{d}\mu_p(\xi)$ is the Riemannian measure on S_pM induced by the Euclidean Lebesgue measure on T_pM. Since $K_M \geq -\kappa^2$, we get from the Bishop-Gromov comparison theorem (cf. [1]) that $\sqrt{g(t;\xi)} \leq S_{-\kappa^2}(t)$ (see Section 1 for the definition of $S_{-\kappa^2}(t)$), $\forall t > 0$. Thus, for any $r \geq R_0$, we have from (3.6) that

$$\begin{aligned}\mathrm{Vol}[B(p,r)] &= \int_{S_pM} \mathrm{d}\mu_p(\xi) \int_0^{\min(c(\xi),r)} \sqrt{g(t;\xi)}\mathrm{d}t \\ &= \int_{\Gamma(\theta_0)} \mathrm{d}\mu_p(\xi) \int_0^{\min(c(\xi),r)} \sqrt{g(t;\xi)}\mathrm{d}t \\ &\quad + \int_{S_pM-\Gamma(\theta_0)} \mathrm{d}\mu_p(\xi) \int_0^{\min(c(\xi),r)} \sqrt{g(t;\xi)}\mathrm{d}t \\ &\leq \int_{\Gamma(\theta_0)} \mathrm{d}\mu_p(\xi) \int_0^{R_0} S_{-\kappa^2}(t)\mathrm{d}t \\ &\quad + \int_{S_pM-\Gamma(\theta_0)} \mathrm{d}\mu_p(\xi) \int_0^{r} S_{-\kappa^2}(t)\mathrm{d}t \\ &\leq \mathrm{Vol}[B(R_0)] + \nu\omega_{n-1} \int_0^{r} S_{-\kappa^2}(t)\mathrm{d}t \\ &= \mathrm{Vol}[B(R_0)] + \nu\alpha_n(r, -\kappa^2),\end{aligned}$$

where $B(R_0)$ denotes the R_0-ball in the space form of constant curvature $-\kappa^2$.

Dividing both sides of the above inequality by $\alpha_n(r, -\kappa^2)$ and letting $r \to \infty$, we obtain

$$\lim_{r\to\infty} \frac{\mathrm{Vol}[B(p,r)]}{\alpha_n(r,-\kappa^2)} \leq \nu < \nu_{-\kappa^2}(M).$$

This is a contradiction, completing the proof of Theorem 1.4.

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