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Delay-Dependent Exponential Stability for Nonlinear Reaction-Diffusion Uncertain Cohen-Grossberg Neural Networks with Partially Known Transition Rates via Hardy-Poincaré Inequality*

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Abstract In this paper, stochastic global exponential stability criteria for delayed impulsive Markovian jumping reaction-diffusion Cohen-Grossberg neural networks (CGNNs for short) are obtained by using a novel Lyapunov-Krasovskii functional approach, linear matrix inequalities (LMIs for short) technique, Itô formula, Poincaré inequality and Hardy-Poincaré inequality, where the CGNNs involve uncertain parameters, partially unknown Markovian transition rates, and even nonlinear p-Laplace diffusion ($p > 1$). It is worth mentioning that ellipsoid domains in $\mathbb{R}^m$ ($m \geq 3$) can be considered in numerical simulations for the first time owing to the synthetic applications of Poincaré inequality and Hardy-Poincaré inequality. Moreover, the simulation numerical results show that even the corollaries of the obtained results are more feasible and effective than the main results of some recent related literatures in view of significant improvement in the allowable upper bounds of delays.

Keywords Hardy-Poincaré inequality, Laplace diffusion, Linear matrix inequality
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1 Introduction

It is well-known that Cohen-Grossberg in [1] proposed originally the CGNNs. Since then the CGNNs have founded its extensive applications in pattern recognition, image and signal processing, quadratic optimization, and aritifical intelligence (see [2–11]). However, these successful applications are greatly dependent on the stability of the neural networks, which is also a crucial feature in the design of the neural networks. In practice, both time delays and impulse are always inevitable, and cause probably some undesirable dynamic network behaviors such as oscillation and instability. Therefore, the stability analysis for delayed impulsive CGNNs has become a topic of great theoretic and practical importance in recent years (see [2–3, 5–6]). Recently, the CGNNs with Markovian jumping parameters have been extensively studied, for the systems with Markovian jumping parameters are useful in modeling abrupt phenomena, such

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as random failures, operating in different points of a nonlinear plant, and changing in the interconnections of subsystems (see [5–8]). Noise disturbance is unavoidable in real nervous systems, which is a major source of instability and poor performances in neural networks. A neural network can be stabilized or destabilized by certain stochastic inputs. The synaptic transmission in real neural networks can be viewed as a noisy process introduced by random fluctuations from the release of neurotransmitters and other probabilistic causes (see [12]). Hence, noise disturbance should be also taken into consideration in discussing the stability of neural networks (see [13–17]). On the other hand, diffusion phenomena can not be unavoidable in real world. Usually diffusion phenomena is simulated by linear Laplace diffusion for simplicity in many previous literatures (see [2, 18–20]). However, diffusion behavior is so complicated that the nonlinear reaction-diffusion models were considered in several papers (see [3, 21–24]). Even the nonlinear p-Laplace diffusion ($p > 1$) was considered in simulating some diffusion behaviors (see [3, 6, 10]). Particularly, if $p = 2$, the p-Laplace diffusion was just the conventional linear Laplace diffusion in many previous papers (see [2, 18–20]). In addition, neural networks may encounter various other factors and problems in the factual operations. In fact, there exist also parameter errors unavoidable in factual systems owing to aging of electronic component, external disturbance and parameter perturbations. It is equally important to ensure that system is stable with respect to these uncertainties (see [25–26]). Naturally, aging of electronic component, external disturbance and parameter perturbations always result in a side-effect of partially unknown Markovian transition rates. Some of recent literatures investigated the stability of neural networks with partially unknown Markovian transition rates (see [27–28]). As far as we know, stochastic stability for the delayed impulsive Markovian jumping Laplace diffusion CGNN with uncertain parameters has rarely been considered. Besides, the stochastic exponential stability always remains the key factor of concern owing to its importance in designing a neural network, and such a situation motivates our present study. Motivated by the above-mentioned literature, particularly by [2, 29–30], we shall investigate the stochastic global exponential stability criteria for the above-mentioned CGNN via the LMIs approach.

The rest of this paper is organized as follows. In Section 2, new CGNN models are formulated, and some preliminaries are given. In Section 3, new LMI-based stochastic global exponential stability criterion are presented for the CGNNs. And in Section 4, three numerical examples are provided to show the higher feasibility and less conservatism of the new criterion compared with those of [2–3, 29–30]. Finally, some conclusions are presented in Section 5.

2 Model Description and Preliminaries

In 2011, Zhang, Wu and Li in [2] considered the following Cohen-Grossberg neural networks under Dirichlet boundary condition:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nabla \cdot (\mathcal{D} \circ \nabla u(t,x)) - \widetilde{A}(u(x,t))[\widetilde{B}(u(t,x)) - C\widetilde{f}(u(t,x)) \\ \qquad - D\widetilde{g}(u(t-\tau(t),x)) + J] \quad \text{for all } t \geq t_0,\ t \neq t_k,\ x \in \Omega, \\ u(t_k,x) = M_k u(t_k^-,x) + N\widetilde{h}(u(t_k^- - \tau(t),x)), \quad k = 1,2,\cdots, \end{cases} \tag{2.1}$$

where $u = u(t,x) = (u_1(t,x), u_2(t,x), \cdots, u_n(t,x))^{\mathrm{T}}$, $\widetilde{f}(u) = (\widetilde{f}_1(u_1), \widetilde{f}_2(u_2), \cdots, \widetilde{f}_n(u_n))^{\mathrm{T}}$, $\widetilde{g}(u) = (\widetilde{g}_1(u_1), \widetilde{g}_2(u_2), \cdots, \widetilde{g}_n(u_n))^{\mathrm{T}}$.

Generally, there exist the following assumptions for the system (2.1) (see [2]):

(H1) $\widetilde{A}(u(t,x))$ is a bounded, positive and continuous diagonal matrix, i.e., there exist two positive definite diagonal matrices $\underline{A}$ and $\overline{A}$, such that $0 < \underline{A} \le \widetilde{A}(u(t,x)) \le \overline{A}$.

(H2) $\widetilde{B}(u(t,x)) = (\widetilde{b}_1(u_1(t,x)), \widetilde{b}_2(u_2(t,x)), \cdots, \widetilde{b}_n(u_n(t,x)))^{\mathrm{T}}$ such that there exists a positive definite matrix $\widetilde{B} = \mathrm{diag}(\widetilde{B}_1, \widetilde{B}_2, \cdots, \widetilde{B}_n)$ satisfying

$$\frac{\widetilde{b}_j(r_1) - \widetilde{b}_j(r_2)}{r_1 - r_2} \ge B_j, \quad \forall r_1, r_2 \in R,\ r_1 \ne r_2,\ j = 1, 2, \cdots.$$

(H3) There exist diagonal matrices

$$\widetilde{F} = \mathrm{diag}(\widetilde{F}_1, \widetilde{F}_2, \cdots, \widetilde{F}_n) > 0, \quad \widetilde{G}_k = \mathrm{diag}(\widetilde{G}_1, \widetilde{G}_2, \cdots, \widetilde{G}_n) > 0, \quad H = \mathrm{diag}(H_1, H_2, \cdots, H_n),$$

such that

$$0 \le \frac{\widetilde{h}_j(r_1) - \widetilde{h}_j(r_2)}{r_1 - r_2} \le H_j, \quad \forall r_1, r_2 \in R,\ r_1 \ne r_2$$

and

$$0 \le \frac{\widetilde{f}_j(r_1) - \widetilde{f}_j(r_2)}{r_1 - r_2} \le \widetilde{F}_j, \quad 0 \le \frac{\widetilde{g}_j(r_1) - \widetilde{g}_j(r_2)}{r_1 - r_2} \le \widetilde{G}_j, \quad \forall j = 1, 2, \cdots, n. \tag{2.2}$$

In this paper, we always assume $\widetilde{h} \equiv 0$ for some rational reason (see Remark 2.3), and consider to replace (H3) with the following more flexible condition:

(H3*) There exist constant diagonal matrices

$$G_k = \mathrm{diag}(G_1^{(k)}, G_2^{(k)}, \cdots, G_n^{(k)}), \quad F_k = \mathrm{diag}(F_1^{(k)}, F_2^{(k)}, \cdots, F_n^{(k)}), \quad k = 1, 2$$

with

$$|F_j^{(1)}| \le F_j^{(2)}, \quad |G_j^{(1)}| \le G_j^{(2)}, \quad j = 1, 2, \cdots, n,$$

such that

$$F_j^{(1)} \le \frac{\widetilde{f}_j(r_1) - \widetilde{f}_j(r_2)}{r_1 - r_2} \le F_j^{(2)}, \quad G_j^{(1)} \le \frac{\widetilde{g}_j(r_1) - \widetilde{g}_j(r_2)}{r_1 - r_2} \le G_j^{(2)}, \quad j = 1, 2, \cdots, n.$$

According to [2, Definition 2.1], a constant vector $u^* \in \mathbb{R}^n$ is said to be an equilibrium point of system (2.1) if

$$\widetilde{B}(u^*) - C\widetilde{f}(u^*) + D\widetilde{g}(u^*) + J = 0, \quad (M_k - I)u^* + N\widetilde{h}(u^*) = 0. \tag{2.3}$$

Let $v = u - u^*$, then the system (2.1) with $\widetilde{h} \equiv 0$ can be transformed into

$$\begin{cases} \dfrac{\partial v}{\partial t} = \nabla \cdot (\mathcal{D} \circ \nabla v(t,x)) - A(v(x,t))[B(v(t,x)) - Cf(v(t,x)) - Dg(v(t-\tau(t),x))] \\ \qquad\qquad \text{for all } t \ge t_0,\ t \ne t_k,\ x \in \Omega, \\ v(t_k, x) = M_k v(t_k^-, x), \quad k = 1, 2, \cdots, \end{cases} \tag{2.4}$$

where $v = v(t,x) = (v_1(t,x), v_2(t,x), \cdots, v_n(t,x))^{\mathrm{T}}$, $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^{\mathrm{T}}$, $A(v(t,x)) = \widetilde{A}(v(t,x) + u^*) = \widetilde{A}(u(t,x))$,

$$\begin{aligned} B(v(t,x)) &= \widetilde{B}(u(t,x)) - \widetilde{B}(u^*), \\ f(v(t,x)) &= \widetilde{f}(u(t,x)) - \widetilde{f}(u^*), \\ g(v(t,x)) &= \widetilde{g}(u(t,x)) - \widetilde{g}(u^*) \end{aligned} \tag{2.5}$$

and

$$f(v)=(f_1(v_1),f_2(v_2),\cdots,f_n(v_n))^{\rm T},\quad g(v)=(g_1(v_1),g_2(v_2),\cdots,g_n(v_n))^{\rm T}.$$

Then, according to [2, Definition 2.1], $v\equiv 0$ is an equilibrium point of system (2.4). Hence, below we only need consider the stability of the null solution of Cohen-Grossberg neural networks. Naturally we propose the following hypotheses on the system (2.4) with $h\equiv 0$, which are perhaps derived by the assumptions (H1)–(H2) and (H3*).

(A1) $A(v(t,x))$ is a bounded, positive and continuous diagonal matrix, i.e., there exist two positive diagonal matrices $\underline{A}$ and $\overline{A}$, such that $0<\underline{A}\le A(v(t,x))\le\overline{A}$.

(A2) $B(v(t,x))=(b_1(v_1(t,x)),b_2(v_2(t,x)),\cdots,b_n(v_n(t,x)))^{\rm T}$, such that there exists a positive definite matrix $B=\mathrm{diag}(B_1,B_2,\cdots,B_n)^{\rm T}\in\mathbb{R}^n$ satisfying

$$\frac{b_j(r)}{r}\ge B_j,\quad \forall j=1,2,\cdots,n,\ r\in\mathbb{R}.$$

(A3) There exist constant diagonal matrices

$$G_k=\mathrm{diag}(G_1^{(k)},G_2^{(k)},\cdots,G_n^{(k)}),\quad F_k=\mathrm{diag}(F_1^{(k)},F_2^{(k)},\cdots,F_n^{(k)}),\quad k=1,2$$

with $|F_j^{(1)}|\le F_j^{(2)},\ |G_j^{(1)}|\le G_j^{(2)},\ j=1,2,\cdots,n$, such that

$$F_j^{(1)}\le\frac{f_j(r)}{r}\le F_j^{(2)},\quad G_j^{(1)}\le\frac{g_j(r)}{r}\le G_j^{(2)},\quad \forall j=1,2,\cdots,n,\ r\in\mathbb{R}.$$

Remark 2.1 In many previous literatures, e.g. [2], authors always assume

$$0\le\frac{f_j(r)}{r}\le F_j,\quad 0\le\frac{g_j(r)}{r}\le G_j,\quad \forall i=1,2,\cdots,n,$$

which may be correspond to (H3). However, $F_j^{(1)},G_j^{(1)}$ in (A3) may not be positive constants, and hence the functions f,g are more generic.

Remark 2.2 It is obvious from (2.5) that $B(0)=0=f(0)=g(0)$, and then $B(0)-Cf(0)-Dg(0)=0$.

Very recently, Wang, Rao and Zhong [3] studied stochastic CGNN with nonlinear p-Laplace diffusion ($p>1$) under Neumann boundary condition:

$$\begin{cases} \mathrm{d}v(t,x)=\{\nabla\cdot\mathcal{D}(t,x,v)\circ\nabla_p v)-A(v(t,x))[B(v(t,x))-Cf(v(t,x))\\ \qquad\qquad +Dg(v(t-\tau(t),x))]\}\mathrm{d}t+\sigma(v(t,x))\mathrm{d}w(t),\quad t\in[t_k,t_{k+1}),\\ v(t_k^+,x)=M_k v(t^-,x),\quad t=t_k,\\ v(t_0+\theta,x)=\varphi(\theta,x),\quad (\theta,x)\in[-\tau,0]\times\Omega,\\ \dfrac{\partial v_i(t,x)}{\partial\nu}=0,\quad (t,x)\in[-\tau,+\infty)\times\partial\Omega,\quad i=1,2,\cdots,n. \end{cases}\tag{2.6}$$

Since stochastic noise disturbance and parameter errors are unavoidable in the practical neural networks, it is necessary to consider the stability of the null solution of the following Markovian jumping CGNN:

$$\begin{cases} \mathrm{d}v(t,x)=\{\nabla\cdot(\mathcal{D}(t,x,v)\circ\nabla_p v(t,x))-A(v(x,t))[B(v(t,x))-C(r(t),t)f(v(t,x))\\ \qquad\qquad -D(r(t),t)g(v(t-\tau(t),x))]\}\mathrm{d}t+\sigma(t,v(t,x),v(t-\tau(t),x),r(t))\mathrm{d}w(t),\\ \qquad\qquad \text{for all } t\ge t_0,\ t\ne t_k,\ x\in\Omega,\\ v(t_k,x)=M_k(r(t))v(t_k^-,x),\quad k=1,2,\cdots. \end{cases}\tag{2.7}$$

The initial conditions and the boundary conditions are given by

$$v(\theta,x)=\phi(\theta,x),\quad (\theta,x)\in[-\tau,0]\times\Omega \tag{2.7b}$$

and

$$\mathfrak{B}[v_i(t,x)]=0,\quad (t,x)\in[-\tau,+\infty)\times\partial\Omega,\quad i=1,2,\cdots,n, \tag{2.7c}$$

respectively, where $p>1$ is a given scalar, and $\Omega\in\mathbb{R}^m$ is a bounded domain with a smooth boundary $\partial\Omega$ of class $\mathcal{C}^2$ by Ω, $v(t,x)=(v_1(t,x),v_2(t,x),\cdots,v_n(t,x))^{\mathrm{T}}\in\mathbb{R}^n$, where $v_i(t,x)$ is the state variable of the ith neuron at time t and in space variable x. Matrix $\mathcal{D}(t,x,v)=(\mathcal{D}_{jk}(t,x,v))_{n\times m}$ satisfies $\mathcal{D}_{jk}(t,x,v)\ge d>0$ for all $j,k,(t,x,v)$, where the smooth functions $\mathcal{D}_{jk}(t,x,v)$ are diffusion operators. Similarly as that of [3], we denote $\nabla_p v=(\nabla_p v_1,\cdots,\nabla_p v_n)^{\mathrm{T}}$ with $\nabla_p v_i=\big(|\nabla v_i|^{p-2}\frac{\partial v_i}{\partial x_1},\cdots,|\nabla v_i|^{p-2}\frac{\partial v_i}{\partial x_m}\big)^{\mathrm{T}}$. And $\mathcal{D}(t,x,v)\circ\nabla_p v=\big(\mathcal{D}_{jk}(t,x,v)|\nabla v_i|^{p-2}\cdot\frac{\partial v_i}{\partial x_k}\big)_{n\times m}$ denotes the Hadamard product of matrix $\mathcal{D}(t,x,v)$ and $\nabla_p v$. For matrix $\mathbb{Y}=(Y_1,\cdots,Y_n)^{\mathrm{T}}$ with $Y_i=(y_{i1},\cdots,y_{im})^{\mathrm{T}}$ $(i=1,2,\cdots,n)$, we denote $\nabla\cdot\mathbb{Y}=(\nabla\cdot Y_1,\nabla\cdot Y_2,\cdots,\nabla\cdot Y_n)^{\mathrm{T}}$, where $\nabla\cdot Y_i=\sum\limits_{k=1}^{m}\frac{\partial y_{ik}}{\partial x_k}$. Particularly, $\nabla_p v=\nabla v$ if $p=2$.

Denote $w(t)=(w^{(1)}(t),w^{(2)}(t),\cdots,w^{(n)}(t))^{\mathrm{T}}$, where $w^{(j)}(t)$ is scalar standard Brownian motion defined on a complete probability space $(\Omega*,\mathcal{F},\mathcal{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\ge0}$. Noise perturbations $\sigma:\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n\times S\to\mathbb{R}^{n\times n}$ is a Borel measurable function. $\{r(t),\,t\ge0\}$ is a right-continuous Markov process on the probability space which takes values in the finite space $S=\{1,2,\cdots,s\}$ with the generator $\Pi=\{\pi_{ij}\}$ given by

$$\mathcal{P}(r(t+\delta)=j\mid r(t)=i)=\begin{cases}\pi_{ij}\delta+o(\delta), & i\ne j,\\ 1+\pi_{ij}\delta+o(\delta), & i=j,\end{cases}$$

where $\pi_{ij}\ge0$ is transition probability rate from i to j $(j\ne i)$ and $\pi_{ii}=-\sum\limits_{\substack{j=1\\ j\ne i}}^{s}\pi_{ij}$, $\delta>0$ and $\lim\limits_{\delta\to0}\frac{o(\delta)}{\delta}=0$. In addition, the transition rates of the Markovian chain are considered to be partially available, namely, some elements in transition rates matrix Π are time-invariant but unknown. For instance, a system with three operation modes may have the transition rate matrix Π as follows:

$$\Pi=\begin{bmatrix}\pi_{11} & ? & ?\\ ? & \pi_{22} & ?\\ \pi_{31} & \pi_{32} & \pi_{33}\end{bmatrix},$$

where "?" represents the inaccessible element. For natation clarity, we denote $S=S^i_{kn}\cup S^i_{un}$ with $S^i_{kn}\triangleq\{j,\text{if}\,\pi_{ij}\text{ is known}\}$ and $S^i_{un}\triangleq\{j,\text{if}\,\pi_{ij}\text{ is unknown, and } j\ne i\}$ for a given $i\in S$. Denote $\widetilde{\alpha}_i\ge\max\limits_{j\in S^i_{un}}\pi_{ij}$. The time-varying delay $\tau(t)$ satisfies $0\le\tau(t)\le\tau$ with $\dot{\tau}(t)\le\kappa<1$.

$$A(v(t,x))=\mathrm{diag}(a_1(v_1(t,x)),a_2(v_2(t,x)),\cdots,a_n(v_n(t,x))),$$
$$B(v(t,x))=(b_1(v_1(t,x)),b_2(v_2(t,x)),\cdots,b_n(v_n(t,x)))^{\mathrm{T}},$$

where $a_j(v_j(t,x))$ represents an amplification function, and $b_j(v_j(t,x))$ is an appropriately behavior function. $C(r(t),t)$, $D(r(t),t)$ are denoted by $C_i(t)$, $D_i(t)$ with $C_i(t)=(c^i_{lk}(t))_{n\times n}$, $D_i(t)=(d^i_{lk}(t))_{n\times n}$, respectively, and $c^i_{lk}(t)$, $d^i_{lk}(t)$ denote the connection strengths of the kth neuron on the lth neuron at time t in the mode $r(t)=i$, respectively. Denote vector functions $f(v(t,x))=(f_1(v_1(t,x)),f_2(v_2(t,x)),\cdots,f_n(v_n(t,x)))^{\mathrm{T}}$, $g(v(t,x))=(g_1(v_1(t,x)),\cdots,$

$g_n(v_n(t,x)))^{\mathrm{T}}$, where $f_j(v_2(t,x))$, $g_j(v_j(t,x))$ are neuron activation functions of the jth unit at time t and in space variable x.

For any mode $r(t)=i\in S$, we assume that C_i, D_i are real constant matrices of appropriate dimensions, and $\Delta C_i(t), \Delta D_i(t)$ are real-valued matrix functions which represent time-varying parameter uncertainties, satisfying

$$C_i(t)=C_i+\Delta C_i(t), \quad D_i(t)=D_i+\Delta D_i(t). \tag{2.8}$$

In addition, we always assume that $t_0=0$, and $v(t_k^+,x)=v(t_k,x)$ for all $k=1,2,\cdots$, where $v(t_k^-,x)$ and $v(t_k^+,x)$ represent the left-hand and right-hand limits of $v(t,x)$ at t_k, respectively. And each t_k $(k=1,2,\cdots)$ is an impulsive moment, satisfying $0<t_1<t_2<\cdots<t_k<\cdots$ and $\lim\limits_{k\to\infty} t_k=+\infty$. The boundary condition (2.7c) is called Dirichlet boundary condition if $\mathfrak{B}[v_i(t,x)]=v_i(t,x)$, and Neumann boundary condition if $\mathfrak{B}[v_i(t,x)]=\frac{\partial v_i(t,x)}{\partial\nu}$, where $\frac{\partial v_i(t,x)}{\partial\nu}=\big(\frac{\partial v_i(t,x)}{\partial x_1},\frac{\partial v_i(t,x)}{\partial x_2},\cdots,\frac{\partial v_i(t,x)}{\partial x_m}\big)^{\mathrm{T}}$ denotes the outward normal derivative on $\partial\Omega$. It is well-known that the stability of neural networks with Neumann boundary condition has been widely studied. The Dirichlet boundary conditions describe the situation, where the space is totally surrounded by a region in which the states of the neuron equal zero on the boundary. And the stability analysis of delayed reaction-diffusion neural networks with the Dirichlet boundary conditions is very important in theories and applications, and also has attracted much attention (see [2, 31–34]). So in this paper, we consider the CGNN under Neumann boundary condition and Dirichlet boundary condition, respectively.

Remark 2.3 If all stochastic factors and uncertain factors are ignored, the system (2.7) with $p=2$ was studied by [2] though there is a little difference between Dirichlet boundary condition and Neumann boundary condition. However, our impulsive assumption $u(t_k^+,x)=Mu(t_k^-,x)$ is more natural than that of [2], which will result in some difference in methods (see, e.g. [3]).

Particularly, if $p=2$, the system (2.7) is reduced to the following CGNN:

$$\begin{cases} \mathrm{d}v(t,x)=\{\nabla\cdot(\mathcal{D}(t,x,v)\circ\nabla v(t,x))-A(v(x,t))[B(v(t,x))-C(r(t),t)f(v(t,x)) \\ \qquad -D(r(t),t)g(v(t-\tau(t),x))]\}\mathrm{d}t+\sigma(t,v(t,x),v(t-\tau(t),x),r(t))\mathrm{d}w(t), \\ \qquad \text{for all } t\geq t_0,\ t\neq t_k,\ x\in\Omega, \\ v(t_k,x)=M_k(r(t))v(t_k^-,x), \quad k=1,2,\cdots. \end{cases} \tag{2.9}$$

For convenience's sake, we need introduce some standard notations.

(1) $L^2(\mathbb{R}\times\Omega)$: The space of real Lebesgue measurable functions of $\mathbb{R}\times\Omega$, it is a Banach space for the 2-norm $\|v(t)\|_2=\big(\sum\limits_{i=1}^n\|v_i(t)\|\big)^{\frac12}$ with $\|v_i(t)\|=\big(\int_\Omega|v_i(t,x)|^2\mathrm{d}x\big)^{\frac12}$, where $|\cdot|$ is the Euclid norm.

(2) $L^2_{\mathcal{F}_0}([-\tau,0]\times\Omega;\mathbb{R}^n)$: The family of all $\mathcal{F}_0$-measurable $C([-\tau,0]\times\Omega;\mathbb{R}^n)$-value random variable $\xi=\{\xi(\theta,x):-\tau\leq\theta\leq0,x\in\Omega\}$ such that $\sup\limits_{-\tau\leq\theta\leq0}\mathbb{E}\|\xi(\theta)\|_2^2<\infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$.

(3) $Q=(q_{ij})_{n\times n}>0$ (<0): A positive (negative) definite matrix, i.e., $y^{\mathrm{T}}Qy>0$ (<0) for any $0\neq y\in\mathbb{R}^n$.

(4) $Q=(q_{ij})_{n\times n}\geq0$ (≤0): A semi-positive (semi-negative) definite matrix, i.e., $y^{\mathrm{T}}Qy\geq0$ (≤0) for any $y\in\mathbb{R}^n$.

(5) $Q_1 \geq Q_2$ ($Q_1 \leq Q_2$): This means $Q_1 - Q_2$ is a semi-positive (semi-negative) definite matrix.

(6) $Q_1 > Q_2$ ($Q_1 < Q_2$): This means $Q_1 - Q_2$ is a positive (negative) definite matrix.

(7) $\lambda_{\max}(\Phi)$ and $\lambda_{\min}(\Phi)$ denote the largest and the smallest eigenvalues of matrix Φ, respectively.

(8) Denote $|C| = (|c_{ij}|)_{n\times n}$ for any matrix $C = (c_{ij})_{n\times n}$;

$$|u(t,x)| = (|u_1(t,x)|, |u_2(t,x)|, \cdots, |u_n(t,x)|)^{\mathrm{T}}$$

for any $u(t,x) = (u_1(t,x), u_2(t,x), \cdots, u_n(t,x))^{\mathrm{T}}$.

(9) I: Identity matrix with compatible dimension.

(10) The symmetric terms in a symmetric matrix are denoted by $*$.

Throughout this paper, we assume (A1)–(A3) and the following conditions hold:

(A4) For any mode $i \in S$, the parameter uncertainties considered here are norm-bounded and of the following forms:

$$(\Delta C_i(t) \quad \Delta D_i(t)) = E_i \mathcal{K}(t)(N_{i1} \quad N_{i2}), \quad \forall i \in S,$$

where $\mathcal{K}(t)$ is an unknown matrix function satisfying $|\mathcal{K}^{\mathrm{T}}(t)||\mathcal{K}(t)| \leq I$, and E_i, N_{i1}, N_{i2} are known real constant matrices.

(A5) There exist symmetrical matrices $R_j \geq 0$ with $|R_j| = R_j$, $j = 1, 2$, such that for any mode $i \in S$,

$$\begin{aligned} &\mathrm{trace}[\sigma^{\mathrm{T}}(t, v(t,x), v(t-\tau(t),x), i)\sigma(t, v(t,x), v(t-\tau(t),x), i)] \\ &\leq v^{\mathrm{T}}(t,x)R_1 v(t,x) + v^{\mathrm{T}}(t-\tau(t),x)R_2 v(t-\tau(t),x). \end{aligned} \tag{2.10}$$

(A6) $\sigma(t, 0, 0, i) = 0$ for all $i \in S$.

Remark 2.4 The condition $|H| = H$ is not too stringent for a semi-positive definite matrix $H = (h_{ij})_{n\times n} \geq 0$. Indeed, all $h_{ij} \geq 0$ imply $|H| = H$.

Similarly as that of [2, Definition 2.1], we can see from (A6) that the system (2.7) has the null solution as its equilibrium point. Let $v(t,x;\phi,i_0)$ be the state trajectory from the initial condition $r(0) = i_0$, $v(\theta,x;\phi) = \phi(\theta,x)$ on $-\tau \leq \theta \leq 0$ in $L^2_{\mathcal{F}_0}([-\tau,0]\times\Omega;\mathbb{R}^n)$. Below, we always assume that $v(t,x;\phi,i_0)$ is a solution of system (2.7).

Definition 2.1 *For any given scalar $p > 1$, the null solution of system* (2.7) *is said to be stochastically globally exponentially stable in the mean square if for every initial condition $\phi \in L^2_{\mathcal{F}_0}([-\tau,0]\times\Omega;\mathbb{R}^n)$, $r(0) = i_0$, there exist scalars $\beta > 0$ and $\gamma > 0$ such that for any solution $v(t,x;\phi,i_0)$,*

$$\mathbb{E}(\|v(t,x;\phi,i_0)\|_2^2) \leq \gamma \mathrm{e}^{-\beta t}\Big[\sup_{-\tau\leq\theta\leq 0}\mathbb{E}(\|\phi(\theta,x)\|_2^2)\Big], \quad t \geq t_0.$$

Notice that if $p = 2$, the system (2.7) is just the system (2.9). And the following Poincaré inequality lemma and Hardy-Poincaré inequality lemma may play role in judging the stability of system (2.9).

Lemma 2.1 (see [35]) (Poincaré Inequality) *Let Ω be a bounded domain of $\mathbb{R}^m$ with a smooth boundary $\partial\Omega$ of class $\mathcal{C}^2$ by Ω. $\psi(x)$ is a real-valued function belonging to $H_0^1(\Omega)$ and satisfies $\mathfrak{B}[\psi(x)]|_{\partial\Omega} = 0$. Then*

$$\lambda_1 \int_\Omega |\psi(x)|^2 \mathrm{d}x \le \int_\Omega |\nabla \psi(x)|^2 \mathrm{d}x,$$

where λ_1 is the lowest positive eigenvalue of the boundary value problem

$$\begin{cases} -\Delta\psi(x) = \lambda\psi(x), & x \in \Omega, \\ \mathfrak{B}[\psi(x)] = 0, & x \in \partial\Omega. \end{cases}$$

Notice that $H_0^1(\Omega)$ is the Sobolev space $W_0^{1,p}(\Omega)$ with $p = 2$, and $W_0^{1,p}(\Omega)$ is the completion of $C_0^\infty(\Omega)$ with respect to the norm $\|\psi\|_{W_0^{1,p}(\Omega)} = \big(\int_\Omega |\nabla\psi|^p \mathrm{d}x\big)^{\frac{1}{p}}$. Thereby, the norm of $H_0^1(\Omega)$ is $\|\psi\|_{H_0^1(\Omega)} = \big(\int_\Omega |\nabla\psi|^2 \mathrm{d}x\big)^{\frac{1}{2}}$. Note that we always denote the $L^2(\Omega)$-norm by $\|\psi\| = \big(\int_\Omega \psi^2 \mathrm{d}x\big)^{\frac{1}{2}}$.

Lemma 2.2 (see [36]) (Hardy-Poincaré Inequality) *For any bounded domain Ω in $\mathbb{R}^m$, any dimension $m \ge 2$ and for every $\psi(x) \in H_0^1(\Omega)$, we have*

$$\int_\Omega |\nabla\psi|^2 \mathrm{d}x - \frac{(m-2)^2}{4} \int_\Omega \frac{\psi^2}{|x|^2} \mathrm{d}x \ge \Lambda_2 \Big(\frac{\omega_m}{\mathrm{meas}(\Omega)}\Big)^{\frac{2}{m}} \int_\Omega \psi^2 \mathrm{d}x,$$

where the constant Λ_2 is the first eigenvalue of the Laplacian in the unit ball in $m = 2$, and ω_m denotes the measure of the unit ball in $\mathbb{R}^m$.

Lemma 2.3 *Let $P_i = \mathrm{diag}(p_{i1}, p_{i2}, \cdots, p_{in})$ be a positive definite matrix for a given i, and v be a solution of system* (2.7)*. Then we have*

$$\begin{aligned} \int_\Omega v^{\mathrm{T}} P_i (\nabla \cdot (\mathcal{D}(t,x,v) \circ \nabla_p v)) \mathrm{d}x &= -\sum_{k=1}^m \sum_{j=1}^n \int_\Omega p_{ij} \mathcal{D}_{jk}(t,x,v) |\nabla v_j|^{p-2} \Big(\frac{\partial v_j}{\partial x_k}\Big)^2 \mathrm{d}x \\ &= \int_\Omega (\nabla \cdot (\mathcal{D}(t,x,v) \circ \nabla_p v))^{\mathrm{T}} P_i v \mathrm{d}x\,. \end{aligned}$$

Proof Since v is a solution of system (2.7), we can see it by Gauss formula and Dirichlet or Neumann boundary condition that

$$\begin{aligned} &\int_\Omega v^{\mathrm{T}} P_i (\nabla \cdot (\mathcal{D}(t,x,v) \circ \nabla_p v)) \mathrm{d}x \\ &= \int_\Omega v^{\mathrm{T}} P_i \Big(\sum_{k=1}^m \frac{\partial}{\partial x_k} \Big(\mathcal{D}_{1k}(t,x,v) |\nabla v_1|^{p-2} \frac{\partial v_1}{\partial x_k} \Big), \cdots, \sum_{k=1}^m \frac{\partial}{\partial x_k} \Big(\mathcal{D}_{nk}(t,x,v) |\nabla v_n|^{p-2} \frac{\partial v_n}{\partial x_k} \Big) \Big)^{\mathrm{T}} \mathrm{d}x \\ &= \int_\Omega \sum_{j=1}^n p_{ij} v_j \sum_{k=1}^m \frac{\partial}{\partial x_k} \Big(\mathcal{D}_{jk}(t,x,v) |\nabla v_j|^{p-2} \frac{\partial v_j}{\partial x_k} \Big) \mathrm{d}x \\ &= -\sum_{k=1}^m \sum_{j=1}^n \int_\Omega p_{ij} \mathcal{D}_{jk}(t,x,v) |\nabla v_j|^{p-2} \Big(\frac{\partial v_j}{\partial x_k}\Big)^2 \mathrm{d}x. \end{aligned}$$

Similarly, we can prove

$$-\sum_{k=1}^m \sum_{j=1}^n \int_\Omega p_{ij} \mathcal{D}_{jk}(t,x,v) |\nabla v_j|^{p-2} \Big(\frac{\partial v_j}{\partial x_k}\Big)^2 \mathrm{d}x = \int_\Omega (\nabla \cdot (\mathcal{D}(t,x,v) \circ \nabla_p v))^{\mathrm{T}} P_i v \mathrm{d}x.$$

Lemma 2.4 (see [37]) *Let $\varepsilon > 0$ be any given scalar, and $\mathcal{M}, \mathfrak{E}$ and $\mathcal{K}$ be matrices with appropriate dimensions. If $\mathcal{K}^{\mathrm{T}}\mathcal{K} \le I$, then we have*

$$\mathcal{M}\mathcal{K}\mathfrak{E} + \mathfrak{E}^{\mathrm{T}}\mathcal{K}^{\mathrm{T}}\mathcal{M}^{\mathrm{T}} \le \varepsilon^{-1}\mathcal{M}\mathcal{M}^{\mathrm{T}} + \varepsilon\mathfrak{E}^{\mathrm{T}}\mathfrak{E}.$$

3 Main Results

In order to compare with the main results of [2], we may give a prior consideration on the conventional linear Laplace diffusion system (2.9).

Theorem 3.1 *The null solution of system* (2.9) *is stochastically globally exponential stable in the mean square if there exist positive scalars $\lambda \le \lambda_1$, $\beta > 0$, a sequence of positive scalars $\underline{\alpha}_i$, $\overline{\alpha}_i\,(i \in S)$ and positive definite diagonal matrices $P_i = \mathrm{diag}(p_{i1}, p_{i2}, \cdots, p_{in})\,(i \in S)$, L_1, L_2 and Q such that the following LMI conditions hold*:

$$\begin{pmatrix} \mathbb{A}_{i1} & 0 & (F_1+F_2)L_1 + P_i\overline{A}|C_i| & P_i\overline{A}|D_i| & P_i\overline{A}|E_i| & 0 \\ * & \mathbb{A}_{i2} & 0 & (G_1+G_2)L_2 & 0 & 0 \\ * & * & -2L_1 & 0 & 0 & |N_{i1}^{\mathrm{T}}| \\ * & * & * & -2L_2 & 0 & |N_{i2}^{\mathrm{T}}| \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0, \tag{3.1}$$

$\forall i \in S,$

$$P_i > \underline{\alpha}_i I, \quad \forall i \in S, \tag{3.2}$$

$$P_i < \overline{\alpha}_i I, \quad \forall i \in S, \tag{3.3}$$

$$M_{jk}^{\mathrm{T}} P_r M_{jk} - P_l < 0, \quad \forall r, j, l \in S, \tag{3.4}$$

where

$$\mathbb{A}_{i1} = -2\lambda\underline{\alpha}_i dI - 2P_i\underline{A}B + \overline{\alpha}_i R_1 + \sum_{j\in S_{kn}^i} \pi_{ij}P_j + \widetilde{\alpha}_i \sum_{j\in S_{un}^i} P_j + \beta P_i + Q - 2F_1L_1F_2,$$

$$\mathbb{A}_{i2} = \overline{\alpha}_i R_2 - (1-\kappa)\mathrm{e}^{-\tau\beta}Q - 2G_1L_2G_2, \quad d = \min_{i,j}\Big\{\inf_{[t_0,+\infty)\times\Omega\times\mathbb{R}} \mathcal{D}_{ij}(t,x,v)\Big\}.$$

Proof Consider the Lyapunov-Krasovskii functional

$$V(t, v(t,x), i) = V_{1i} + V_{2i}, \quad \forall i \in S,$$

where

$$V_{1i} = \mathrm{e}^{\beta t}\int_\Omega v^{\mathrm{T}}(t,x)P_i v(t,x)\mathrm{d}x,$$

$$V_{2i} = \mathrm{e}^{\beta t}\int_\Omega \int_{-\tau(t)}^0 \mathrm{e}^{\beta\theta} v^{\mathrm{T}}(t+\theta,x)Qv(t+\theta,x)\mathrm{d}\theta\mathrm{d}x.$$

Then by Itô formula, we get the following stochastic differential:

$$\mathrm{d}V(t,v(t,x),i) = \mathcal{L}V(t,v(t,x),i)\mathrm{d}t + V_v(t,v(t,x),i)\sigma(t,v(t,x),v(t-\tau(t),x),i)\mathrm{d}w(t),$$

$$V_v(t,v(t,x),i) = \Big(\frac{\partial V(t,v(t,x),i)}{\partial v_1}, \cdots, \frac{\partial V(t,v(t,x),i)}{\partial v_n}\Big).$$

$\mathcal{L}$ is the weak infinitesimal operator such that $\mathcal{L}V(t, v(t,x), i) = \mathcal{L}V_{1i} + \mathcal{L}V_{2i}$ for any given $i \in S$. Next, it follows by Lemma 2.3 ($p = 2$) and (2.9) that for $t \neq t_k$,

$$\begin{aligned}\mathcal{L}V_{1i} = \mathrm{e}^{\beta t}\Big\{ &- \sum_{k=1}^{m}\sum_{j=1}^{n}\int_\Omega 2p_{ij}\mathcal{D}_{jk}(t,x,v)\Big(\frac{\partial v_j}{\partial x_k}\Big)^2 \mathrm{d}x - 2\int_\Omega v^{\mathrm{T}} P_i A(v(t,x))B(v(t,x))\mathrm{d}x \\ &+ 2\int_\Omega [v^{\mathrm{T}} P_i A(v(t,x))C_i(t)f(v(t,x)) + v^{\mathrm{T}} P_i A(v(t,x))D_i(t)g(v(t-\tau(t),x))]\mathrm{d}x \\ &+ \int_\Omega v^{\mathrm{T}}\Big(\sum_{j\in S}\pi_{ij}P_j\Big)v^{\mathrm{T}}\mathrm{d}x \\ &+ \int_\Omega \mathrm{trace}[\sigma^{\mathrm{T}}(t,v(t,x),v(t-\tau(t),x),i)P_i\sigma(t,v(t,x),v(t,x),v(t-\tau(t),x),i)]\mathrm{d}x\Big\} \\ &+ \beta\mathrm{e}^{\beta t}\int_\Omega v^{\mathrm{T}} P_i v\mathrm{d}x. \end{aligned} \tag{3.5}$$

Here, $v = v(t,x)$ is a solution of system (2.9). And for $t \neq t_k$,

$$\begin{aligned} &\mathcal{L}V_{2i} \\ &= \mathrm{e}^{\beta t}\int_\Omega v^{\mathrm{T}}(t,x)Qv(t,x)\mathrm{d}x - (1-\dot{\tau}(t))\mathrm{e}^{\beta(t-\tau(t))}\int_\Omega v^{\mathrm{T}}(t-\tau(t),x)Qv(t-\tau(t),x)\mathrm{d}x \\ &\le \mathrm{e}^{\beta t}\Big[\int_\Omega v^{\mathrm{T}}(t,x)Qv(t,x)\mathrm{d}x - (1-\kappa)\mathrm{e}^{-\tau\beta}\int_\Omega v^{\mathrm{T}}(t-\tau(t),x)Qv(t-\tau(t),x)\mathrm{d}x\Big] \\ &= \mathrm{e}^{\beta t}\Big[\int_\Omega |v^{\mathrm{T}}(t,x)|Q|v(t,x)|\mathrm{d}x - (1-\kappa)\mathrm{e}^{-\tau\beta}\int_\Omega |v^{\mathrm{T}}(t-\tau(t),x)|Q|v(t-\tau(t),x)|\mathrm{d}x\Big], \end{aligned} \tag{3.6}$$

Moreover, we can get by Poincaré inequality and $0 < \lambda \le \lambda_1$,

$$\begin{aligned} &\int_\Omega - \sum_{k=1}^{m}\sum_{j=1}^{n}\int_\Omega 2p_{ij}\mathcal{D}_{jk}(t,x,v)\Big(\frac{\partial v_j}{\partial x_k}\Big)^2\mathrm{d}x \\ &\le -\int_\Omega \sum_{j=1}^{n}(2\lambda_1\underline{\alpha}_i dI)v_j^2\mathrm{d}x \le -\int_\Omega |v^{\mathrm{T}}|(2\lambda\underline{\alpha}_i dI)|v|\mathrm{d}x. \end{aligned} \tag{3.7}$$

It follows by (A1)–(A2) that

$$\int_\Omega v^{\mathrm{T}} P_i A(v(t,x))B(v(t,x))\mathrm{d}x \ge \int_\Omega v^{\mathrm{T}} P_i \underline{A} B v\mathrm{d}x = \int_\Omega |v^{\mathrm{T}}|P_i\underline{A}B|v|\mathrm{d}x. \tag{3.8}$$

In addition, we have

$$\begin{aligned} &2\int_\Omega v^{\mathrm{T}} P_i A(v(t,x))C_i(t)f(v(t,x))\mathrm{d}x \\ &\le 2\int_\Omega |v^{\mathrm{T}}|P_i A(v(t,x))|C_i(t)||f(v(t,x))|\mathrm{d}x \\ &\le 2\int_\Omega [|v^{\mathrm{T}}|P_i\overline{A}(|C_i| + |\Delta C_i(t)|)|f(v(t,x))|]\mathrm{d}x \\ &\le 2\int_\Omega [|v^{\mathrm{T}}|P_i\overline{A}(|C_i| + |E_i||\mathcal{K}(t)||N_{i1}|)|f(v(t,x))|]\mathrm{d}x. \end{aligned} \tag{3.9}$$

Similarly,

$$\begin{aligned}&2\int_\Omega v^{\mathrm{T}}P_iA(v(t,x))D_i(t)g(v(t-\tau(t),x))\mathrm{d}x\\&\le 2\int_\Omega (|v^{\mathrm{T}}|P_i\overline{A}(|D_i|+|E_i||\mathcal{K}(t)||N_{i2}|)|g(v(t-\tau(t),x))|)\mathrm{d}x.\end{aligned}\tag{3.10}$$

From (A3), we have

$$\begin{aligned}&2|f^{\mathrm{T}}(v(t,x))|L_1|f(v(t,x))|-2|v^{\mathrm{T}}(t,x)|(F_1+F_2)L_1|f(v(t,x))|\\&+2|v^{\mathrm{T}}(t,x)|F_1L_1F_2|v(t,x)|\le 0,\end{aligned}\tag{3.11}$$

$$\begin{aligned}&2|g^{\mathrm{T}}(v(t-\tau(t),x))|L_2|g(v(t-\tau(t),x))|\\&-2|v^{\mathrm{T}}(t-\tau(t),x)|(G_1+G_2)L_2|g(v(t-\tau(t),x))|\\&+2|v^{\mathrm{T}}(t-\tau(t),x)|G_1L_2G_2|v(t-\tau(t),x)|\le 0.\end{aligned}\tag{3.12}$$

From $\pi_{ii}<0$ and the definition of $\widetilde{\alpha}_i$, it is clear that

$$\sum_{j\in S}\pi_{ij}P_j\le\sum_{j\in S^i_{kn}}\pi_{ij}P_j+\widetilde{\alpha}_i\sum_{j\in S^i_{un}}P_j.\tag{3.13}$$

(A5) derives

$$\begin{aligned}&\int_\Omega \mathrm{trace}[\sigma^{\mathrm{T}}(t,v(t,x),v(t-\tau(t),x),i)P_i\sigma(t,v(t,x),v(t,x),v(t-\tau(t),x),i)]\mathrm{d}x\\&\le\int_\Omega(v^{\mathrm{T}}(t,x)\overline{\alpha}_iR_1v(t,x)+v^{\mathrm{T}}(t-\tau(t),x)\overline{\alpha}_iR_2v(t-\tau(t),x))\mathrm{d}x\\&\le\int_\Omega(|v^{\mathrm{T}}(t,x)|\overline{\alpha}_iR_1|v(t,x)|+|v^{\mathrm{T}}(t-\tau(t),x)|\overline{\alpha}_iR_2|v(t-\tau(t),x)|)\mathrm{d}x.\end{aligned}\tag{3.14}$$

Combining (3.5)–(3.14) results in

$$\mathcal{L}V(t,v(t,x),i)\le \mathrm{e}^{\beta t}\int_\Omega\zeta^{\mathrm{T}}(t,x)\mathfrak{A}_i\zeta(t,x)\mathrm{d}x,\tag{3.15}$$

where

$$\mathfrak{A}_i=\begin{pmatrix}\mathbb{A}_{i1}&0&(F_1+F_2)L_1+P_i\overline{A}(|C_i|+|E_i||\mathcal{K}(t)||N_{i1}|)&P_i\overline{A}(|D_i|+|E_i||\mathcal{K}(t)||N_{i2}|)\\ *&\mathbb{A}_{i2}&0&(G_1+G_2)L_2\\ *&*&-2L_1&0\\ *&*&*&-2L_2\end{pmatrix}$$

and $\zeta(t,x)=(|v^{\mathrm{T}}(t,x)|,|v^{\mathrm{T}}(t-\tau(t),x)|,|f^{\mathrm{T}}(v(t,x))|,|g^{\mathrm{T}}(v(t-\tau(t),x))|)^{\mathrm{T}}$.

Next we claim that $\mathfrak{A}_i<0$.

For convenience, we denote

$$\Omega_{i1}=\begin{pmatrix}\mathbb{A}_{i1}&0&(F_1+F_2)L_1+P_i\overline{A}|C_i|&P_i\overline{A}|D_i|\\ *&\mathbb{A}_{i2}&0&(G_1+G_2)L_2\\ *&*&-2L_1&0\\ *&*&*&-2L_2\end{pmatrix},$$

$$\Delta\Omega_{i1}=\begin{pmatrix}0&0&P_i\overline{A}|E_i||\mathcal{K}(t)||N_{i1}|&P_i\overline{A}|E_i||\mathcal{K}(t)||N_{i2}|\\ *&0&0&0\\ *&*&0&0\\ *&*&*&0\end{pmatrix},$$

$$\mathcal{M}=\begin{pmatrix} P_i\overline{A}|E_i| \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathfrak{E}=\begin{pmatrix} 0 \\ 0 \\ |N_{i1}^{\mathrm{T}}| \\ |N_{i2}^{\mathrm{T}}| \end{pmatrix}^{\mathrm{T}}.$$

By applying Schur complement to (3.1), we can get from Lemma 2.4,

$$\mathfrak{A}_i=\Omega_{i1}+\Delta\Omega_{i1}=\Omega_{i1}+\mathcal{M}|\mathcal{K}(t)|\mathfrak{E}+\mathfrak{E}^{\mathrm{T}}|\mathcal{K}^{\mathrm{T}}(t)|\mathcal{M}^{\mathrm{T}}\le\Omega_{i1}+\mathcal{M}\mathcal{M}^{\mathrm{T}}+\mathfrak{E}^{\mathrm{T}}\mathfrak{E}<0,$$

which proves our claim. And then $\mathcal{L}V(t,v(t,x),i)\le 0$. Define

$$\mathcal{V}(t,v(t,x),i)=\int_\Omega v^{\mathrm{T}}(t,x)P_iv(t,x)\mathrm{d}x+\int_\Omega\int_{-\tau(t)}^0 \mathrm{e}^{\beta\theta}v^{\mathrm{T}}(t+\theta,x)Qv(t+\theta,x)\mathrm{d}\theta\mathrm{d}x.$$

Then we have $V(t,v(t,x),i)=\mathrm{e}^{\beta t}\mathcal{V}(t,v(t,x),i)$, satisfying

$$\mathcal{L}(\mathrm{e}^{\beta t}\mathcal{V}(t,v(t,x),i))=\mathcal{L}V(t,v(t,x),i)\le 0.$$

Now, by applying the Dynkin formula, we can derive that for any $i\in S$,

$$\mathrm{e}^{\beta t}\mathbb{E}\mathcal{V}(t)-\mathbb{E}\mathcal{V}(t_0)=\mathbb{E}\int_{t_0}^t\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s\le 0,\quad \forall\beta>0,\ t\ne t_k. \tag{3.16}$$

In fact, due to $v(t_k^+,x)=v(t_k,x)$, we might as well assume $t_{k-1}\le t<t_k$ for any given $k\in\{1,2,\cdots\}$. And then we have

$$\begin{aligned}
0\ge\mathbb{E}\int_{t_0}^t\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s&=\mathbb{E}\int_{t_0}^{t_1^-}\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s+\mathbb{E}\int_{t_1^-}^{t_1}\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s+\mathbb{E}\int_{t_1}^{t_2^-}\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s\\
&\quad+\cdots+\mathbb{E}\int_{t_{k-2}}^{t_{k-1}^-}\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s+\mathbb{E}\int_{t_{k-1}^-}^{t_{k-1}}\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s+\mathbb{E}\int_{t_{k-1}}^{t}\mathcal{L}(\mathrm{e}^{\beta s}\mathcal{V}(s))\mathrm{d}s\\
&=\mathrm{e}^{\beta t_1}\mathbb{E}\mathcal{V}(t_1^-)-\mathbb{E}\mathcal{V}(t_0)+\mathrm{e}^{\beta t_1}[\mathbb{E}\mathcal{V}(t_1)-\mathbb{E}\mathcal{V}(t_1^-)]+\mathrm{e}^{\beta t_2}\mathbb{E}\mathcal{V}(t_2^-)-\mathrm{e}^{\beta t_1}\mathbb{E}\mathcal{V}(t_1)\\
&\quad+\cdots+\mathrm{e}^{\beta t_{k-1}}\mathbb{E}\mathcal{V}(t_{k-1}^-)-\mathrm{e}^{\beta t_{k-2}}\mathbb{E}\mathcal{V}(t_{k-2})\\
&\quad+\mathrm{e}^{\beta t_{k-1}}[\mathbb{E}\mathcal{V}(t_{k-1})-\mathbb{E}\mathcal{V}(t_{k-1}^-)]+\mathrm{e}^{\beta t}\mathbb{E}\mathcal{V}(t)-\mathrm{e}^{\beta t_{k-1}}\mathbb{E}\mathcal{V}(t_{k-1})\\
&=\mathrm{e}^{\beta t}\mathbb{E}\mathcal{V}(t)-\mathbb{E}\mathcal{V}(t_0),
\end{aligned}$$

which proves (3.16). On the other hand, we claim

$$\mathcal{V}(t_k,v(t_k,x),j)\le\mathcal{V}(t_k^-,v(t_k^-,x),i)\quad \text{for all }\ i,j\in S,\ k=1,2,\cdots. \tag{3.17}$$

Indeed, we can get by the assumptions on M_{jk}:

$$\begin{aligned}
&\int_\Omega(v^{\mathrm{T}}(t_k,x)P_jv(t_k,x)-v^{\mathrm{T}}(t_k^-,x)P_iv(t_k^-,x))\mathrm{d}x\\
\le&\int_\Omega(v^{\mathrm{T}}(t_k^-,x)(M_{ik}^{\mathrm{T}}P_jM_{ik}-P_i)v(t_k^-,x))\mathrm{d}x\le 0,\\
&\int_\Omega\int_{-\tau(t_k)}^0\mathrm{e}^{\beta\theta}v^{\mathrm{T}}(t_k+\theta,x)Qv(t_k+\theta,x)\mathrm{d}\theta\mathrm{d}x\\
&-\int_\Omega\int_{-\tau(t_k^-)}^0\mathrm{e}^{\beta\theta}v^{\mathrm{T}}(t_k^-+\theta,x)Qv(t_k^-+\theta,x)\mathrm{d}\theta\mathrm{d}x
\end{aligned}$$

$$\begin{aligned}&= \int_\Omega \int_{t_k-\tau(t_k)}^{t_k} \mathrm{e}^{\beta(\xi-t_k)} v^{\mathrm{T}}(\xi,x) Q v(\xi,x)\mathrm{d}\xi\mathrm{d}x \\ &\quad - \int_\Omega \int_{t_k^- -\tau(t_k^-)}^{t_k^-} \mathrm{e}^{\beta(\xi-t_k^-)} v^{\mathrm{T}}(\xi,x) Q v(\xi,x)\mathrm{d}\xi\mathrm{d}x \\ &= 0.\end{aligned}$$

Thus, we prove the claim (3.17). Owing to (3.16)–(3.17), we get

$$\mathrm{e}^{\beta t_k}\mathbb{E}\mathcal{V}(t_k, v(t_k,x), j) \le \mathrm{e}^{\beta t_k^-}\mathbb{E}\mathcal{V}(t_k^-, v(t_k^-,x), i) \le \mathbb{E}\mathcal{V}(t_0), \quad \forall i,j \in S. \tag{3.18}$$

Hence, combining (3.16) and (3.18) implies

$$\mathrm{e}^{\beta t}\mathbb{E}\mathcal{V}(t) \le \mathbb{E}\mathcal{V}(t_0) \quad \text{for all } t \ge t_0. \tag{3.19}$$

Now, for any $\phi(\theta,x) \in L^2_{\mathcal{F}_0}([-\tau,0]\times\Omega;\mathbb{R}^n)$ and any system mode $i \in S$, the solution $v(t,x,\phi,i_0)$ of system (2.9) with the initial value ϕ satisfies

$$\begin{aligned}&\min_{i\in S}\{\underline{\alpha}_i\}\mathrm{e}^{\beta t}\mathbb{E}(\|v(t,x,\phi,i_0)\|_2^2) \le \underline{\alpha}_i \mathrm{e}^{\beta t}\mathbb{E}(\|v(t,x,\phi,i_0)\|_2^2) \\ &\le \mathrm{e}^{\beta t}\mathbb{E}\mathcal{V}(t,v(t,x),i) \le \mathbb{E}\mathcal{V}(t_0, v(t_0,x), i) \\ &\le \mathbb{E}\big(\overline{\alpha}_i\|\phi(0)\|_2^2\big) + \mathbb{E}\Big(\int_{-\tau}^0\int_\Omega \mathrm{e}^{\beta\theta}[\phi^{\mathrm{T}}(\theta,x)Q\phi(\theta,x)]\mathrm{d}x\mathrm{d}\theta\Big) \\ &\le \Big(\max_{i\in S}\{\overline{\alpha}_i\} + \lambda_{\max}Q\Big)\sup_{-\tau\le\theta\le 0}\mathbb{E}(\|\phi(\theta)\|_2^2), \quad t \ge t_0\end{aligned} \tag{3.20}$$

or

$$\mathbb{E}(\|v(t,x\,;\phi,i_0)\|_2^2) \le \gamma \mathrm{e}^{-\beta t}\sup_{-\tau\le\theta\le 0}\mathbb{E}(\|\phi(\theta,x)\|_2^2), \quad t \ge t_0, \tag{3.21}$$

where scalars $\gamma = \frac{1}{\min_{i\in S}\{\underline{\alpha}_i\}}\big(\max_{i\in S}\{\overline{\alpha}_i\} + \lambda_{\max}Q\big) > 0$, $\beta > 0$. Therefore, we can see it by (3.21) and Definition 2.1 that the null solution of system (2.9) is globally stochastically exponentially stable in the mean square.

Remark 3.1 In Theorem 3.1, the magnitude of λ_1 is determined by the bounded domain $\Omega \in \mathbb{R}^m$. However, if $m \ge 3$, the exact computation of λ_1 is usually not possible. Nevertheless, we can estimate the value of λ_1. For instance, under the Dirichlet boundary assumption, we may fix $\lambda = \Lambda_2\big(\frac{\omega_m}{\mathrm{meas}(\Omega)}\big)^{\frac{2}{m}}$ in Theorem 3.1 due to Hardy-Poincaré inequality. In fact, from $\lambda_1 = \inf_{\substack{\psi\in H_0^1(\Omega)\\ \|\psi\|=1}} \int_\Omega |\nabla\psi|^2\mathrm{d}x$, we know that $0 < \lambda \le \lambda_1$, satisfying $\int_\Omega |\nabla\psi|^2\mathrm{d}x \ge \lambda\int_\Omega \psi^2\mathrm{d}x$ for all $\psi \in H_0^1(\Omega)$. In many recent literatures (see [2, 31–34]), $\Omega \in \mathbb{R}^m$ is restricted to be a cube. Moreover, in their numerical examples, the dimension m is restricted to be 1 or 2. Now, in this paper, we abolish these limitations thanks to the synthetic application of Poincaré inequality and Hardy-Poincaré inequality. Below, Example 4.3 will show the effectiveness of Theorem 3.1, where Ω is assumed to be a spheroid and not a sphere. Notice that if Ω is a ball, the constants of Hardy-Poincaré inequality are optimal (see [36, Theorem 4.1]). But Theorem 3.1 admits actually $\lambda < \lambda_1$, and then we may fix $\lambda = \Lambda_2\big(\frac{\omega_m}{\mathrm{meas}(\Omega)}\big)^{\frac{2}{m}}$. So we need not assume in numerical examples that Ω is the similar ball as that of [29–30]. To the best of our knowledge, it is the first time to apply both Poincaré inequality and Hardy-Poincaré inequality to stability analysis of the reaction-diffusion neural networks.

Remark 3.2 Below, Example 4.3 is given to show that Theorem 3.1 is more effective and less conservative than some existing results due to significant improvement in the allowable upper bounds of delays.

If $\mathcal{D}(t,x,v)\equiv\mathcal{D}$ is a diagonal constant matrix, the system (2.9) is perhaps reduced to the following system:

$$\begin{cases} \mathrm{d}v(t,x)=\{\mathcal{D}\Delta v(t,x))-A(v(x,t))[B(v(t,x))-C(r(t),t)f(v(t,x)) \\ \qquad\qquad -D(r(t),t)g(v(t-\tau(t),x))]\}\mathrm{d}t+\sigma(t,v(t,x),v(t-\tau(t),x),r(t))\mathrm{d}w(t) \\ \qquad\qquad\quad \text{for all } t\ge t_0,\ t\ne t_k,\ x\in\Omega, \\ v(t_k,x)=M_k(r(t))v(t_k^-,x),\quad k=1,2,\cdots, \end{cases} \tag{3.22}$$

where $\Delta v(t,x)=(\Delta v_1(t,x),\Delta v_2(t,x),\cdots,\Delta v_n(t,x))^{\mathrm{T}}$, and $\Delta v_j(t,x)=\sum\limits_{k=1}^{m}\frac{\partial}{\partial x_k}\big(\frac{\partial v_j(t,x)}{\partial x_k}\big)$.

Similarly to (3.7), we have

$$\begin{aligned} 2\int_\Omega v^{\mathrm{T}}(t,x)P\mathcal{D}\Delta v(t,x)\mathrm{d}x &\le -2\int_\Omega\sum_{k=1}^{n}p_{kk}\mathcal{D}_{kk}\sum_{j=1}^{m}\Big(\frac{\partial v_k(t,x)}{\partial x_j}\Big)^2\mathrm{d}x \\ &\le -2\lambda\int_\Omega|v^{\mathrm{T}}|(P\mathcal{D})|v|\mathrm{d}x, \end{aligned} \tag{3.7*}$$

where both constant matrices $\mathcal{D}=\mathrm{diag}(\mathcal{D}_{11},\mathcal{D}_{22},\cdots,\mathcal{D}_{nn})$ and $P=\mathrm{diag}(p_{11},p_{22},\cdots,p_{nn})$ are positive definite.

Hence, similarly to the proof of Theorem 3.1, we can prove the following similar result.

Theorem 3.2 *The null solution of system* (3.22) *is stochastically globally exponential stable in the mean square if there exist positive scalars* $\lambda\le\lambda_1,\ \beta>0$, *a sequence of positive scalars* $\underline{\alpha}_i$, $\overline{\alpha}_i\,(i\in S)$ *and positive definite diagonal matrices* $P_i=\mathrm{diag}(p_{i1},p_{i2},\cdots,p_{in})\,(i\in S)$, L_1, L_2 *and* Q *such that the following LMI conditions hold*:

$$\begin{pmatrix} \overline{\mathbb{A}}_{i1} & 0 & (F_1+F_2)L_1+P_i\overline{A}|C_i| & P_i\overline{A}|D_i| & P_i\overline{A}|E_i| & 0 \\ * & \mathbb{A}_{i2} & 0 & (G_1+G_2)L_2 & 0 & 0 \\ * & * & -2L_1 & 0 & 0 & |N_{i1}^{\mathrm{T}}| \\ * & * & * & -2L_2 & 0 & |N_{i2}^{\mathrm{T}}| \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{pmatrix}<0,\quad \forall i\in S,$$

$$P_i>\underline{\alpha}_i I,\quad \forall i\in S,$$

$$P_i<\overline{\alpha}_i I,\quad \forall i\in S,$$

$$M_{jk}^{\mathrm{T}}P_rM_{jk}-P_l<0,\quad \forall r,j,l\in S,$$

where

$$\overline{\mathbb{A}}_{i1}=-2\lambda P_i\mathcal{D}-2P_i\underline{A}B+\overline{\alpha}_iR_1+\sum_{j\in S_{kn}^i}\pi_{ij}P_j+\widetilde{\alpha}_i\sum_{j\in S_{un}^i}P_j+\beta P_i+Q-2F_1L_1F_2.$$

Consider the deterministic system (2.4) with $h\equiv 0$,

$$\begin{cases} \dfrac{\partial v}{\partial t}=\mathcal{D}\Delta v(t,x)-A(v(x,t))[B(v(t,x))-Cf(v(t,x))-Dg(v(t-\tau(t),x))] \\ \qquad\qquad \text{for all } t\ge t_0,\ t\ne t_k,\ x\in\Omega, \\ v(t_k,x)=M_kv(t_k^-,x),\quad k=1,2,\cdots. \end{cases} \tag{3.23}$$

Then, from Theorem 3.2, we can deduce the following corollary.

Corollary 3.3 *The null solution of system* (3.23) *is stochastically globally exponential stable in the mean square if there exist positive scalars* $\lambda \le \lambda_1$, $\beta > 0$, *and positive definite diagonal matrices* P, L_1, L_2 *and* Q *such that the following LMI conditions hold*:

$$\begin{pmatrix} \widehat{A}_1 & 0 & (F_1+F_2)L_1 + P\overline{A}|C| & P\overline{A}|D| \\ * & \widehat{A}_2 & 0 & (G_1+G_2)L_2 \\ * & * & -2L_1 & 0 \\ * & * & * & -2L_2 \end{pmatrix} < 0, \tag{3.24}$$

$$M_k^{\mathrm{T}} P M_k - P < 0, \tag{3.25}$$

where

$$\widehat{A}_1 = -2\lambda P\mathcal{D} - 2P\underline{A}B + \beta P + Q - 2F_1L_1F_2, \quad \widehat{A}_2 = -(1-\kappa)\mathrm{e}^{-\tau\beta}Q - 2G_1L_2G_2.$$

Remark 3.3 In [2, Theorem 3.1], Ω is restricted to be a cube $\Omega = \{(x_1, x_2, \cdots, x_m)^{\mathrm{T}} \in \mathbb{R}^m : |x_j| < l_j, j = 1, 2, \cdots, m\}$, and $F_1 = G_1$ are assumed to be 0. Under the Dirichlet boundary condition, the null solution of system (3.23) is exponentially stable if all (C1)–(C3) (see [2, Theorem 3.1]) are satisfied, where

$$\begin{pmatrix} -2lP\mathcal{D} - 2P\underline{A}B + F_2^2 & P\overline{A}|C| & P\overline{A}|D| \\ * & -I & 0 \\ * & * & -I \end{pmatrix} < 0 \tag{C1}$$

and $l = \sum\limits_{j=1}^{m} \frac{1}{l_j^2}$. Here, we point out that in comparison with Corollary 3.3, conditions (C1)–(C3) (see [2, Theorem 3.1]) are too complicated to be satisfied. LMI condition (3.24) is more feasible than (C1) of [2, Theorem 2.1]. Below we shall give a numerical example for it (see Example 4.1).

Finally, we consider the LMI criterion for the system (2.7) with p-Laplace diffusion ($p > 1$).

Theorem 3.4 *The null solution of system* (2.7) *is stochastically globally exponential stable in the mean square if there exists a positive scalar* $\beta > 0$, *a sequence of positive scalars* $\underline{\alpha}_i$, $\overline{\alpha}_i$ ($i \in S$) *and positive definite diagonal matrices* $P_i = \mathrm{diag}(p_{i1}, p_{i2}, \cdots, p_{in})$ ($i \in S$), L_1, L_2 *and* Q *such that the following LMI conditions hold*:

$$\begin{pmatrix} \widetilde{\mathbb{A}}_{i1} & 0 & (F_1+F_2)L_1 + P_i\overline{A}|C_i| & P_i\overline{A}|D_i| & P_i\overline{A}|E_i| & 0 \\ * & \mathbb{A}_{i2} & 0 & (G_1+G_2)L_2 & 0 & 0 \\ * & * & -2L_1 & 0 & 0 & |N_{i1}^{\mathrm{T}}| \\ * & * & * & -2L_2 & 0 & |N_{i2}^{\mathrm{T}}| \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0, \tag{3.1*}$$

$\forall i \in S$,

$$P_i > \underline{\alpha}_i I, \quad \forall i \in S, \tag{3.2*}$$

$$P_i < \overline{\alpha}_i I, \quad \forall i \in S, \tag{3.3*}$$

$$M_{jk}^{\mathrm{T}} P_r M_{jk} - P_l < 0, \quad \forall r, j, l \in S, \tag{3.4*}$$

where

$$\widetilde{\mathbb{A}}_{i1} = -2P_i\underline{A}B + \overline{\alpha}_i R_1 + \sum_{j\in S^i_{kn}} \pi_{ij}P_j + \widetilde{\alpha}_i \sum_{j\in S^i_{un}} P_j + \beta P_i + Q - 2F_1L_1F_2.$$

Proof First, we may construct the same Lyapunov-Krasovskii functional as that of the proof for Theorem 3.1. Second, we can get by Lemma 2.3:

$$\int_\Omega v^{\mathrm{T}} P_i(\nabla\cdot(D(t,x,v)\circ\nabla_p v))\mathrm{d}x = \int_\Omega (\nabla\cdot(D(t,x,v)\circ\nabla_p v))^{\mathrm{T}} P_i u\mathrm{d}x \le 0.$$

And then we have the similar inequality as (3.5):

$$\begin{aligned}\mathcal{L}V_{1i} \le \mathrm{e}^{\beta t}\Big\{&0 - 2\int_\Omega v^{\mathrm{T}} P_i A(v(t,x))B(v(t,x))\mathrm{d}x\\ &+ 2\int_\Omega [v^{\mathrm{T}} P_i A(v(t,x))C_i(t)f(v(t,x)) + v^{\mathrm{T}} P_i A(v(t,x))D_i(t)g(v(t-\tau(t),x))]\mathrm{d}x\\ &+ \int_\Omega v^{\mathrm{T}}\Big(\sum_{j\in S}\pi_{ij}P_j\Big)v\mathrm{d}x\\ &+ \int_\Omega \mathrm{trace}[\sigma^{\mathrm{T}}(t,v(t,x),v(t-\tau(t),x),i)P_i\sigma(t,v(t,x),v(t,x),v(t-\tau(t),x),i)]\mathrm{d}x\Big\}\\ &+ \beta\mathrm{e}^{\beta t}\int_\Omega v^{\mathrm{T}} P_i v\mathrm{d}x. \end{aligned} \tag{3.5*}$$

The rest of the proof is completely similar as that of Theorem 3.1. We can derive those similar inequalities as (3.6)–(3.21). And then, based on Definition 2.1, the null solution of system (2.7) is globally stochastically exponentially stable in the mean square.

If Markovian jumping phenomena and parametric uncertainties are ignored, the system (2.7) is reduced to the following system:

$$\begin{cases} \mathrm{d}v(t,x) = \{\nabla\cdot(\mathcal{D}(t,x,v)\circ\nabla_p v(t,x)) - A(v(x,t))[B(v(t,x)) - Cf(v(t,x)) \\ \qquad\qquad - Dg(v(t-\tau(t),x))]\}\mathrm{d}t + \sigma(t,v(t,x),v(t-\tau(t),x),r(t))\mathrm{d}w(t) \\ \qquad\qquad \text{for all } t\ge t_0,\ t\ne t_k,\ x\in\Omega, \\ v(t_k,x) = M_k(r(t))v(t_k^-,x), \quad k=1,2,\cdots. \end{cases} \tag{3.26}$$

Then we get the following lemma from Theorem 3.4.

Corollary 3.5 *The null solution of system* (3.26) *is stochastically globally exponential stable in the mean square if there exist positive scalars* $\beta > 0$, $\underline{\alpha}$, $\overline{\alpha}$, *and positive definite diagonal matrices* P, L_1, L_2 *and* Q *such that the following LMI conditions hold*:

$$\begin{pmatrix} A_1^* & 0 & (F_1+F_2)L_1 + P\overline{A}|C| & P\overline{A}|D| \\ * & A_2^* & 0 & (G_1+G_2)L_2 \\ * & * & -2L_1 & 0 \\ * & * & * & -2L_2 \end{pmatrix} < 0, \tag{3.1**}$$

$$P > \underline{\alpha}I, \tag{3.2**}$$

$$P < \overline{\alpha}I, \tag{3.3**}$$

$$M_k^{\mathrm{T}} P M_k - P < 0, \tag{3.4**}$$

where

$$A_1^* = -2P\underline{A}B + \overline{\alpha}R_1 + \beta P + Q - 2F_1L_1F_2, \quad A_2^* = \overline{\alpha}R_2 - (1-\kappa)\mathrm{e}^{-\tau\beta}Q - 2G_1L_2G_2. \tag{3.27}$$

Remark 3.4 In [3, Theorem 2.1], R_2 in (2.10) is assumed to be 0. In addition, $F_1 = G_1$ is also assumed to be 0. If there exist positive definite diagonal matrices P_1, P_2 such that the following LMI holds:

$$\begin{pmatrix} -2P_1\underline{A}B + P_1R_1 + P_2 + F_2^2 & P_1\overline{A}|C| & P_1\overline{A}|D| \\ * & -I & 0 \\ * & * & -I \end{pmatrix} < 0, \tag{C1*}$$

and other two complicated conditions similar to (C2) and (C3) in [2, Theorem 3.1]. Below, Example 4.2 shows that Corollary 3.5 is better than [3, Theorem 2.1] due to less conservativeness and more feasibility.

Remark 3.5 The nonlinear p-Laplace diffusions in Theorem 3.4 bring a great difficulty establishing LMI conditions for the stability criterion. However, it is the first attempt to present the LMI-based criterion for the uncertain CGNNs with nonlinear p-Laplace diffusion. Below, Example 4.3 is given to show that Theorem 3.4 possesses less conservatism due to significant improvement in the allowable upper bounds of delays.

4 Numerical Examples and Comparisons

In this section, we shall give three numerical examples (Examples 4.1–4.3) for Corollaries 3.3 and 3.5 in comparison with [2, Theorem 3.1] and [3, Theorem 2.1]. Finally, Example 4.3 is presented to illustrate that Theorems 3.1 and 3.4 possess more effectiveness and less conservatism due to significant improvement in the allowable upper bounds of delays.

Example 4.1 Comparing Corollary 3.3 with the main result of [2].

Under the Dirichlet boundary condition, we consider the following system:

$$\begin{cases} \begin{pmatrix} \dfrac{\partial v_1}{\partial t} \\ \dfrac{\partial v_2}{\partial t} \end{pmatrix} \\ = \mathcal{D}\Delta v(t,x) - \begin{pmatrix} a_1(v_1) & 0 \\ 0 & a_2(v_2) \end{pmatrix} \left[\begin{pmatrix} b_1(v_1) \\ b_2(v_2) \end{pmatrix} - Cf(v(t,x)) - Dg(v(t-0.65,x)) \right] \\ \quad \text{for all } t \geq t_0,\ t \neq t_k,\ x \in \Omega, \\ v(t_k,x) = M_k v(t_k^-,x), \quad k = 1,2,\cdots, \end{cases} \tag{4.1}$$

$$\phi(s,x) = \begin{pmatrix} x^2(1-\cos(5\pi x^2))\cos^{189}(x^2-0.25)\mathrm{e}^{-100s} \\ (1-x)\sin^2(4\pi x^2)\cos^{201}(x^2-0.55)\mathrm{e}^{-100s} \end{pmatrix}, \quad -0.65 \leq s \leq 0, \tag{4.2}$$

where $v = (v_1^{\mathrm{T}}(t,x), v_2^{\mathrm{T}}(t,x))^{\mathrm{T}} \in \mathbb{R}^2$, $\Omega = \{(x_1,x_2)^{\mathrm{T}} \in \mathbb{R}^2 : |x_j| < \sqrt{2},\ j = 1,2\}$, and then $l = 1$, $\lambda_1 = \pi^2 = 9.8696$ (see [35]). In addition, $a_1(v_1) = 0.13 + 0.07\sin^2(tx^2)$, $a_2(v_2) = 0.14 + 0.06\cos^2(tx^2)$, $b_1(v_1) = 0.02v_1 + 2v_1\sin^2(t^2+x^2))$, $b_2(v_2) = 0.016v_2 + 12v_2\sin^2(t^2+x^2)$, $f(v) = g(v) = (0.1v_1, 0.1v_2 + 0.1v_2\sin^2(tx^2))^{\mathrm{T}}$, and

$$\mathcal{D} = \begin{pmatrix} 0.003 & 0 \\ 0 & 0.0032 \end{pmatrix}, \quad C = D = \begin{pmatrix} 0.11 & -0.003 \\ -0.003 & 0.12 \end{pmatrix}, \quad M_k = \begin{pmatrix} 0.68 & 0.01 \\ 0.01 & 0.55 \end{pmatrix},$$

and hence

$$\underline{A} = \begin{pmatrix} 0.13 & 0 \\ 0 & 0.14 \end{pmatrix}, \quad \overline{A} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \quad B = \begin{pmatrix} 0.02 & 0 \\ 0 & 0.016 \end{pmatrix},$$
$$F_1 = G_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad F_2 = G_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}.$$

We might as well assume that $\lambda = 9.8 < \lambda_1$, $\beta = 0.01$, $t_0 = 0$, $t_k - t_{k-1} = 0.525$, $\tau(t) \equiv 0.65 = \tau$ and then $\kappa = 0$ for all $t \geq t_0$. We may take $\lambda = 9.8$. Now, by using Matlab LMI toolbox to solve the LMI (C1), we get $t_{\min} = 0.0144 > 0$, which implies the LMI (C1) is found infeasible. But by using Matlab LMI toolbox to solve the LMIs (3.24) and (3.25), the result is $t_{\min} = -0.1182 < 0$, and

$$P = \begin{pmatrix} 86.1682 & 0 \\ 0 & 83.8211 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 35.2742 & 0 \\ 0 & 33.7609 \end{pmatrix},$$
$$L_2 = \begin{pmatrix} 35.5378 & 0 \\ 0 & 34.6527 \end{pmatrix}, \quad Q = \begin{pmatrix} 2.8991 & 0 \\ 0 & 2.7795 \end{pmatrix}.$$

Hence, Corollary 3.3 derives that the null solution of system (4.1) is stochastically globally exponential stable in the mean square (see Figures 1–3).

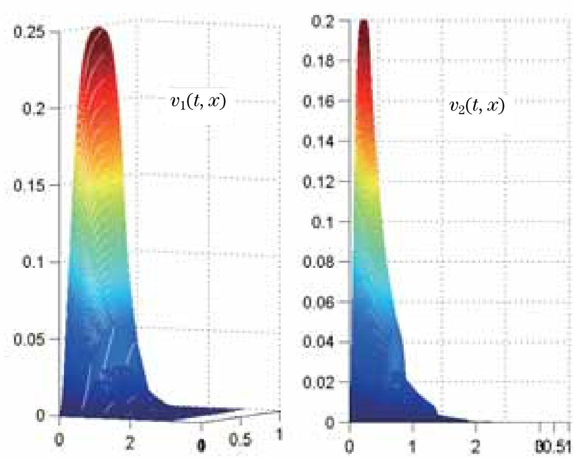

Figure 1 Computer simulations of the states $v_1(t,x)$ and $v_2(t,x)$

Remark 4.1 The stability of the null solution of system (4.1) can not be judged by [2, Theorem 3.1], for the first LMI (C1) of three conditions (C1)–(C3) is found infeasible. But all LMI conditions are only sufficient ones, not necessary for the stability. Corollary 3.3 shows that the null solution of system (4.1) is stochastically globally exponential stable in the mean square. Hence, Corollary 3.3 is really effective and less conservative than [2, Theorem 3.1].

Example 4.2 Comparing Corollary 3.5 with the main result of [3].

Under the Neumann boundary condition and the initial condition (4.2), we consider the system (3.26) with the following parameters:

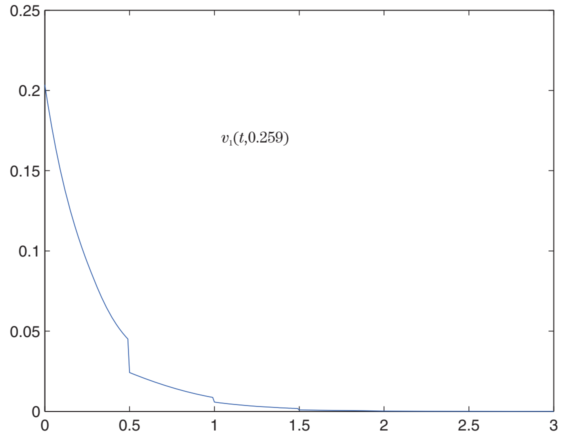

Figure 2 Sectional curve of the state variable $v_1(t,x)$

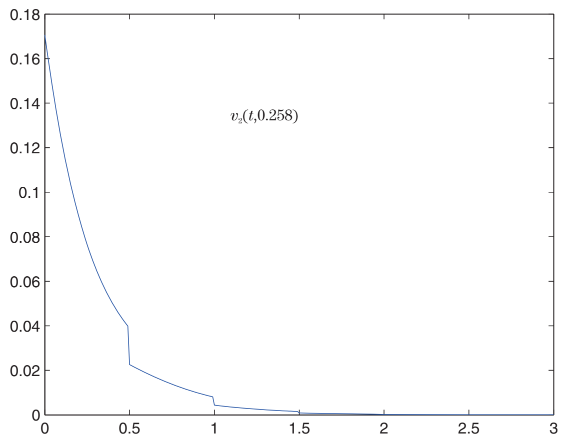

Figure 3 Sectional curve of the state variable $v_2(t,x)$

$$\underline{A}=\begin{pmatrix}1.3 & 0\\ 0 & 1.4\end{pmatrix},\quad \overline{A}=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix},\quad B=\begin{pmatrix}1.8 & 0\\ 0 & 1.88\end{pmatrix},$$
$$F_1=G_1=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix},\quad F_2=\begin{pmatrix}7.38 & 0\\ 0 & 7.48\end{pmatrix},\quad G_2=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix},$$
$$\mathcal{D}(t,x,v)=\begin{pmatrix}0.0007 & 0.0006\\ 0.0008 & 0.0009\end{pmatrix},\quad C=D=\begin{pmatrix}0.11 & -0.003\\ -0.003 & 0.12\end{pmatrix},$$
$$M_k=\begin{pmatrix}0.57 & 0.01\\ 0.01 & 0.65\end{pmatrix},\quad R_1=\begin{pmatrix}0.01 & 0.0012\\ 0.0012 & 0.01\end{pmatrix},\quad R_2=\begin{pmatrix}0.01 & 0.001\\ 0.001 & 0.015\end{pmatrix}.$$

Assume, in addition, $\beta=0.01$, $\tau=0.65$, $k=0$.

By using Matlab LMI toolbox to solve the LMI (C1*), the result is $t_{\min}=0.0050>0$, which implies the LMI (C1*) is found infeasible. But by solving LMIs (3.1**)–(3.4**), one can obtain

$t_{\min} = -0.0037 < 0$, and $\underline{\alpha} = 2.1189,\ \overline{\alpha} = 7.6303$,

$$P = \begin{pmatrix} 4.4085 & 0 \\ 0 & 4.0900 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 0.1571 & 0 \\ 0 & 0.1393 \end{pmatrix},$$
$$L_2 = \begin{pmatrix} 0.8179 & 0 \\ 0 & 0.4227 \end{pmatrix}, \quad Q = \begin{pmatrix} 3.7133 & 0 \\ 0 & 3.9282 \end{pmatrix}.$$

Hence, Corollary 3.5 derives that the null solution of system (3.26) is stochastically globally exponential stable in the mean square.

Remark 4.2 The stability of the null solution of system (3.26) with the above mentioned data can not be judged by [3, Theorem 2.1], for the first LMI (C1) of three conditions (C1)–(C3) is found infeasible. But all LMI conditions are only sufficient ones, not necessary for the stability. Hence, Corollary 3.5 is really more effective and less conservative than [3, Theorem 2.1] for the same reason as that of Remark 4.1.

Example 4.3 Comparing the allowable upper bound of Theorem 3.1 ($p > 1$) with that of Theorem 3.4 ($p = 2$).

Under the Dirichlet boundary condition, we consider the system (2.7) with the following parameters:

$$\mathcal{D}(t,x,v) = \begin{pmatrix} 0.003 & 0.005 & 0.003 \\ 0.004 & 0.0006 & 0.005 \end{pmatrix}, \quad \underline{A} = \begin{pmatrix} 0.13 & 0 \\ 0 & 0.14 \end{pmatrix}, \quad \overline{A} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix},$$
$$B = \begin{pmatrix} 5.160 & 0 \\ 0 & 5.160 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 0.11 & -0.003 \\ -0.003 & 0.12 \end{pmatrix} = D_1,$$
$$C_2 = \begin{pmatrix} 0.15 & -0.003 \\ -0.003 & 0.15 \end{pmatrix} = D_2, \quad C_3 = \begin{pmatrix} 0.16 & -0.003 \\ -0.003 & 0.16 \end{pmatrix} = D_3,$$
$$E_1 = E_2 = E_3 = \begin{pmatrix} 0.11 & 0.003 \\ 0.003 & 0.12 \end{pmatrix}, \quad N_{11} = \begin{pmatrix} 0.1 & -0.1 \\ -0.1 & 0.12 \end{pmatrix}, \quad N_{12} = \begin{pmatrix} 0.12 & 0.1 \\ 0.1 & 0.1 \end{pmatrix},$$
$$N_{21} = \begin{pmatrix} 0.1 & -0.13 \\ -0.13 & 0.12 \end{pmatrix}, \quad N_{22} = \begin{pmatrix} 0.12 & 0.1 \\ 0.1 & 0.15 \end{pmatrix},$$
$$N_{31} = \begin{pmatrix} 0.13 & -0.1 \\ -0.1 & 0.12 \end{pmatrix}, \quad N_{32} = \begin{pmatrix} 0.125 & 0.1 \\ 0.1 & 0.1 \end{pmatrix},$$
$$F_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = G_1, \quad F_2 = \begin{pmatrix} 0.100 & 0 \\ 0 & 0.200 \end{pmatrix} = G_2,$$
$$R_1 = \begin{pmatrix} 0.01 & 0.0001 \\ 0.0001 & 0.012 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 0.01 & 0.0001 \\ 0.0001 & 0.015 \end{pmatrix},$$
$$M_k(r(t)) = \begin{pmatrix} 0.91 & 0.01 \\ 0.01 & 0.95 \end{pmatrix}, \quad \forall r(t) = i \in S = \{1,2,3\},\ k = 1,2,\cdots.$$

The transition matrix is considered as

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix} = \begin{pmatrix} -0.6 & ? & ? \\ 0.2 & ? & ? \\ ? & 0.3 & ? \end{pmatrix}. \tag{4.3}$$

Then we have $d = 0.003$, $\widetilde{\alpha}_1 = 0.6$, $\widetilde{\alpha}_2 = 0.8$, $\widetilde{\alpha}_3 = 0.7$. Assume, in addition, $\beta = 0.01$. Denote $v = v(t,x) = (v_1(t,x), v_2(t,x))^{\mathrm{T}}$, and $x = (x_1, x_2, x_3)^{\mathrm{T}} \in \Omega = \big\{x \in \mathbb{R}^3 : \frac{x_1^2}{1.03} + \frac{x_2^2}{1.2} +$

$\frac{x_3^2}{1.1} \leq 1\}$. A direct computation yields $\Lambda_2 = 5.7832$, $\omega_3 = \frac{4}{3}\pi$, $\text{meas}(\Omega) = 4.8842$, and then $\lambda = \Lambda_2\big(\frac{\omega_3}{\text{meas}(\Omega)}\big)^{\frac{2}{3}} = 5.2203$.

Let $\tau(t) \equiv 100.29$, and then $\kappa = 0$. Now we use the Matlab LMI toolbox to solve the LMIs (3.1*)–(3.4*). The results show $t_{\min} = -0.0418 < 0$, and $\overline{\alpha}_1 = 1.8714$, $\underline{\alpha}_1 = 0.7246$, $\overline{\alpha}_2 = 1.9114$, $\underline{\alpha}_2 = 0.7669$, $\overline{\alpha}_3 = 1.8892$, $\underline{\alpha}_3 = 0.7450$,

$$P_1 = \begin{pmatrix} 1.4688 & 0 \\ 0 & 1.4296 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1.5544 & 0 \\ 0 & 1.5133 \end{pmatrix},$$
$$P_3 = \begin{pmatrix} 1.5082 & 0 \\ 0 & 1.4716 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.2288 & 0 \\ 0 & 0.3194 \end{pmatrix},$$
$$L_1 = \begin{pmatrix} 0.6175 & 0 \\ 0 & 0.6072 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0.6194 & 0 \\ 0 & 0.6110 \end{pmatrix}.$$

Then we can conclude from Theorem 3.4 that the null solution of system (2.7) is stochastically globally exponential stable in the mean square for the maximum allowable upper bounds $\tau = 100.29$. This shows that the approach developed in Theorem 3.4 is effective and less conservative than some existing results.

Particularly, if $p = 2$ in the system (2.7), $\tau(t) \equiv 100.59$, and $\kappa = 0$, one can solve LMIs (3.1)–(3.4), and obtain $t_{\min} = -0.0426 < 0$, and $\overline{\alpha}_1 = 1.8760$, $\underline{\alpha}_1 = 0.7331$, $\overline{\alpha}_2 = 1.9165$, $\underline{\alpha}_2 = 0.7825$, $\overline{\alpha}_3 = 1.8945$, $\underline{\alpha}_3 = 0.7616$,

$$P_1 = \begin{pmatrix} 1.4738 & 0 \\ 0 & 1.4344 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1.5605 & 0 \\ 0 & 1.5191 \end{pmatrix},$$
$$P_3 = \begin{pmatrix} 1.5148 & 0 \\ 0 & 1.4779 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.2454 & 0 \\ 0 & 0.3364 \end{pmatrix},$$
$$L_1 = \begin{pmatrix} 0.6186 & 0 \\ 0 & 0.6082 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0.6205 & 0 \\ 0 & 0.6121 \end{pmatrix}.$$

Then we can conclude from Theorem 3.1 that the null solution of system (2.9) (or system (2.7) with $p = 2$) is stochastically globally exponential stable in the mean square for the maximum allowable upper bounds $\tau = 100.59$, which shows that Theorem3.1 is effective and less conservative than some existing results.

Table 1 Allowable upper bound of τ for Theorems 3.1 and 3.4

p	Theorem 3.1	Theorem 3.4
$= 2$	100.59	100.29
> 1		100.29

Remark 4.3 In this numerical example, Ω is an ellipsoid in $\mathbb{R}^3$. But in recent related literatures (see [29–30]), only the sphere is considered in their numerical examples. Moreover, in many recent literatures (see [32–36]), Ω is restricted to be a cube in $\mathbb{R}^1$ or $\mathbb{R}^2$ in their numerical examples. Now in this paper, due to the synthetic application of Poincaré inequality and Hardy-Poincaré inequality, we abolish these limitations. As far as we know, it is the first time to consider an ellipsoid in numerical simulation.

Remark 4.4 Table 1 in this numerical example shows that the allowable upper bound of τ for Theorem 3.1 is bigger than that of Theorem 3.4 (with $p = 2$), which implies the diffusion item plays an active role in the stability criterion.

Remark 4.5 Example 4.3 illustrates that the allowable upper bound of time delays for Theorem 3.1 or Theorem 3.4 is far greater than that of any recent literatures related to delay-dependent stability criteria (see [27, 38–43]).

5 Conclusions

In this paper, the stochastic global exponential stability for delayed impulsive Markovian jumping reaction-diffusion Cohen-Grossberg neural networks is investigated, in which uncertain parameters and partially unknown transition rates and even the nonlinear p-Laplace diffusion bring a great difficulty in judging the stability. By using a novel Lyapunov-Krasovskii functional approach, linear matrix inequality technique, Itô formula, some new stability criteria are obtained. Particularly, the synthetic application of Poincaré inequality and Hardy-Poincaré inequality admits ellipsoid domains to be considered in numeral simulation (see Remarks 3.1 and 4.3). Note that if $p = 2$, the p-Laplace diffusion is just the conventional linear Laplace diffusion studied by many previous literatures. And even if $p = 2$, the LMI-based criteria have advantages over some previous ones thanks to the less conservatism and higher computational efficiency (see Remark 4.3). The diffusion item plays an active role in judging the stability (see Remark 4.4). As pointed out in Remarks 3.1 and 4.3, Poincaré inequality and Hardy-Poincaré inequality are linked judiciously in judging the stability of reaction-diffusion neural networks for the first time so that Ω can be a spheroid and not a sphere in numerical examples. In addition, the feasibility of the LMI conditions of new criteria can be easily checked by the Matlab LMI toolbox. Examples 4.1–4.2 show that corollaries of the main results obtained in this paper are more feasible and effective than the main results of some recent related literatures (see Remarks 4.1–4.2). Finally, Example 4.3 illustrates that the allowable upper bound of time delays for Theorem 3.1 or Theorem 3.4 is far greater than that of any previous related literature (see Remark 4.5). All these numerical examples show the effectiveness and the less conservatism of all the proposed methods.

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References

[1] Cohen, M. and Grossberg, S., Absolute stability and global pattern formation and parallel memory storage by competitive neural networks, *IEEE Trans. Systems Man Cybernt.*, **13**, 1983, 815–826.

[2] Zhang, X., Wu, S. and Li, K., Delay-dependent exponential stability for impulsive Cohen-Grossberg neural networks with time-varying delays and reaction-diffusion terms, *Commun. Nonlinear Sci. Numer. Simulat.*, **16**, 2011, 1524–1532.

[3] Wang, X. R., Rao, R. F. and Zhong, S. M., LMI approach to stability analysis of Cohen-Grossberg neural networks with p-Laplace diffusion, *J. App. Math.*, 2012, 523812, 12 pages.

[4] Rong, L. B., Lu, W. L. and Chen, T. P., Global exponential stability in Hopfield and bidirectional associative memory neural networks with time delays, *Chin. Ann. Math. Ser. B*, **25**(2), 2004, 255–262.

[5] Rakkiyappan, R. and Balasubramaniam, P., Dynamic analysis of Markovian jumping impulsive stochastic Cohen-Grossberg neural networks with discrete interval and distributed time-varying delays, *Nonlinear Anal. Hybrid Syst.*, **3**, 2009, 408–417.

[6] Rao, R. F. and Pu, Z. L., Stability analysis for impulsive stochastic fuzzy p-Laplace dynamic equations under Neumann or Dirichlet boundary condition, *Bound. Value Probl.*, 2013, 2013:133, 14 pages.

[7] Balasubramaniam, P. and Rakkiyappan, R., Delay-dependent robust stability analysis for Markovian jumping stochastic Cohen-Grossberg neural networks with discrete interval and distributed time-varying delays, *Nonlinear Anal. Hybrid Syst.*, **3**, 2009, 207–214.

[8] Zhang, H. and Wang, Y., Stability analysis of Markovian jumping stochastic Cohen-Grossberg neural networks with mixed time delays, *IEEE Trans. Neural Networks*, **19**, 2008, 366–370.

[9] Song, Q. K. and Cao J. D., Stability analysis of Cohen-Grossberg neural network with both time-varying and continuously distributed delays, *J. Comp. Appl. Math.*, **197**, 2006, 188–203.

[10] Rao, R. F., Wang, X. R., Zhong, S. M. and Pu, Z. L., LMI approach to exponential stability and almost sure exponential stability for stochastic fuzzy Markovian jumping Cohen-Grossberg neural networks with nonlinear p-Laplace diffusion, *J. Appl. Math.*, 2013, 396903, 21 pages.

[11] Jiang, M., Shen, Y. and Liao, X., Boundedness and global exponential stability for generalized Cohen-Grossberg neural networks with variable delay, *Appl. Math. Comp.*, **172**, 2006, 379–393.

[12] Haykin, S., Neural Networks, Prentice-Hall, Upper Saddle River, NJ, USA, 1994.

[13] Zhu, Q. and Cao, J., Exponential stability of stochastic neural networks with both Markovian jump parameters and mixed time delays, *IEEE Trans. System, Man, and Cybernt.*, **41**, 2011, 341–353.

[14] Zhu, Q. and Cao, J., Stability analysis for stochastic neural networks of neutral type with both Markovian jump parameters and mixed time delays, *Neurocomp.*, **73**, 2010, 2671–2680.

[15] Zhu, Q., Yang, X. and Wang, H., Stochastically asymptotic stability of delayed recurrent neural networks with both Markovian jump parameters and nonlinear disturbances, *J. Franklin Inst.*, **347**, 2010, 1489–1510.

[16] Zhu, Q. and Cao, J., Stochastic stability of neural networks with both Markovian jump parameters and continuously distributed delays, *Discrete Dyn. Nat. Soc.*, 2009, 490515, 20 pages.

[17] Zhu, Q. and Cao, J., Robust exponential stability of Markovian jump impulsive stochastic Cohen-Grossberg neural networks with mixed time delays, *IEEE Trans. Neural Networks*, **21**, 2010, 1314–1325.

[18] Liang X. and Wang, L. S., Exponential stability for a class of stochastic reaction-diffusion Hopfield neural networks with delays, *J. Appl. Math.*, 2012, 693163, 12 pages.

[19] Zhang, Y. T., Asymptotic stability of impulsive reaction-diffusion cellular neural networks with time-varying delays, *J. Appl. Math.*, 2012, 501891, 17 pages.

[20] Abdelmalek, S., Invariant regions and global existence of solutions for reaction-diffusion systems with a tridiagonal matrix of diffusion coefficients and nonhomogeneous boundary conditions, *J. Appl. Math.*, 2007, 12375, 15 pages.

[21] Higham, D. J. and Sardar, T., Existence and stability of fixed points for a discretised nonlinear reaction-diffusion equation with delay, *Appl. Numer. Math.*, **18**, 1995, 155–173.

[22] Baranwal, V. K., Pandey, R. K., Tripathi, M. P. and Singh, O. P., An analytic algorithm for time fractional nonlinear reaction-diffusion equation based on a new iterative method, *Commun. Nonlinear Sci. Numer. Simul.*, **17**, 2012, 3906–3921.

[23] Meral, G. and Tezer-Sezgin, M., The comparison between the DRBEM and DQM solution of nonlinear reaction-diffusion equation, *Commun. Nonlinear Sci. Numer. Simul.*, **16**, 2011, 3990–4005.

[24] Liang, G., Blow-up and global solutions for nonlinear reaction-diffusion equations with nonlinear boundary condition, *Appl. Math. Comput.*, **218**, 2011, 3993–3999.

[25] Chen, H., Zhang, Y. and Zhao, Y., Stability analysis for uncertain neutral systems with discrete and distributed delays, *Appl. Math. Comput.*, **218**, 2012, 11351–11361.

[26] Sheng, L. and Yang, H., Novel global robust exponential stability criterion for uncertain BAM neural networks with time-varying delays, *Chaos, Sol.* & *Frac.*, **40**, 2009, 2102–2113.

[27] Tian, J. K., Li, Y., Zhao, J. and Zhong, S. M., Delay-dependent stochastic stability criteria for Markovian jumping neural networks with mode-dependent time-varying delays and partially known transition rates, *Appl. Math. Comput.*, **218**, 2012, 5769–5781.

[28] Rao, R. F., Zhong, S. M. and Wang, X. R., Delay-dependent exponential stability for Markovian jumping stochastic Cohen-Grossberg neural networks with p-Laplace diffusion and partially known transition rates via a differential inequality, *Adv. Diff. Equations*, 2013, 2013:183.

[29] Zhang, Y. and Luo, Q., Novel stability criteria for impulsive delayed reaction-diffusion Cohen-Grossberg neural networks via Hardy-Poincaré inequality, *Chaos, Sol.* & *Frac.*, **45**, 2012, 1033–1040.

[30] Zhang, Y. and Luo, Q., Global exponential stability of impulsive delayed reaction-diffusion neural networks via Hardy-Poincaré inequality, *Neurocomp.*, **83**, 2012, 198–204.

[31] Li, Y. and Zhao, K., Robust stability of delayed reaction-diffusion recurrent neural networks with Dirichlet boundary conditions on time scales, *Neurocomp.*, **74**, 2011, 1632–1637.

[32] Wang, K., Teng, Z. and Jiang, H., Global exponential synchronization in delayed reaction-diffusion cellular neural networks with the Dirichlet boundary conditions, *Math. Comp. Modelling*, **52**, 2010, 12–24.

[33] Wang, Z., Zhang, H. and Li, P., An LMI approach to stability analysis of reaction-diffusion Cohen-Grossberg neural networks concerning Dirichlet boundary conditions and distributed delays, *IEEE Trans. System, Man, and Cybern.*, **40**, 2010, 1596–1606.

[34] Wu, A. L. and Fu, C. J., Global exponential stability of non-autonomous FCNNs with Dirichlet boundary conditions and reaction-diffusion terms, *Appl. Math. Modelling*, **34**, 2010, 3022–3029.

[35] Pan, J. and Zhong, S. M., Dynamic analysis of stochastic reaction-diffusion Cohen-Grossberg neural networks with delays, *Adv. Diff. Equations*, 2009, 410823, 18 pages.

[36] Brezis, H. and Vazquez, J. L., Blow-up solutions of some nonlinear elliptic problems, *Rev. Mat. Univ. Comp. Mad.*, **10**, 1997, 443–469.

[37] Wang, Y., Xie, L. and de Souza, C. E., Robust control of a class of uncertain nonlinear system, *Systems Control Lett.*, **19**, 1992, 139–149.

[38] Kao, Y. G., Guo, J. F., Wang C. H. and Sun, X. Q., Delay-dependent robust exponential stability of Markovian jumping reaction-diffusion Cohen-Grossberg neural networks with mixed delays, *J. Franklin Inst.*, **349**(6), 2012, 1972–1988.

[39] Rakkiyappan, R. and Balasubramaniam, P., Dynamic analysis of Markovian jumping impulsive stochastic Cohen-Grossberg neural networks with discrete interval and distributed time-varying delays, *Nonlinear Anal. Hyb. Syst.*, **3**, 2009, 408–417.

[40] Rao, R. F., Pu, Z. L., Zhong, S. M. and Huang, J. L., On the role of diffusion behaviors in stability criterion for p-Laplace dynamical equations with infinite delay and partial fuzzy parameters under Dirichlet boundary value, *J. Appl. Math.*, 2013, 940845, 8 pages.

[41] Rao, R. F., Zhong, S. M. and Wang, X. R., Stochastic stability criteria with LMI conditions for Markovian jumping impulsive BAM neural networks with mode-dependent time-varying delays and nonlinear reaction-diffusion, *Commun. Nonlinear Sci. Numer. Simulat.*, **19**(1), 2014, 258–273.

[42] Rao, R. F. and Pu, Z. L., LMI-based stability criterion of impulsive TS fuzzy dynamic equations via fixed point theory, *Abstract and Applied Analysis*, 2013, 261353, 9 pages.

[43] Pu, Z. L. and Rao, R. F., Exponential robust stability of TS fuzzy stochastic p-Laplace PDEs under zero-boundary condition, *Bound. Value Probl.*, 2013, 2013: 264,14 pages.