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The Riemann Problem with Delta Data for Zero-Pressure Gas Dynamics*

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Abstract In this paper, the Riemann problem with delta initial data for the one-dimensional system of conservation laws of mass, momentum and energy in zero-pressure gas dynamics is considered. Under the generalized Rankine-Hugoniot conditions and the entropy condition, we constructively obtained the global existence of generalized solutions which contains delta-shock. Moreover, the author obtains the stability of generalized solutions by making use of the perturbation of the initial data.

Keywords Zero-pressure gas dynamics, Generalized Rankine-Hugoniot conditions, Delta-shock
2000 MR Subject Classification 35L65, 35L67

1 Introduction

In this paper, we are concerned with the one-dimensional system of conservation laws of mass, momentum and energy in zero-pressure gas dynamics characterized by

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2)_x = 0, \\ \Big(\rho \dfrac{u^2}{2} + H\Big)_t + \Big(\Big(\rho \dfrac{u^2}{2} + H\Big) u\Big)_x = 0, \end{cases} \tag{1.1}$$

where ρ and u represent the density and velocity, respectively, $H = \rho\tau$ is the internal energy, τ is the internal energy per unit mass. The zero-pressure gas dynamics system is a very important one to approach the full Euler equations, which can describe the motion of free particles which stick under collision and explain the formation of large-scale structures in the universe (see [1–2]). The zero-pressure gas dynamics system consisting of conservation laws of mass and momentum has been investigated extensively. For related results, see [3–8] and the papers cited therein. However, it is well known that for the media which can be considered as having no pressure, we must take into account energy transport. Therefore it is very necessary to consider the energy conservation law in zero-pressure gas dynamics (see [9–10]).

For the Riemann problem of system (1.1), it is not difficult to see that the delta shock and vacuum do occur (see [9]). In this paper, we will investigate the Riemann problem with delta initial data and possible interactions of delta shock waves and contact vacuum state for system

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(1.1). As for delta shock waves, we refer readers to [11–17] and the references cited therein for more details.

In this article, we first consider the Riemann problem for (1.1) with initial data

$$(\rho, u, H)(x, 0) = \begin{cases} (\rho_-, u_-, H_-), & x < -\epsilon, \\ \Big(\dfrac{\omega_0}{\epsilon}, u_0, \dfrac{h_0}{\epsilon}\Big), & -\epsilon < x < \epsilon, \\ (\rho_+, u_+, H_+), & x > \epsilon, \end{cases} \tag{1.2}$$

where $\epsilon > 0$ is sufficiently small. We constructively obtain the solutions for the problem (1.1)–(1.2). Moreover, a new kind of nonclassical wave is obtained, namely, a delta contact discontinuity, which is a Dirac delta function supported on a contact discontinuity. Let $\epsilon \to 0$, under the stability theory of weak solutions, we obtain solutions for the system (1.1) with delta initial data

$$(\rho, u, H)(x, 0) = \begin{cases} (\rho_-, u_-, H_-), & x < 0, \\ (\omega_0\delta, u_0, h_0\delta), & x = 0, \\ (\rho_+, u_+, H_+), & x > 0, \end{cases} \tag{1.3}$$

where δ is the standard Dirac delta function. With the help of generalized Rankine-Hugoniot conditions and the entropy condition, we obtain the global existence of generalized solutions for the problems (1.1) with (1.3). Moreover, if we let $\omega_0 = 0$, $u_0 = 0$ and $h_0 = 0$, the solutions of (1.1) with (1.3) correspond to the solutions of Riemann problem for (1.1). This method has been used in [18] to study the Riemann problem with delta initial data for the 1-D Chaplygin gas equations.

This paper is organized as follows. In Section 2, we recall and present some known results about the system (1.1) and its Riemann solution with constant initial data. In Section 3, we construct solutions of (1.1)–(1.2) case by case. Then letting $\epsilon \to 0$, we obtain the generalized solutions of (1.1) with (1.3).

2 The Riemann Problem with Constant Initial Data

In this section, we study solutions of system (1.1) by considering the Riemann problem with initial data

$$(\rho, u, H)(x, 0) = \begin{cases} (\rho_-, u_-, H_-), & x < 0, \\ (\rho_+, u_+, H_+), & x > 0. \end{cases} \tag{2.1}$$

The details can be found in [9]. The system (1.1) has a triple eigenvalue $\lambda = u$ and two right eigenvectors $r_1 = (1, 0, 0)^{\mathrm{T}}$ and $r_2 = (0, 0, 1)^{\mathrm{T}}$, which satisfy $\nabla\lambda \cdot r_i = 0,\ i = 1, 2$. Thus the system (1.1) is linearly degenerate. Furthermore, we obtain the Riemann solutions of (1.1) with (2.1) containing contact discontinuities, vacuum or delta shock wave.

For the case $u_- \le u_+$, the solution can be expressed as

$$(\rho, u, H)(\xi) = \begin{cases} (\rho_-, u_-, H_-), & -\infty < \xi < u_-, \\ (0, u(\xi), 0), & u_- \le \xi \le u_+, \\ (\rho_+, u_+, H_+), & u_+ < \xi < +\infty, \end{cases} \tag{2.2}$$

where $u(\xi)$ satisfies that $u(u_-) = u_-$ and $u(u_+) = u_+$ (see Figure 2.1).

For the case $u_- > u_+$, the overlap of the characteristic results in a δ-wave connecting the two states (ρ_-, u_-, H_-) and (ρ_+, u_+, H_+). The solution of (1.1) with (2.1) is

$$(\rho, u, H)(t,x) = \begin{cases} (\rho_-, u_-, H_-)(t,x), & x < x(t), \\ (\omega(t)\delta(x - x(t)), u_\delta(t), h(t)\delta(x - x(t))), & x = x(t), \\ (\rho_+, u_+, H_+)(t,x), & x > x(t), \end{cases} \tag{2.3}$$

where $x(t)$, $\omega(t)$, $h(t)$ and u_δ satisfy the generalized R-H conditions

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = u_\delta(t), \\ \dfrac{\mathrm{d}\omega(t)}{\mathrm{d}t} = [\rho]u_\delta(t) - [\rho u], \\ \dfrac{\mathrm{d}\omega(t)u_\delta(t)}{\mathrm{d}t} = [\rho u]u_\delta(t) - [\rho u^2], \\ \dfrac{\mathrm{d}\big(\omega(t)\frac{{u_\delta}^2(t)}{2} + h(t)\big)}{\mathrm{d}t} = \Big[\dfrac{\rho u^2}{2} + H\Big]u_\delta(t) - \Big[\Big(\dfrac{\rho u^2}{2} + H\Big)u\Big], \end{cases} \tag{2.4}$$

where $[\rho] = \rho_+ - \rho_-$, with initial data $x(0) = 0,\ \omega(0) = 0,\ h(0) = 0$. By simple calculation, we obtain

$$\begin{cases} x(t) = \dfrac{\sqrt{\rho_-}u_- + \sqrt{\rho_+}u_+}{\sqrt{\rho_-} + \sqrt{\rho_+}}t, \\ \omega(t) = \sqrt{\rho_-\rho_+}(u_- - u_+)t, \quad \text{if } \rho_- \neq \rho_+, \\ u_\delta = \dfrac{\sqrt{\rho_-}u_- + \sqrt{\rho_+}u_+}{\sqrt{\rho_-} + \sqrt{\rho_+}}, \\ h(t) = \dfrac{\rho_-\rho_+(u_- - u_+)^2 + 2(\sqrt{\rho_-} + \sqrt{\rho_+})(H_-\sqrt{\rho_+} + H_+\sqrt{\rho_-})}{2(\sqrt{\rho_-} + \sqrt{\rho_+})^2}(u_- - u_+)t \end{cases} \tag{2.5}$$

and

$$\begin{cases} x(t) = \dfrac{u_- + u_+}{2}t, \\ \omega(t) = \rho_-(u_- - u_+)t, \quad \text{if } \rho_- = \rho_+, \\ u_\delta = \dfrac{u_- + u_+}{2}, \\ h(t) = \dfrac{\rho_-(u_- - u_+)^2 + 4(H_- + H_+)}{8}(u_- - u_+)t. \end{cases} \tag{2.6}$$

Thus, we have obtained the solutions of system (1.1) with (2.1).

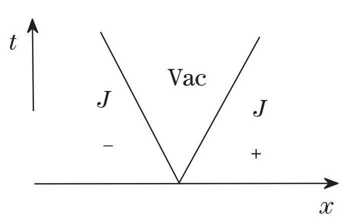

Figure 2.1 $u_- \leq u_+$

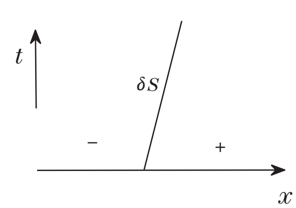

Figure 2.2 $u_- > u_+$

3 The Riemann Problem With Delta Initial Data

In this section, we first consider the solution to system (1.1) with (1.2). Then letting $\epsilon \to 0$, we get the solution of (1.1) with (1.3) under the stability theory of weak solutions. According to the relations among u_-, u_+ and u_0, we discuss the Riemann problem case by case.

Case 1 $u_- \le u_0 \le u_+$

In this case, when t is small enough, the solution of the initial value problem (1.1)–(1.2) can be expressed briefly as follows (see Figure 3.1(a)):

$$(\rho_-, u_-, H_-) + J^- + \text{Vac} + J_1^0 + \Big(\frac{\omega_0}{\epsilon}, u_0, \frac{h_0}{\epsilon}\Big) + J_2^0 + \text{Vac} + J^+ + (\rho_+, u_+, H_+),$$

where "+" means "followed by". The propagation speed of the J_1^0 and J_2^0 is u_0. Thus we see that the J_1^0 can not overtake J_2^0 at a finite time. So far, the solutions of (1.1) with (1.2) have been constructed completely. Letting $\epsilon \to 0$, we obtain a solution of (1.1) with (1.3) as follows (see Figure 3.1(b)):

$$(\rho_-, u_-, H_-) + J^- + \text{Vac} + \delta S + \text{Vac} + J^+ + (\rho_+, u_+, H_+),$$

where the propagation speed of the δS is u_0.

If $\omega_0 = 0$, $u_0 = 0$ and $h_0 = 0$, it is easy to see that the solution of (1.1) with (1.3) is consistent with the Riemann solutions of (1.1) with (2.1), which implies that the solution is stable with respect to the perturbation in this case.

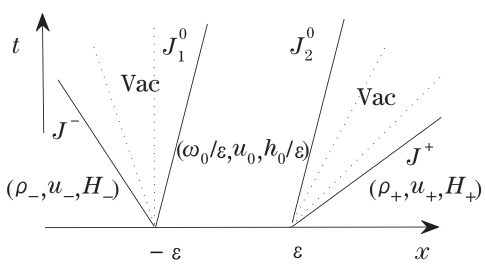

Figure 3.1 (a)

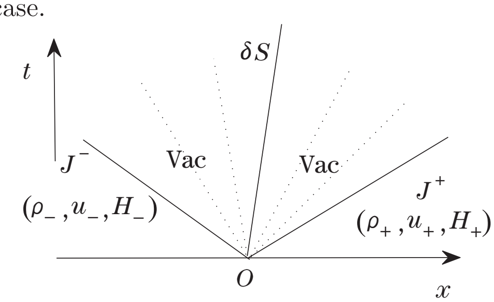

Figure 3.1 (b)

Case 2 $u_0 < u_- < u_+$ (if $u_- < u_+ < u_0$, then the structure of the solution is similar)

Similarly to the analysis in Case 1, we seek the solution of the following form (see Figure 3.2):

$$(\rho, u, H)(x,t) = \begin{cases} (\rho_-, u_-, H_-)(t,x), & x < x(t), \\ (\omega(t)\delta(x - x(t)), u_\delta(t), h(t)\delta(x - x(t))), & x = x(t), \\ (0, u_\delta(t), 0)(t,x), & x(t) < x < u_+ t, \\ (\rho_+, u_+, H_+)(t,x), & x > u_+ t \end{cases} \tag{3.1}$$

and δS satisfies the following generalized Rankine-Hugoniot conditions:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = u_\delta(t), \\ \dfrac{\mathrm{d}\omega(t)}{\mathrm{d}t} = [\rho] u_\delta(t) - [\rho u], \\ \dfrac{\mathrm{d}\omega(t) u_\delta(t)}{\mathrm{d}t} = [\rho u] u_\delta(t) - [\rho u^2], \\ \dfrac{\mathrm{d}\big(\omega(t)\frac{u_\delta{}^2(t)}{2} + h(t)\big)}{\mathrm{d}t} = \Big[\rho \dfrac{u^2}{2} + H\Big] u_\delta(t) - \Big[\Big(\rho \dfrac{u^2}{2} + H\Big) u\Big], \end{cases} \tag{3.2}$$

where $[\rho] = 0 - \rho_-$, with initial data

$$(x, \omega, u_\delta, h)(0) = (0, \omega_0, u_0, h_0). \tag{3.3}$$

For more details, we refer readers to [18–19] and the references cited therein.

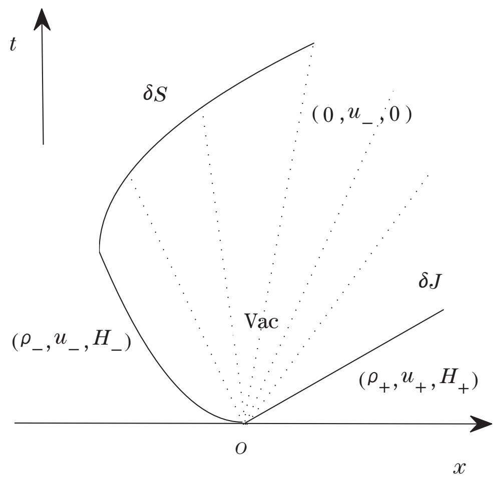

Figure 3.2

Next, we only need to solve the initial value problem (3.2) with (3.3). From (3.2), we have

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\rho_- u_\delta + \rho_- u_-, \tag{3.4}$$

$$\frac{\mathrm{d}wu_\delta}{\mathrm{d}t} = -\rho_- u_- u_\delta + \rho_- u_-^2 \tag{3.5}$$

and

$$\frac{\mathrm{d}w\frac{u_\delta^2}{2} + h}{\mathrm{d}t} = -\Big(\frac{\rho_- u_-^2}{2} + H_-\Big)u_\delta + \Big(\frac{\rho_- u_-^2}{2}u_- + H_- u_-\Big). \tag{3.6}$$

By virtue of (3.4)–(3.5), we have

$$w\mathrm{d}w = \omega_0\rho_-(u_- - u_0)\mathrm{d}t. \tag{3.7}$$

Solving (3.7) with $\omega(0) = \omega_0$, we have

$$w(t) = \sqrt{\omega_0^2 + 2\omega_0\rho_-(u_- - u_0)t}. \tag{3.8}$$

From (3.4) and (3.7), we have

$$u_\delta = u_- - \frac{\omega_0(u_- - u_0)}{\sqrt{\omega_0^2 + 2\omega_0\rho_-(u_- - u_0)t}}. \tag{3.9}$$

Using (3.5)–(3.6) and (3.8)–(3.9), we obtain

$$h(t) = h(0) + \frac{H_-}{\rho_-}\sqrt{\omega_0^2 + 2\omega_0\rho_-(u_- - u_0)t} - \frac{\omega_0^2(u_- - u_0)}{2\sqrt{\omega_0^2 + 2\omega_0\rho_-(u_- - u_0)t}}. \tag{3.10}$$

Remark 3.1 From (3.9), we have

$$\lim_{t\to\infty} u_\delta \to u_- < u_+. \tag{3.11}$$

This implies that δS does not penetrate vacuum completely and it converts to δJ when $t \to \infty$.

Remark 3.2 From (3.4), (3.6) and (3.8), if $\omega_0 = 0$, $u_0 = 0$, $h_0 = 0$, then $(\omega, u, h) = (0, u_-, 0)$. This is consistent with the results of the Riemann problem (1.1) with (2.1). It implies that the solution constructed here is stable under some perturbations.

Case 3 $u_+ < u_0 < u_-$

Similarly to the analysis in Case 1, we seek the solution of the following form (see Figure 3.3):

$$(\rho, u, H)(x,t) = \begin{cases} (\rho_-, u_-, H_-)(t,x), & x < x(t), \\ (\omega(t)\delta(x - x(t)), u_\delta(t), h(t)\delta(x - x(t))), & x = x(t), \\ (\rho_+, u_+, H_+)(t,x), & x > x(t) \end{cases} \tag{3.12}$$

and δS satisfies the following generalized Rankine-Hugoniot conditions:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = u_\delta(t), \\ \dfrac{\mathrm{d}\omega(t)}{\mathrm{d}t} = [\rho]u_\delta(t) - [\rho u], \\ \dfrac{\mathrm{d}\omega(t)u_\delta(t)}{\mathrm{d}t} = [\rho u]u_\delta(t) - [\rho u^2], \\ \dfrac{\mathrm{d}\big(\omega(t)\frac{u_\delta{}^2(t)}{2} + h(t)\big)}{\mathrm{d}t} = \Big[\rho\dfrac{u^2}{2} + H\Big]u_\delta(t) - \Big[\Big(\rho\dfrac{u^2}{2} + H\Big)u\Big], \end{cases} \tag{3.13}$$

where $[\rho] = \rho_+ - \rho_-$, with initial data

$$(x, \omega, u_\delta, h)(0) = (0, \omega_0, u_0, h_0). \tag{3.14}$$

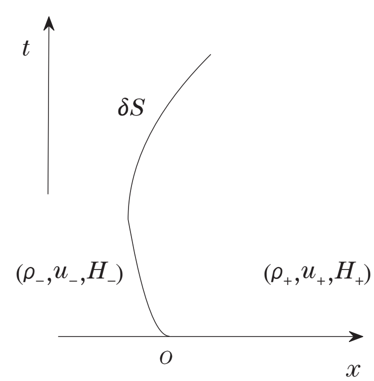

Figure 3.3

Now, we are going to solve the initial value problem (3.13) with (3.14).

Integrating (3.13) from 0 to t with initial data (3.14), we have

$$\begin{cases} \omega - \omega_0 = [\rho]x - [\rho u]t, \\ \omega u_\delta - \omega_0 u_0 = [\rho u]x - [\rho u^2]t, \\ \dfrac{\omega u_\delta^2}{2} - \dfrac{\omega_0 u_0^2}{2} + h - h_0 = \Big[\dfrac{\rho u^2}{2} + H\Big]x - \Big[\dfrac{\rho u^2}{2}u + Hu\Big]t. \end{cases} \tag{3.15}$$

Canceling ω in the first and second equation of (3.5), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big\{\frac{1}{2}[\rho]x^2+(\omega_0-[\rho u]t)x+\frac{1}{2}[\rho u^2]t^2-\omega_0u_0t\Big\}=0. \tag{3.16}$$

Integrating (3.16) from 0 to t, we have

$$\frac{1}{2}[\rho]x^2+(\omega_0-[\rho u]t)x+\frac{1}{2}[\rho u^2]t^2-\omega_0u_0t=0. \tag{3.17}$$

Solving (3.17), we get

$$x(t)=\begin{cases}\dfrac{\omega_0u_0t-\frac{1}{2}[\rho u^2]t^2}{\omega_0-[\rho u]t}, & [\rho]=0,\\ \dfrac{-\omega_0+[\rho u]t+\omega(t)}{[\rho]}, & [\rho]\neq 0\end{cases} \tag{3.18}$$

and

$$\omega(t)=\sqrt{\rho_-\rho_+[u]^2t^2+2\omega_0([\rho]u_0-[\rho u])t+\omega_0^2}. \tag{3.19}$$

From (3.15), we have

$$u_\delta=\frac{[\rho u]x-[\rho u^2]t+\omega_0u_0}{w(t)} \tag{3.20}$$

and

$$h(t)=h_0+\Big[\frac{\rho u^2}{2}+H\Big]x-\Big[\frac{\rho u^2}{2}u+Hu\Big]t+\frac{\omega_0u_0^2}{2}-\frac{\omega u_\delta^2}{2}. \tag{3.21}$$

Remark 3.3 It is seen that

$$\lim_{t\to\infty}u_\delta(t)=\begin{cases}\dfrac{u_++u_-}{2}, & [\rho]=0,\\ \dfrac{\sqrt{\rho_-}u_-+\sqrt{\rho_+}u_+}{\sqrt{\rho_-}+\sqrt{\rho_+}}, & [\rho]\neq 0.\end{cases} \tag{3.22}$$

So from (3.22), we have

$$u_+<\lim_{t\to\infty}u_\delta(t)<u_-. \tag{3.23}$$

Remark 3.4 If $\omega_0=0$, $u_0=0$, $h_0=0$, then

$$\begin{aligned}&(x,\omega,u_\delta,h)\\ &=\begin{cases}\Big(\dfrac{u_++u_-}{2}t,\rho_-(u_--u_+)t,\dfrac{u_-+u_+}{2},\dfrac{\rho_-(u_--u_+)^2+4(H_-+H_+)}{8}(u_--u_+)t\Big), & [\rho]=0,\\ \Big(\dfrac{\sqrt{\rho_-}u_-+\sqrt{\rho_+}u_+}{\sqrt{\rho_-}+\sqrt{\rho_+}}t,\sqrt{\rho_-\rho_+}(u_--u_+)t,\dfrac{\sqrt{\rho_-}u_-+\sqrt{\rho_+}u_+}{\sqrt{\rho_-}+\sqrt{\rho_+}}, & \\ \dfrac{\rho_-\rho_+(u_--u_+)^2+2(\sqrt{\rho_-}+\sqrt{\rho_+})(H_-\sqrt{\rho_+}+H_+\sqrt{\rho_-})}{2(\sqrt{\rho_-}+\sqrt{\rho_+})^2}(u_--u_+)t\Big), & [\rho]\neq 0.\end{cases}\end{aligned} \tag{3.24}$$

This is consistent with the results of the Riemann problems (1.1) with (2.1). It implies that the solution constructed here is stable under some perturbations.

Case 4 $u_0 < u_+ < u_-$ (if $u_+ < u_- < u_0$, then the structure of the solution is similar)

Similarly to the analysis in Case 2, we know that, in this case, when t is small enough, the solution is the same as that in Case 2. More precisely, according to (3.9), there exists a unique t_1, such that $u_\delta(t_1) = u_+$.

When $0 \le t \le t_1$, the solution is the same as that in Case 2. When $t > t_1$, the δ-shock wave will overtake J^+ in finite time $t = t_2$, and we seek the solution of the following form (see Figure 3.4):

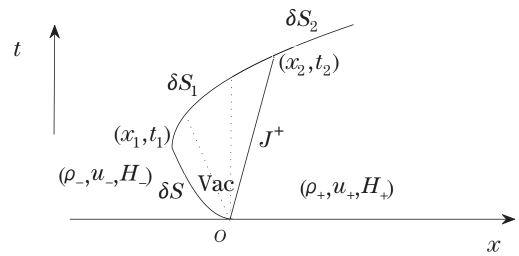

Figure 3.4

For $0 \le t \le t_1$, the solution is the same as that in Case 2, and here we omit the details.

For $t_1 \le t \le t_2$,

$$(\rho, u, H)(x,t) = \begin{cases} (\rho_-, u_-, H_-)(t,x), & x < x^1(t), \\ (\omega^1(t)\delta(x - x^1(t)), u_\delta^1(t), h^1(t)\delta(x - x^1(t))), & x = x^1(t), \\ (0, u_\delta^1(t), 0)(t,x), & x^1(t) < x < u_+ t, \\ (\rho_+, u_+, H_+)(t,x), & x > u_+ t, \end{cases} \tag{3.25}$$

and δS_1 satisfies the following generalized Rankine-Hugoniot conditions:

$$\begin{cases} \dfrac{\mathrm{d}x^1(t)}{\mathrm{d}t} = u_\delta^1(t), \\ \dfrac{\mathrm{d}\omega^1(t)}{\mathrm{d}t} = [\rho] u_\delta^1(t) - [\rho u], \\ \dfrac{\mathrm{d}\omega^1(t) u_\delta^1(t)}{\mathrm{d}t} = [\rho u] u_\delta^1(t) - [\rho u^2], \\ \dfrac{\mathrm{d}\big(\omega^1(t) \frac{{u_\delta^1}^2(t)}{2} + h^1(t)\big)}{\mathrm{d}t} = \Big[\rho \dfrac{u^2}{2} + H\Big] u_\delta^1(t) - \Big[\Big(\rho \dfrac{u^2}{2} + H\Big) u\Big], \end{cases} \tag{3.26}$$

where $[\rho] = 0 - \rho_-$, with initial data

$$(x^1, \omega^1, u_\delta^1, h^1)(0) = (x_1, \omega_1, u_1, h_1), \tag{3.27}$$

where $x_1 = x(t_1)$, $\omega_1 = \omega(t_1)$, $u_1 = u(t_1)$ and $h_1 = h(t_1)$. The solution of (3.26)–(3.27) $(x^1, \omega^1, u_\delta^1, h^1)(t)$ can be obtained similarly as in Case 2.

For $t > t_2$, where t_2 is determined by $x^1(t_2) = u_+ t_2$,

$$(\rho, u, H)(x,t) = \begin{cases} (\rho_-, u_-, H_-)(t,x), & x < x^{\mathbf{2}}(t), \\ (\omega^{\mathbf{2}}(t)\delta(x - x^{\mathbf{2}}(t)), u_\delta^{\mathbf{2}}(t), h^{\mathbf{2}}(t)\delta(x - x^{\mathbf{2}}(t))), & x = x^{\mathbf{2}}(t), \\ (\rho_+, u_+, H_+)(t,x), & x > x^{\mathbf{2}}(t), \end{cases} \tag{3.28}$$

and δS_2 satisfies the following generalized Rankine-Hugoniot conditions

$$\begin{cases} \dfrac{\mathrm{d}x^{\mathbf{2}}(t)}{\mathrm{d}t} = u_\delta^{\mathbf{2}}(t), \\ \dfrac{\mathrm{d}\omega^{\mathbf{2}}(t)}{\mathrm{d}t} = [\rho]u_\delta^{\mathbf{2}}(t) - [\rho u], \\ \dfrac{\mathrm{d}\omega^{\mathbf{2}}(t)u_\delta^{\mathbf{2}}(t)}{\mathrm{d}t} = [\rho u]u_\delta^{\mathbf{2}}(t) - [\rho u^2], \\ \dfrac{\mathrm{d}(\omega^{\mathbf{2}}(t)\frac{u_\delta^{\mathbf{2}\,2}(t)}{2} + h^{\mathbf{2}}(t))}{\mathrm{d}t} = \Big[\rho\dfrac{u^2}{2} + H\Big]u_\delta^{\mathbf{2}}(t) - \Big[\Big(\rho\dfrac{u^2}{2} + H\Big)u\Big], \end{cases} \tag{3.29}$$

where $[\rho] = \rho_+ - \rho_-$, with initial data

$$(x^{\mathbf{2}}, \omega^{\mathbf{2}}, u_\delta^{\mathbf{2}}, h^{\mathbf{2}})(0) = (x_2, \omega_2, u_2, h_2), \tag{3.30}$$

where $x_2 = x^1(t_2),\ \omega_2 = \omega^1(t_2),\ u_2 = u^1(t_2)$ and $h_2 = h^1(t_2)$. The solution of (3.29)–(3.30) $(x^{\mathbf{2}}, \omega^{\mathbf{2}}, u_\delta^{\mathbf{2}}, h^{\mathbf{2}})(t)$ can be obtained similarly as in Case 3.

So far, we have finished the discussion for all kinds of interactions. We summarize our results in the following.

Theorem 3.1 *The solution of* (1.1) *with* (1.3) *can be obtained by letting* $\epsilon \to 0$ *for the solutions of* (1.1) *with* (1.2). *If we let* $\omega_0 = 0,\ u_0 = 0,\ h_0 = 0$, *the solution of* (1.1) *with* (1.3) *corresponds exactly to the solution of Riemann problem* (1.1) *with* (2.1). *It implies that the solution constructed here is stable under some perturbation.*

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