Chin. Ann. Math. Ser. B
40(3), 2019, 331–338
DOI: 10.1007/s11401-019-0136-6

Chinese Annals of Mathematics, Series B

Meromorphic Function Sharing Sets with Its Difference Operator or Shifts*

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Abstract Let f be a nonconstant meromorphic function, $c \in \mathbb{C}$, and let $a(z)(\not\equiv 0) \in S(f)$ be a meromorphic function. If $f(z)$ and $P(z, f(z))$ share the sets $\{a(z), -a(z)\}$, $\{0\}$ CM almost and share $\{\infty\}$ IM almost, where $P(z, f(z))$ is defined as (1.1), then $f(z) \equiv \pm P(z, f(z))$ or $f(z)P(z, f(z)) \equiv \pm a^2(z)$. This extends the results due to Chen and Chen (2013), Liu (2009) and Yi (1987).

Keywords Meromorphic function, Difference operator, Shared sets
2010 MR Subject Classification 30D35

1 Introduction

In this paper, a meromorphic function always means meromorphic in the whole complex plane, and we assume that the reader is familiar with Nevanlinna theory of meromorphic functions. For a meromorphic function $f(z)$, we denote by $S(f)$ the set of all meromorphic functions $a(z)$ such that $T(r, a) = o(T(r, f))$ for all r outside of a set with finite logarithmic measure (see [6, 8]).

For a meromorphic function f and a set $S \subseteq \mathbb{C}$, we define

$$E_f(S) = \bigcup_{a\in S}\{z \mid f(z) - a = 0, \text{ counting multiplicities}\},$$

$$\overline{E}_f(S) = \bigcup_{a\in S}\{z \mid f(z) - a = 0, \text{ ignoring multiplicities}\}.$$

If $E_f(S) = E_g(S)$, then we say that f and g share S CM.

If $\overline{E}_f(S) = \overline{E}_g(S)$, then we say that f and g share S IM.

Let $a(z)$ be a common small function of both $f(z)$ and $g(z)$, and set $N(r, a)$ be a counting function of both zeros of $f(z) - a(z)$ and $g(z) - a(z)$ with same multiplicity. If

$$N\Big(r, \frac{1}{f-a}\Big) + N\Big(r, \frac{1}{g-a}\Big) - 2N(r, a) = S(r, f) + S(r, g),$$

then we call that $f(z)$ and $g(z)$ share $a(z)$ CM almost (see [3]).

Manuscript received May 23, 2016, Revised December 23, 2016.
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*This work was supported by the National Natural Science Foundation of China (No. 11701188).

Set $\overline{N}(r,a)$ be a counting function of both zeros of $f(z)-a(z)$ and $g(z)-a(z)$ ignoring multiplicity. If

$$\overline{N}\Big(r,\frac{1}{f-a}\Big)+\overline{N}\Big(r,\frac{1}{g-a}\Big)-2\overline{N}(r,a)=S(r,f)+S(r,g),$$

then we call that $f(z)$ and $g(z)$ share $a(z)$ IM almost (see [3]).

Specially, $N(r,1)$ ($\overline{N}(r,1)$) denote the counting function of both zeros of $f(z)-1$ and $g(z)-1$ with same multiplicity (ignoring multiplicity).

For a meromorphic function $f(z)$, $c\in\mathbb{C}$, we denote its shift and difference operator by $f(z+c)$ and $\Delta_c f := f(z+c)-f(z)$, respectively.

The classical results in the uniqueness theory of meromorphic functions are the five values and four values theorems due to Nevanlinna (see [6, 8]). Corresponding to sharing sets, Gross and Osgood [4] obtained the following result.

Theorem 1.1 *Let f and g be two nonconstant entire functions of finite order. If f and g share the sets $\{1,-1\}$ and $\{0\}$ CM, then $f\equiv\pm g$ or $fg\equiv\pm1$.*

In 1987, Yi [9] improved Theorem 1.1 as follows.

Theorem 1.2 *Let f and g be two nonconstant meromorphic functions. If f and g share the sets $\{1,-1\}$, $\{0\}$ and $\{\infty\}$ CM, then $f\equiv\pm g$ or $fg\equiv\pm1$.*

Recently, a number of papers (including [1, 2, 5, 7, 10]) have focused on value distribution of difference analogues of meromorphic functions. Liu [7] investigated the cases that $f(z)$ shares sets with its shift $f(z+c)$ or difference operator $\Delta_c f := f(z+c)-f(z)$, and proved the following result.

Theorem 1.3 *Let f be a nonconstant entire function of finite order, $c\in\mathbb{C}$, and let $a(z)\in S(f)$ be a non-vanishing periodic entire function with period c. If $f(z)$ and $f(z+c)$ share the sets $\{a(z),-a(z)\}$ and $\{0\}$ CM, then $f(z)\equiv\pm f(z+c)$.*

In 2013, Chen and Chen [1] extended Theorem 1.3 as follows.

Theorem 1.4 *Let f be a nonconstant entire function of finite order, $c\in\mathbb{C}$, let $a(z)\in S(f)$ be a non-vanishing periodic entire function with period c, and let*

$$P(z,f(z))=b_k(z)f(z+kc)+\cdots+b_1(z)f(z+c)+b_0(z)f(z), \tag{1.1}$$

where $b_k(z)\not\equiv0,\ b_0(z),\cdots,b_k(z)\in S(f)$ and k is a nonnegative integer. If $f(z)$ and $P(z,f(z))$ share the sets $\{a(z),-a(z)\}$ and $\{0\}$ CM, then $f(z)\equiv\pm P(z,f(z))$.

Now one may ask the following questions which are the motivation of the paper:

(I) In Theorem 1.2, can 3CM be replaced by 2CM + 1IM?

(II) In Theorems 1.3–1.4, is the condition "$f(z)$ has finite order" necessary?

(III) What will happen in Theorems 1.3–1.4 if $f(z)$ is a meromorphic function?

(IV) In Theorems 1.3–1.4, can the condition "$a(z)\in S(f)$ be a non-vanishing periodic entire function with period c" be replaced by "$a(z)\in S(f)$"?

In this paper we investigate the above problems, and prove the following results.

Theorem 1.5 *Let f and g be two nonconstant meromorphic functions, $c \in \mathbb{C}$, and let $a(z)(\not\equiv 0)$ be a common small function related to f and g. If $f(z)$ and $g(z)$ share the sets $\{a(z), -a(z)\}$, $\{0\}$ CM almost and share $\{\infty\}$ IM almost, then $f(z) \equiv \pm g(z)$ or $f(z)g(z) \equiv \pm a^2(z)$.*

With Theorem 1.5, it is easy to get Theorem 1.6.

Theorem 1.6 *Let f be a nonconstant meromorphic function, $c \in \mathbb{C}$, and let $a(z)(\not\equiv 0) \in S(f)$ be a meromorphic function. If $f(z)$ and $P(z, f(z))$ share the sets $\{a(z), -a(z)\}$, $\{0\}$ CM almost and share $\{\infty\}$ IM almost, where $P(z, f(z))$ is defined as* (1.1), *then $f(z) \equiv \pm P(z, f(z))$ or $f(z)P(z, f(z)) \equiv \pm a^2(z)$.*

From Theorem 1.6, we have the corollary as follows.

Corollary 1.1 *Let f be a nonconstant entire function, $c \in \mathbb{C}$, and let $a(z)(\not\equiv 0) \in S(f)$ be a meromorphic function. If $f(z)$ and $P(z, f(z))$ share the sets $\{a(z), -a(z)\}$, $\{0\}$ CM almost, where $P(z, f(z))$ is defined as* (1.1), *then $f(z) \equiv \pm P(z, f(z))$ or $f(z)P(z, f(z)) \equiv \pm a^2(z)$.*

For the meromorphic function share three sets with its shift, we obtain the following result.

Theorem 1.7 *Let f be a nonconstant meromorphic function, $c \in \mathbb{C}$. If $f(z)$ and $\Delta_c f$ share the sets $\{1, -1\}$, $\{0\}$ CM and share $\{\infty\}$ IM almost, then $f(z + c) \equiv 2f(z)$.*

For the meromorphic function with finite order, we prove the following result.

Theorem 1.8 *Let f be a nonconstant meromorphic function of finite order, $c \in \mathbb{C}$, and let $a(z)(\not\equiv 0) \in S(f)$ be a meromorphic function. If $f(z)$ and $P(z, f(z))$ share the sets $\{a(z), -a(z)\}$, $\{0\}$ CM almost and share $\{\infty\}$ IM almost, where $P(z, f(z))$ is defined as* (1.1), *then $f(z) \equiv \pm P(z, f(z))$.*

From Theorem 1.8, we can deduce Theorems 1.3–1.4 immediately.

Example 1.1 Let $f(z) = \mathrm{e}^{\mathrm{e}^z}$, and $P(z, f(z)) = f(z + \pi\mathrm{i})$, $a(z) \equiv 1$, then $P(z, f(z)) = \mathrm{e}^{-\mathrm{e}^z}$. Obviously $f(z)$ and $P(z, f(z))$ share the sets $\{a(z), -a(z)\}$, $\{0\}$ CM almost and share $\{\infty\}$ IM almost, and $f(z)P(z, f(z)) \equiv 1$. Thus, the case "$f(z)P(z, f(z)) \equiv \pm a^2(z)$" in Theorem 1.6 can not be deleted.

2 Some Lemmas

For the proof of our results, we need the following results.

Lemma 2.1 *Let $F(z)$ and $G(z)$ be two nonconstant meromorphic functions with $\overline{N}(r, F) = S(r, F)$. Supposed that $F(z)$, $G(z)$ share $0, 1$ CM almost, and share ∞ IM almost. If*

$$N(r, 1) - \overline{N}(r, 1) \neq S(r, F) + S(r, G),$$

then $F(z) \equiv G(z)$.

Proof Let

$$\phi(z) = \frac{F'}{F} - \frac{G'}{G}. \tag{2.1}$$

If $\phi(z) \not\equiv 0$, then $m(r,\phi) = S(r,F) + S(r,G)$. Since $F(z)$, $G(z)$ share $0,1$ CM almost, share ∞ IM almost, and $\overline{N}(r,F) = S(r,F)$, we get $N(r,\phi) \le \overline{N}(r,F) + S(r,F) + S(r,G) \le S(r,F) + S(r,G)$. So from (2.1) we have

$$N(r,1) - \overline{N}(r,1) \le N\Big(r,\frac{1}{\phi}\Big) \le T(r,\phi) + O(1) \le S(r,F) + S(r,G),$$

a contradiction. Thus $\phi(z) \equiv 0$. From (2.1), it is easy to obtain $F(z) \equiv cG(z)$, where c is a constant. Since $N(r,1) - \overline{N}(r,1) \ne S(r,F) + S(r,G)$, there exists z_0 such that $F(z_0) = G(z_0) = 1$. So $c = 1$, that is $F(z) \equiv G(z)$.

This completed the proof of Lemma 2.1.

Lemma 2.2 *Let $F(z)$ and $G(z)$ be two nonconstant meromorphic functions with $\overline{N}(r,F) = S(r,F)$. Suppose that $F(z)$, $G(z)$ share $0,1$ CM almost, and share ∞ IM almost. If $F(z)$ is not a Mobius transformation of $G(z)$, then*

$$N(r,1) \le N\Big(r,\frac{1}{F'}\Big) + N\Big(r,\frac{1}{G'}\Big) + S(r,F) + S(r,G). \tag{2.2}$$

Proof By Lemma 2.1, we have

$$N(r,1) - \overline{N}(r,1) = S(r,F) + S(r,G). \tag{2.3}$$

Set

$$\varphi(z) = \frac{F''}{F'} - 2\frac{F'}{F-1} - \frac{G''}{G'} + 2\frac{G'}{G-1}. \tag{2.4}$$

If $\varphi(z) \equiv 0$, then from (2.4), we know that $F(z)$ is a Mobius transformation of $G(z)$, a contradiction. Thus $\varphi(z) \not\equiv 0$. From (2.3)–(2.4) and the fact that $F(z)$, $G(z)$ share $0,1$ CM almost, share ∞ IM almost, we obtain

$$\begin{aligned}
N(r,1) &\le N_{1)}(r,1) + S(r,F) + S(r,G)\\
&\le N\Big(r,\frac{1}{\varphi}\Big) + S(r,F) + S(r,G)\\
&\le T(r,\varphi) + S(r,F) + S(r,G) + O(1)\\
&\le N(r,\varphi) + S(r,F) + S(r,G)\\
&\le N\Big(r,\frac{1}{F'}\Big) + N\Big(r,\frac{1}{G'}\Big) + S(r,F) + S(r,G),
\end{aligned}$$

where $N_{1)}(r,1)$ denotes the counting function of both simple zeros of $F(z)-1$ and $G(z)-1$.

This completed the proof of Lemma 2.2.

Lemma 2.3 (see [5]) *Let $f(z)$ be a non-constant meromorphic function of finite order, $c \in \mathbb{C}$, then*

$$m\Big(r,\frac{f(z+c)}{f(z)}\Big) = o\{T(r,f)\}$$

for all r outside of a possible exceptional set E with finite logarithmic measure.

3 Proof of Theorem 1.5

Obviously $f^2(z)$ and $g^2(z)$ share 0, $a^2(z)$ CM almost, and share ∞ IM almost. Set $F(z) = \frac{f^2(z)}{a^2(z)}$ and $G(z) = \frac{G^2(z)}{a^2(z)}$. Then $F(z)$ and $G(z)$ share $0, 1$ CM almost, and share ∞ IM almost. Thus

$$T(r, F) = O(T(r, G)), \quad T(r, G) = O(T(r, F)).$$

So $S(r, F) = S(r, G)$. Define $S(r) = S(r, F) + S(r, G)$.

Obviously $F(z)$ and $G(z)$ have no simple zeros and poles.

Now we assume that both $F(z) \not\equiv G(z)$ and $F(z)G(z) \not\equiv 1$. We claim that

$$N\Big(r, \frac{1}{F}\Big) + \overline{N}(r, F) = S(r), \quad N\Big(r, \frac{1}{G}\Big) + \overline{N}(r, G) = S(r).$$

Firstly, we prove $\overline{N}(r, F) = \overline{N}(r, G) = S(r)$. Set

$$\varphi(z) = \frac{F(G-1)}{G(F-1)}. \tag{3.1}$$

Since $F(z)$ and $G(z)$ share $0, 1$ CM almost, and share ∞ IM, we get

$$N(r, \varphi) + N\Big(r, \frac{1}{\varphi}\Big) = S(r). \tag{3.2}$$

From (3.1) we get

$$G(z) - F(z) = (\varphi(z) - 1)G(z)(F(z) - 1). \tag{3.3}$$

From (3.3) and the fact that F and G share ∞ IM almost, we obtain that all the multiple poles of F and G must be the multiple zeros of $\varphi - 1$. Noting that F and G have no simple pole, if $\varphi' \not\equiv 0$, we get

$$\overline{N}(r, F) = \overline{N}_{(2}(r, F) \leq 2N\Big(r, \frac{1}{\varphi'}\Big) + S(r) \leq 2\Big\{N\Big(r, \frac{1}{\varphi}\Big) + \overline{N}(r, \varphi)\Big\} + S(r). \tag{3.4}$$

Combining (3.2) and (3.4), we get $\overline{N}(r, F) = S(r)$.

If $\varphi'(z) \equiv 0$, then $\varphi(z) \equiv c$, where c is a constant. If $c = 1$, form (3.1) we get $F(z) \equiv G(z)$, a contradiction. If $c \neq 1$, then it follows from (3.3) that $N(r, F) = S(r)$.

Since F and G share ∞ IM almost, $\overline{N}(r, F) = \overline{N}(r, G) = S(r)$.

Next, we prove $N\big(r, \frac{1}{F}\big) = N\big(r, \frac{1}{G}\big) = S(r)$. Let

$$\phi(z) = \frac{F'(z)}{F(z) - 1} - \frac{G'(z)}{G(z) - 1}. \tag{3.5}$$

If $\phi(z) \equiv 0$, then we get $F(z) \equiv G(z)$, a contradiction.

If $\phi(z) \not\equiv 0$, then $m(r, \phi) = S(r, F)$. Since $F(z)$ and $G(z)$ share $0, 1$ CM almost, and share ∞ IM almost, from (3.5), we know that the pole of $\varphi(z)$ must be the pole of $F(z)$ and all the poles of $\phi(z)$ are simple. Thus $N(r, \phi) \leq \overline{N}(r, F) = S(r)$. So $T(r, \phi) = m(r, \phi) + N(r, \phi) = S(r)$. From (3.5) we also get that the multiple pole of $F(z)$ must be the zeros of $\phi(z)$. Since $F(z)$ have no simple zero,

$$N\Big(r, \frac{1}{F}\Big) = N_{(2}\Big(r, \frac{1}{F}\Big) \leq 2N\Big(r, \frac{1}{\phi}\Big) \leq 2T(r, \phi) + O(1) \leq S(r).$$

Since F and G share 0 CM almost, $N\big(r,\frac{1}{F}\big)=N\big(r,\frac{1}{G}\big)=S(r)$. Thus the claim is proved.

If $F(z)$ is not a Mobius transformation of $G(z)$, then from Lemma 2.2, we get

$$N\Big(r,\frac{1}{F-1}\Big)=N(r,1)+O(1)\le\overline{N}\Big(r,\frac{1}{F'}\Big)+\overline{N}(r,\frac{1}{G'})+S(r) \tag{3.6}$$

and

$$N\Big(r,\frac{1}{G-1}\Big)=N(r,1)+O(1)\le\overline{N}\Big(r,\frac{1}{F'}\Big)+\overline{N}\Big(r,\frac{1}{G'}\Big)+S(r). \tag{3.7}$$

On the other hand, by Nevanlinna's second fundamental theorem and

$$N\Big(r,\frac{1}{F}\Big)+\overline{N}(r,F)=S(r),\quad N\Big(r,\frac{1}{G}\Big)+\overline{N}(r,G)=S(r),$$

we have

$$T(r,F)\le N\Big(r,\frac{1}{F-1}\Big)-N\Big(r,\frac{1}{F'}\Big)+S(r,F) \tag{3.8}$$

and

$$T(r,G)\le N\Big(r,\frac{1}{G-1}\Big)-N\Big(r,\frac{1}{G'}\Big)+S(r,G). \tag{3.9}$$

By (3.6)–(3.9), we get

$$\begin{aligned}T(r,F)+T(r,G)&\le 2N(r,1)-N\Big(r,\frac{1}{F'}\Big)-N\Big(r,\frac{1}{G'}\Big)+S(r)\\&\le N(r,1)+S(r)\\&\le\frac{1}{2}\{T(r,F)+T(r,G)\}+S(r).\end{aligned}$$

Then $T(r,F)+T(r,G)\le S(r)$, a contradiction. Thus $F(z)$ is a Mobius transformation of $G(z)$, that is

$$F(z)=\frac{AG(z)+B}{CG(z)+D}, \tag{3.10}$$

where A,B,C,D are constants, and $AD-BC\neq 0$.

Next we discuss following two cases.

Case 1 $C=0$. Thus $AD\neq 0$. From (3.10), we have $F(z)=\frac{A}{D}G(z)+\frac{B}{D}$. If $B\neq 0$, it follows from $N\big(r,\frac{1}{F}\big)=S(r)$ that $N\big(r,\frac{1}{G-\frac{B}{A}}\big)=S(r)$, then we get a contradiction by Nevanlinna's second fundamental theorem. Hence $F(z)=\frac{A}{D}G(z)$. If $F(z)\neq 1$, it is easy to get a contradiction by Nevanlinna's second fundamental theorem. So there exists z_0 such that $F(z_0)=G(z_0)=1$. Thus we get $\frac{A}{D}=1$, that is $F(z)\equiv G(z)$.

Case 2 $C\neq 0$. We consider two subcases.

Case 2.1 $D\neq 0$. Then from (3.10), we obtain $F(z)\neq\infty$, $G(z)\neq\infty$, $G(z)\neq-\frac{D}{C}$. By Nevanlinna's second fundamental theorem, we get a contradiction.

Case 2.2 $D=0$. Then $B\neq 0$. From (3.10), we have $CF(z)G(z)=AG(z)+B$. It is easy to get $F(z)\neq\infty$, and $G(z)\neq\infty$. If $A\neq 0$, we get $G\neq-\frac{B}{A}$, which contradicts Nevanlinna's second fundamental theorem. So $A=0$. Then $F(z)G(z)=\frac{B}{C}$. In the same way as in Case 1, we can get $\frac{B}{C}=1$.

Thus we get $F(z)\equiv G(z)$ or $F(z)G(z)\equiv 1$.

If $F(z)\equiv G(z)$, then $f(z)\equiv\pm g(z)$. If $F(z)G(z)\equiv 1$, then $f(z)g(z)\equiv\pm a^2(z)$.

This completed the proof of Theorem 1.5.

4 Proof of Theorem 1.7

By Theorem 1.5, we get $f(z) \equiv \pm\Delta_c f$ or $f(z)\Delta_c f \equiv \pm 1$.

If $f(z)\Delta_c f \equiv \pm 1$, that is

$$f(z)[f(z+c) - f(z)] \equiv \pm 1. \tag{4.1}$$

From (4.1) and the fact that $f(z), \Delta_c f$ share 0 CM almost and share ∞ IM almost, we obtain $f(z) \neq 0$ and $f(z) \neq \infty$. Thus $f(z) = \mathrm{e}^{h(z)}$, where $h(z)$ be a nonconstant entire function.

By (4.1), we get

$$f(z)[f(z+c) - f(z)] \equiv t, \tag{4.2}$$

where $t^2 = 1$.

From (4.2) and $f(z) = \mathrm{e}^{h(z)}$, we obtain

$$\mathrm{e}^{h(z)}[\mathrm{e}^{h(z+c)} - \mathrm{e}^{h(z)}] \equiv t.$$

That is

$$\mathrm{e}^{h(z)h(z+c)} - \mathrm{e}^{2h(z)} \equiv t.$$

Since $\mathrm{e}^{h(z)h(z+c)} \neq 0$, we easily get $\mathrm{e}^{2h(z)} \neq -t$, and obviously $\mathrm{e}^{2h(z)} \neq 0, \infty$. Then by Picard theorem, we get $\mathrm{e}^{2h(z)} \equiv C_1$, then $h \equiv C_2$, where C_1, C_2 are constants. A contradiction.

So we get $f(z) \equiv \pm[f(z+c) - f(z)]$, that is $f(z) \equiv f(z+c) - f(z)$ or $f(z) \equiv f(z) - f(z+c)$. If $f(z) \equiv f(z) - f(z+c)$, then $f(z+c) \equiv 0$. So $f(z) \equiv 0$, a contradiction. So $f(z) \equiv f(z+c) - f(z)$. Thus $f(z+c) \equiv 2f(z)$.

This completed the proof of Theorem 1.7.

5 Proof of Theorem 1.8

By Theorem 1.5, we get $f(z) \equiv \pm P(z, f(z))$ or $f(z)P(z, f(z)) \equiv \pm a^2(z)$.

If $f(z)P(z, f(z)) \equiv \pm a^2(z)$, we get

$$\frac{1}{f^2(z)} = \pm \frac{1}{a^2(z)} \frac{P(z, f(z))}{f(z)}. \tag{5.1}$$

Since $f(z), P(z, f(z))$ share 0 CM almost, share ∞ IM almost and $T(r, a(z)) = S(r)$, $N\big(r, \frac{1}{f}\big) = S(r)$.

By (5.1) and Lemma 2.3, we get

$$2m\Big(r, \frac{1}{f}\Big) = m\Big(r, \frac{1}{f^2}\Big) = m\Big(r, \pm\frac{P(z, f(z))}{a^2(z)f(z)}\Big) \leq S(r).$$

So $T(r, f) = T\big(r, \frac{1}{f}\big) + O(1) = m\big(r, \frac{1}{f}\big) + N\big(r, \frac{1}{f}\big) + O(1) \leq S(r)$, a contradiction.

This completed the proof of Theorem 1.8.

Acknowledgement The authors thank the referees and editors for several helpful suggestions.

References

[1] Chen, B. Q. and Chen, Z. X., Entire functions sharing sets of small functions with their difference operators or shifts, *Math. Slovaca,* **6**, 2013, 1233–1246.

[2] Chiang, Y. M. and Feng, S. J., On the Nevanlinna characteristic of $f(z+\eta)$ and difference equations in the complex plane, *Ramanujan J.,* **16**, 2008, 105–129.

[3] Fang, M. L., Unicity theorem or meromorphic function and its differential polynomial, *Adv. in Math. (PRC),* **24**, 1995, 244–249.

[4] Gross, F. and Osgood, C. F., Entire Functions with Common Preimages, Factorization Theory of Meromorphic Functions and Related Topics, Lect. Notes Pure Appl. Math., **78**, Marcel Dekker, New York, 1982, 19–24.

[5] Halburd, R. G. and Korhonen, R. J., Difference analogue of the lemma on the logarithmic derivative with applications to difference equations, *J. Math. Anal. Appl.,* **314**, 2006, 477–487.

[6] Hayman, W. K., Meromorphic Functions, Clarendon Press, Oxford, 1964.

[7] Liu, K., Meromorphic functions sharing a set with applications to difference equations, *J. Math. Anal. Appl.,* **358**, 2009, 384–393.

[8] Yang, C. C. and Yi, H. X., Uniqueness Theory of Meromorphic Functions, Science Press, Beijing, 2003.

[9] Yi, H. X., Meromorphic functions with common preimages, *J. of Math. (PRC),* **3**, 1987, 219–224.

[10] Zhang, J., Value distribution and shared sets of differences of meromorphic functions, *J. Math. Anal. Appl.,* **367**, 2010, 401–408.