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FIRST INITIAL-BOUNDARY VALUE PROBLEMS FOR QUASILINEAR HYPERBOLIC-PARABOLIC COUPLED SYSTEMS

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Abstract

In this paper, under some conditions of solvability, the local time existence and uniqueness theorem is proved for general types of first initial-boundary value problems for quasilinear hyperbolic-parabolic coupled systems in two variables.

§ 1. Introduction

We have proved in [1, 2] the local time existence and uniqueness theorems for initial value problems and second initial-boundary value problems for quasilinear hyperbolic-parabolic coupled systems. In this paper we discuss first initial-boundary value problems for this kind of systems.

As in [2] we consider on a rectangular domain

$$R(\delta)=\{(t,\ x)\,|\,0\leqslant t\leqslant\delta,\ 0\leqslant x\leqslant 1\} \tag{1.1}$$

the following quasilinear hyperbolic-parabolic coupled system

$$\begin{cases} \displaystyle\sum_{j=1}^{n}\zeta_{lj}(t,\ x,\ u,\ v)\left(\frac{\partial u_j}{\partial t}+\lambda_l(t,\ x,\ u,\ v,\ v_x)\frac{\partial u_j}{\partial x}\right) \\ \quad =\zeta_l(t,\ x,\ u,\ v)\left(\dfrac{\partial v}{\partial t}+\lambda_l(t,\ x,\ u,\ v,\ v_x)\dfrac{\partial v}{\partial x}\right)+\mu_l(t,\ x,\ u,\ v,\ v_x) \\ \qquad\qquad (l=1,\ \cdots,\ n), & (1.2) \\ \dfrac{\partial v}{\partial t}-a(t,\ x,\ u,\ v,\ v_x)\,\dfrac{\partial^2 v}{\partial x^2}=b(t,\ x,\ u,\ v,\ v_x). & (1.3) \end{cases}$$

Without loss of generality, the inital conditions may be written as

$$t=0:\ u_j=0\quad(j=1,\ \cdots,\ n), \tag{1.4}$$

$$v=0. \tag{1.5}$$

Moreover, we can assume that

$$a(0,\ x,\ 0,\ 0,\ 0)=1, \tag{1.6}$$

$$b(0,\ x,\ 0,\ 0,\ 0)=0, \tag{1.7}$$

$$\zeta_{lj}(0,\ x,\ 0,\ 0)=\delta_{lj}=\begin{cases}1, & \text{if } l\neq j,\\ 0, & \text{if } l\neq j.\end{cases} \tag{1.8}$$

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The boundary conditions are as follows:

on $x=1$:

$$u_{\bar{r}}=G_{\bar{r}}(t,\ u)\ (\bar{r}=1,\ \cdots,\ h,\ h\leqslant n), \tag{1.9}$$

$$v=\varphi_2(t); \tag{1.10}$$

on $x=0$:

$$u_{\hat{s}}=\hat{G}_{\hat{s}}(t,\ u)\quad (\hat{s}=m+1,\ \cdots,\ n,\ m\geqslant 0), \tag{1.11}$$

$$v=\varphi_1(t). \tag{1.12}$$

Here, the boundary conditions for v are of Dirichlet type, so this problem is called the first initial-boundary value problem.

We assume that the following conditions are satisfied:

(1) Conditions of orientability

$$\lambda_{\bar{r}}(0,\ 1,\ 0,\ 0,\ 0)<0,\ \lambda_{\bar{s}}(0,\ 1,\ 0,\ 0,\ 0)>0\begin{pmatrix}\bar{r}=1,\ \cdots,\ h\\ \bar{s}=h+1,\ \cdots,\ n,\end{pmatrix}, \tag{1.13}$$

$$\lambda_{\hat{r}}(0,\ 0,\ 0,\ 0,\ 0)<0,\ \lambda_{\hat{s}}(0,\ 0,\ 0,\ 0,\ 0)>0\begin{pmatrix}\hat{r}=1,\ \cdots,\ m\\ \hat{s}=m+1,\ \cdots,\ n.\end{pmatrix}. \tag{1.14}$$

As usual, the characteristic directions on the boundary are colled departing characteristic directions, if as long as t increases, they point towards the interior of the domain Thus, on the boundary, the number of boundary conditions for u is equal to the number of departing characteristic direction (cf. [3]). For example, on $x=1$ the number of boundary conditions for u is equal to h, the number which appears in (1.13).

(2) Conditions of compatibility

$$G_{\bar{r}}(0,\ 0)=0,\ \hat{G}_{\hat{s}}(0,\ 0)=0\quad (\bar{r}=1,\ \cdots,\ h;\ \hat{s}=m+1,\ \cdots,\ n), \tag{1.15}$$

$$\varphi_i(0)=0\quad (i=1,\ 2), \tag{1.16}$$

$$\frac{\partial G_{\bar{r}}}{\partial t}(0,\ 0)+\sum_{j=1}^{n}\frac{\partial G_{\bar{r}}}{\partial u_j}(0,\ 0)\mu_j(0,\ 1,\ 0,\ 0,\ 0)=\mu_{\bar{r}}(0,\ 1,\ 0,\ 0,\ 0)\quad (\bar{r}=1,\ \cdots,\ h), \tag{1.17}$$

$$\frac{\partial \hat{G}_{\hat{s}}}{\partial t}(0,\ 0)+\sum_{j=1}^{n}\frac{\partial \hat{G}_{\hat{s}}}{\partial u_j}(0,\ 0)\mu_j(0,\ 0,\ 0,\ 0,\ 0)=\mu_{\hat{s}}(0,\ 0,\ 0,\ 0,\ 0)\quad (\hat{s}=m+1,\ \cdots,\ n), \tag{1.18}$$

$$\varphi_i'(0)=0\quad (i=1,\ 2). \tag{1.19}$$

(3) Conditions of solvability

$$\det\left|\delta_{\bar{r}\bar{r}'}-\frac{\partial G_{\bar{r}}}{\partial u_{\bar{r}'}}(0,\ 0)\right|\neq 0\quad (\bar{r},\ \bar{r}'=1,\ \cdots,\ h), \tag{1.20}$$

$$\det\left|\delta_{\hat{s}\hat{s}'}-\frac{\partial \hat{G}_{\hat{s}}}{\partial u_{\hat{s}'}}(0,\ 0)\right|\neq 0\quad (\hat{s},\ \hat{s}'=m+1,\ \cdots,\ n). \tag{1.21}$$

Conditions (1.20), (1.21) imply that boundary conditions (1.9), (1.11) can be rewritten as follows:

on $x=1$:

$$u_{\bar{r}}=H_{\bar{r}}(t,\ u_{\bar{s}})\quad (\bar{r}=1,\ \cdots,\ h;\ \bar{s}=h+1,\ \cdots,\ n), \tag{1.22}$$

on $x=0$:

$$u_{\hat{s}}=\hat{H}_{\hat{s}}(t, u_{\hat{r}}) \quad (\hat{r}=1, \cdots, m; \ \hat{s}=m+1, \cdots, n). \tag{1.23}$$

(4) Conditions of smoothness: The coefficients of the system and the boundary conditions are suitably smooth. The detail will be explained later on.

The main aim of this paper is to prove that under the preceding hypotheses (1)—(4) the first initial boundary value problem (1.2)—(1.5), (1.9)—(1.12) admits a unique classical solution on $R(\delta)$ where $\delta>0$ is suitably small.

§ 2. Some estimations for the solutions of first initial-boundary value problems for heat equations

At first, we consider the following first initial-boundary value problem for heat equations

$$\begin{cases} \dfrac{\partial v}{\partial t}=\dfrac{\partial^2 v}{\partial x^2}+b(t, x), & (2.1)\\ t=0: v=0, & (2.2)\\ x=1: v=0, & (2.3)\\ x=0: v=0. & (2.4)\end{cases}$$

Let $G_0(t, x; \tau, \xi)$ be the fundamental solution of the heat equation

$$G_0(t, x; \tau, \xi)=\frac{1}{2\sqrt{\pi(t-\tau)}}e^{-\frac{(x-\xi)^2}{4(t-\tau)}} \quad (t\geqslant\tau), \tag{2.5}$$

then

$$G(t, x; \tau, \xi)=\sum_{n=-\infty}^{\infty}[G_0(t, x; \tau, 2n+\xi)-G_0(t, x; \tau, 2n-\xi)], \tag{2.6}$$

$$N(t, x; \tau, \xi)=\sum_{n=-\infty}^{\infty}[G_0(t, x; \tau, 2n+\xi)+G_0(t, x; \tau, 2n-\xi)] \tag{2.7}$$

are the Green function and the Neumann function for the first and the second initial-boundary value problem of the heat equation respectively. $G(t, x; \tau, \xi)$, as a function of (t, x), satisfies the heat equation $\frac{\partial G}{\partial t}=\frac{\partial^2 G}{\partial x^2}$ for $t>\tau$ and $G=0$ for $x=0$ or $x=1$; on the other hand, $G(t, x; \tau, \xi)$, as a function of (τ, ξ), satisfies the adjoint equation $\frac{\partial G}{\partial \tau}=-\frac{\partial^2 G}{\partial \xi^2}$ for $\tau<t$ and $G=0$ for $\xi=0$ or $\xi=1$. Besides

$$\frac{\partial G}{\partial x}=-\frac{\partial N}{\partial \xi}, \quad \frac{\partial G}{\partial \xi}=-\frac{\partial N}{\partial x}. \tag{2.8}$$

Let

$$V_\sigma(t, x)=t^{-\frac{\sigma}{2}}\exp\left\{-\frac{x^2}{16t}\right\}, \tag{2.9}$$

a direct calculation gives

$$\int_{-\infty}^{\infty}V_\sigma(t, \xi)d\xi=4\sqrt{\pi}\,t^{\frac{1-\sigma}{2}} \quad (t>0), \tag{2.10}$$

$$\int_0^t\int_{-\infty}^{\infty} V_\sigma(t-\tau,\ \xi)d\xi\,d\tau=\frac{8\sqrt{\pi}}{3-\sigma}t^{\frac{3-\sigma}{2}}\quad (\text{if } \sigma<3), \tag{2.11}$$

$$\int_{t-\gamma}^t\int_{-\infty}^{\infty} V_\sigma(t-\tau,\ \xi)d\xi\,d\tau=\frac{8\sqrt{\pi}}{3-\sigma}\gamma^{\frac{3-\sigma}{2}}\quad (\text{if } \sigma<3 \text{ and } 0\leqslant\gamma\leqslant t), \tag{2.12}$$

$$\int_0^{t_1-\gamma}\int_{-\infty}^{\infty} V_\sigma(t-\tau,\ \xi)d\xi\,d\tau\leqslant\frac{8\sqrt{\pi}}{\sigma-3}\gamma^{-\frac{\sigma-3}{2}}\quad (\text{if } \sigma>3 \text{ and } 0<\gamma\leqslant t_1\leqslant t). \tag{2.13}$$

We can find the following two lemmas in [2].

Lemma 2.1 *It holds on $R(\delta_0)$ that*

$$\left|\frac{\partial^{i+j}G(t,\ x;\ \tau,\ \xi)}{\partial x^i\,\partial t^j}\right|,\ \left|\frac{\partial^{i+j}N(t,\ x;\ \tau,\ \xi)}{\partial x^i\,\partial t^j}\right|\leqslant P_{ij}V_{i+2j+1}(t-\tau,\ x-\xi)\ (t>\tau), \tag{2.14}$$

$$\left|\frac{(x-\xi)^k}{(t-\tau)^s}\frac{\partial^{i+j}G}{\partial x^i\,\partial t^j}\right|,\ \left|\frac{(x-\xi)^k}{(t-\tau)^s}\frac{\partial^{i+j}N}{\partial x^i\partial t^j}\right|\leqslant P_{ijks}V_{i+2j+2s-k+1}(t-\tau,\ x-\xi)\ (t>\tau), \tag{2.15}$$

where P_{ij}, P_{ijks} are positive constants.

Lemma 2.2 *It holds for $(t,\ x)\in R(\delta_0)$ that*

$$\int_0^t\left|\frac{\partial N}{\partial x}(t,\ x;\ \tau,\ 0)\right|d\tau\leqslant C\quad (x\neq 0), \tag{2.16}$$

$$\int_0^t\left|\frac{\partial N}{\partial x}(t,\ x;\ \tau,\ 1)\right|d\tau\leqslant C\quad (x\neq 1), \tag{2.17}$$

where constant C depends only on δ_0.

In what follows we need some a priori estimations for the solutions of first initial-boundary value problems for heat equations.

Lemma 2.3 *Suppose that on $R(\delta_0)$ $b(t,\ x)$ is Hölder continuous with respect to t and to x with the exponents $\frac{\alpha}{2}$ and α respectively $(0<\alpha<1)$ and*

$$b(0,\ x)=0, \tag{2.18}$$

then the first initial-boundary value problem (2.1)—(2.4) admits on $R(\delta_0)$ a unique classical solution

$$v(t,\ x)=\int_0^t\int_0^1 G(t,\ x;\ \tau,\ \xi)b(\tau,\ \xi)d\xi\,d\tau \tag{2.19}$$

and it holds that

$$\frac{\partial v}{\partial x}(t,\ x)=\int_0^t\int_0^1\frac{\partial G}{\partial x}(t,\ x;\ \tau,\ \xi)b(\tau,\ \xi)d\xi\,d\tau, \tag{2.20}$$

$$\begin{aligned}\frac{\partial^2 v}{\partial x^2}(t,\ x)=&\int_0^t\int_0^1\frac{\partial^2 G}{\partial x^2}(t,\ x;\ \tau,\ \xi)(b(\tau,\ \xi)-b(\tau,\ x))d\xi\,d\tau\\ &+\int_0^t\left[\frac{\partial N}{\partial x}(t,\ x;\ \tau,\ 0)-\frac{\partial N}{\partial x}(t,\ x;\ \tau,\ 1)\right]b(\tau,\ x)d\tau,\end{aligned} \tag{2.21}$$

$$\begin{aligned}\frac{\partial v}{\partial t}(t,\ x)=&\int_0^t\int_0^1\frac{\partial G}{\partial t}(t,\ x;\ \tau,\ \xi)(b(\tau,\ \xi)-b(t,\ \xi))d\xi\,d\tau\\ &+\int_0^1 G(t,\ x;0,\ \xi)b(t,\ \xi)d\xi.\end{aligned} \tag{2.22}$$

Moreover, on $R(\delta_0)$, v, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial^2 v}{\partial x^2}$ are continuous, $\frac{\partial v}{\partial x}$ is Hölder continuous with respect to t with the exponent $\frac{1+\alpha}{2}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial^2 v}{\partial x^2}$ are Hölder continuous with respect to t and to x with the exponents $\frac{\alpha}{2}$ and α respectively, denoted by $v\in \bar{C}^{2+\alpha}$ with the notation in [1, 2].

This lemma can be proved by means of the properties of the Green function and the Neumann function. The last part of the conclusion follows from the proof of the following lemmas, as for the rest we refer to [4].

Lemma 2.4 (*first a priori estimation*). *Under the assumption of Lemma 2.3, if $v=v(t, x)$ is the solution of the first initial-boundary value problem (2.1)—(2.4) on $R(\delta_0)$, then for any $\delta\,(0\leqslant\delta\leqslant\delta_0)$ we have*

$$|v|\underset{\text{def}}{=}\|v\|+\left\|\frac{\partial v}{\partial x}\right\|\leqslant C_1\delta^{\frac{1}{2}}\|b\| \quad on\ R(\delta), \tag{2.23}$$

where constant C_1 depends only on δ_0.

Proof Using inequalities (2.14) and (2.10), (2.23) follows from expressions (2.19) and (2.20).

Lemma 2.5 (*second a priori estimation*) *Under the assumption of lemma 2.3, for any $\delta\,(0\leqslant\delta\leqslant\delta_0)$ we have*

$$|v|_1\underset{\text{def}}{=}|v|+\left\|\frac{\partial v}{\partial t}\right\|+\left\|\frac{\partial^2 v}{\partial x^2}\right\|+H_t^{\frac{1}{2}}\left[\frac{\partial v}{\partial x}\right]$$
$$\leqslant C_2(\|b\|+\delta^{\frac{\alpha}{2}}H_x^{\alpha}[b]) \quad on\ R(\delta), \tag{2.24}$$

where constant C_2 depends only on δ_0.

Proof By means of (2.15), (2.11) and Lemma 2.2, (2.21) gives

$$\left\|\frac{\partial^2 v}{\partial x^2}\right\|\leqslant d_1\,(\delta^{\frac{\alpha}{2}}H_x^{\alpha}[b]+\|b\|). \tag{2.25}$$

Noticing that $v(t, x)$ satisfies the heat equation (2.1), we can also get a similar estimation for $\frac{\partial v}{\partial t}$. Here and hereafter, $d_i\,(i=1, 2, \cdots)$ denote constants depending only on δ_0.

In a similar way as in [1], we have

$$H_t^{\frac{1}{2}}\left[\frac{\partial v}{\partial x}\right]\leqslant d_2\|b\|. \tag{2.26}$$

(2.24) follows from the combination of (2.25) and (2.26).

Lemma 2.6 (third a priori estimation). *Under the assumption of lemma 2.3, for any $\delta\,(0\leqslant\delta\leqslant\delta_0)$ we have*

$$|v|_2\underset{\text{def}}{=}|v|_1+H_t^{\frac{1+\alpha}{2}}\left[\frac{\partial v}{\partial x}\right]+H^{\alpha}\left[\frac{\partial v}{\partial t}\right]+H^{\alpha}\left[\frac{\partial^2 v}{\partial x^2}\right]$$
$$\leqslant C_3(\|b\|+H^{\alpha}[b]) \quad on\ R(\delta), \tag{2.27}$$

where C_3 is a constant depending only on δ_0.

Proof First of all, we estimate $H_t^{\frac{1+\alpha}{2}}\left[\frac{\partial v}{\partial x}\right]$. We get from (2.20)

$$\frac{\partial v}{\partial x}(t, x)=\int_0^t\int_0^1\frac{\partial G}{\partial x}(t, x; \tau, \xi)(b(\tau, \xi)-b(t, \xi))d\xi\, d\tau$$
$$+\int_0^t\int_0^1\frac{\partial G}{\partial x}(t, x; \tau, \xi)b(t, \xi)d\xi\, d\tau=I_1+I_2. \tag{2.28}$$

Suppose $\delta\geqslant t_1\geqslant t_2\geqslant 0$ and let $\gamma=t_1-t_2$.

If $t_1-2\gamma=t_2-\gamma\geqslant 0$, we have

$$\begin{aligned}
&I_1(t_1, x)-I_1(t_2, x)\\
&=\int_{t_1-2\gamma}^{t_1}\int_0^1\frac{\partial G}{\partial x}(t_1, x; \tau, \xi)(b(\tau, \xi)-b(t_1, \xi))d\xi\, d\tau\\
&\quad-\int_{t_2-\gamma}^{t_2}\int_0^1\frac{\partial G}{\partial x}(t_2, x; \tau, \xi)(b(\tau, \xi)-b(t_2, \xi))d\xi\, d\tau\\
&\quad+\int_0^{t_2-\gamma}\int_0^1\left(\frac{\partial G}{\partial x}(t_1, x; \tau, \xi)-\frac{\partial G}{\partial x}(t_2, x; \tau, \xi)\right)(b(\tau, \xi)-b(t_2, \xi))d\xi\, d\tau\\
&\quad-\int_0^{t_2-\gamma}\int_0^1\frac{\partial G}{\partial x}(t_1, x; \tau, \xi)(b(t_1, \xi)-b(t_2, \xi))d\xi\, d\tau\\
&=I_{11}+I_{12}+I_{13}+I_{14}.
\end{aligned} \tag{2.29}$$

By means of Lemma 2.1 and (2.10)—(2.13), it is easy to obtain with a similar way as in [1] that

$$|I_{11}|,\ |I_{12}|,\ |I_{13}|\leqslant d_3\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b].$$

Moreover

$$\begin{aligned}
&I_{14}+I_2(t_1, x)-I_2(t_2, x)\\
&=\int_0^{t_1}\int_0^1\frac{\partial G}{\partial x}(t_1, x; \tau, \xi)b(t_1, \xi)d\xi\, d\tau\\
&\quad-\int_0^{t_2}\int_0^1\frac{\partial G}{\partial x}(t_2, x; \tau, \xi)b(t_2, \xi)d\xi\, d\tau\\
&\quad-\int_0^{t_2-\gamma}\int_0^1\frac{\partial G}{\partial x}(t_1, x; \tau, \xi)(b(t_1, \xi)-b(t_2, \xi))d\xi\, d\tau\\
&=\int_{t_2-2\gamma}^{t_2}\int_0^1\frac{\partial G(t_2, x; \tau, \xi)}{\partial x}(b(t_1, \xi)-b(t_2, \xi))d\xi\, d\tau\\
&\quad+\int_{-\gamma}^0\int_0^1\frac{\partial G(t_2, x; \tau, \xi)}{\partial x}b(t_2, \xi)d\xi d\tau=J_1+J_2.
\end{aligned}$$

From Lemma 2.1 and (2.10)—(2.13) it follows that

$$|J_1|\leqslant d_4\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b].$$

Noticing (2.18) we have

$$\begin{aligned}
|J_2|&\leqslant d_5H_t^{\frac{\alpha}{2}}[b]\,t_2^{\frac{\alpha}{2}}\int_{-\gamma}^0\int_0^1 V_2(x-\xi, t_2-\tau)d\xi\, d\tau\\
&\leqslant d_6H_t^{\frac{\alpha}{2}}[b]\,t_2^{\frac{\alpha}{2}}\int_{-\gamma}^0(t_2-\tau)^{-\frac{1}{2}}d\tau\leqslant d_7H_t^{\frac{\alpha}{2}}[b]\,t_2^{\frac{\alpha}{2}}(\sqrt{t_2+\gamma}-\sqrt{t_2})\\
&=d_7H_t^{\frac{\alpha}{2}}[b]\,t_2^{\frac{\alpha}{2}}\frac{\gamma^{\frac{1-\alpha}{2}}\cdot\gamma^{\frac{1+\alpha}{2}}}{\sqrt{t_2+\gamma}+\sqrt{t_2}}.
\end{aligned}$$

Since $t_2\geqslant\gamma$, we get

$$|J_2|\leqslant d_7\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b].$$

The combination of the preceding formulae gives that

$$\left|\frac{\partial v}{\partial x}(t_1, x)-\frac{\partial v}{\partial x}(t_2, x)\right|\leqslant d_8\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b] \quad \text{for } t_1-2\gamma\geqslant 0. \tag{2.30}$$

If $t_1-2\gamma=t_2-\gamma<0$, From Lemma 2.1 and (2.10)—(2.13) it follows directly that

$$|I_1(t_1, x)-I_1(t_2, x)|\leqslant d_9\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b],$$

$$\begin{aligned}
&I_2(t_1, x)-I_2(t_2, x)\\
&=\int_0^{t_1}\int_0^1\frac{\partial G}{\partial x}(t_1, x;\ \tau, \xi)b(t_1, \xi)d\xi\, d\tau\\
&\quad-\int_0^{t_2}\int_0^1\frac{\partial G}{\partial x}(t_2, x;\ \tau, \xi)b(t_2, \xi)d\xi\, d\tau\\
&=\int_0^{t_2}\int_0^1\frac{\partial G}{\partial x}(t_2, x;\ \tau, \xi)(b(t_1, \xi)-b(t_2, \xi))d\xi\, d\tau\\
&\quad+\int_{-\gamma}^0\int_0^1\frac{\partial G}{\partial x}(t_2, x;\ \tau, \xi)b(t_1, \xi)d\xi\, d\tau=S_1+S_2.
\end{aligned}$$

Using Lemma 2.1 and (2.10)—(2.13) we get

$$\begin{aligned}
|S_1|&\leqslant d_{10}\gamma^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b]\int_0^{t_2}(t_2-\tau)^{-\frac{1}{2}}d\tau\\
&\leqslant 2d_{10}\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b].
\end{aligned} \tag{2.31}$$

Moreover, (2.18) gives

$$|S_2|\leqslant d_{11}H_t^{\frac{\alpha}{2}}[b]\, t_1^{\frac{\alpha}{2}}\int_{-\gamma}^0(t_2-\tau)^{-\frac{1}{2}}d\tau\leqslant d_{12}\gamma^{\frac{1+\alpha}{2}}H_t^{\frac{\alpha}{2}}[b]. \tag{2.32}$$

Thus, (2.30) stays still valid for $t_1-2\gamma<0$. Finally, we obtain

$$H_t^{\frac{1+\alpha}{2}}\left[\frac{\partial v}{\partial x}\right]\leqslant d_{13}H_t^{\frac{\alpha}{2}}[b] \quad \text{on } R(\delta).$$

Now we estimate $H^\alpha\left[\frac{\partial v}{\partial t}\right]$. (2.22) gives

$$\frac{\partial v}{\partial t}(t, x)=L_1(t, x)+L_2(t, x),$$

where

$$L_1(t, x)=\int_0^t\int_0^1\frac{\partial G}{\partial t}(t, x;\ \tau, \xi)(b(\tau, \xi)-b(t, \xi))d\xi\, d\tau,$$

$$L_2(t, x)=\int_0^1 G(t, x;\ 0, \xi)b(t, \xi)d\xi.$$

As above, suppose $\delta\geqslant t_1\geqslant t_2\geqslant 0$ and let $\gamma=t_1-t_2$.

If $t_1-2\gamma=t_2-\gamma\geqslant 0$, we have

$$\begin{aligned}
L_1(t_1, x)-L_2(t_2, x)&=\int_{t_1-2\gamma}^{t_1}\int_0^1\frac{\partial G}{\partial t}(t_1, x;\ \tau, \xi)(b(\tau, \xi)-b(t_1, \xi))d\xi\, d\tau\\
&\quad-\int_{t_1-\gamma}^{t_2}\int_0^1\Big(\frac{\partial G}{\partial t}(t_2, x;\ \tau, \xi)(b(\tau, \xi)-b(t_2, \xi))d\xi\, d\tau
\end{aligned}$$

$$+\int_0^{t_2-\gamma}\int_0^1\left(\frac{\partial G}{\partial t}(t_1, x; \tau, \xi)-\frac{\partial G}{\partial t}(t_2, x; \tau, \xi)\right)(b(\tau, \xi)-b(t_2, \xi))d\xi\, d\tau$$

$$-\int_0^{t_2-\gamma}\int_0^1\frac{\partial G}{\partial t}(t_1, x; \tau, \xi)(b(t_1, \xi)-b(t_2, \xi))d\xi\, d\tau$$

$$=L_1^{(1)}+L_1^{(2)}+L_1^{(3)}+L_1^{(4)}.$$

It follows from Lemma 2.1 and (2.10)—(2.13) that

$$|L_1^{(1)}|,\ |L_1^{(2)}|\leqslant d_{14}\gamma^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b],$$

and

$$|L_1^{(3)}|\leqslant d_{15}H_t^{\frac{\alpha}{2}}[b]\int_{t_2}^{t_1}\int_0^{t_2-\gamma}\int_0^1\frac{\partial^2 G}{\partial t^2}(t, x; \tau_1, \xi)(t_2-\tau)^{\frac{\alpha}{2}}d\xi\, d\tau\, dt$$

$$\leqslant d_{16}\gamma^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b].$$

Moreover

$$L_1^{(4)}=\int_0^{t_2-\gamma}\int_0^1\frac{\partial G}{\partial \tau}(t_1, x; \tau, \xi)(b(t_1, \xi)-b(t_2, \xi))d\xi\, d\tau$$

$$=\int_0^1(G(t_1, x; t_2-\gamma, \xi)-G(t_1, x; 0, \xi))(b(t_1, \xi)-b(t_2, \xi))d\xi,$$

hence

$$L_1^{(4)}+L_2(t_1, x)-L_2(t_2, x)$$

$$=\int_0^1 G(2\gamma, x; 0, \xi)(b(t_1, \xi)-b(t_2, \xi))d\xi$$

$$+\int_0^1(G(t_1, x; 0, \xi)-G(t_2, x; 0, \xi))b(t_2, \xi)d\xi$$

$$=k_1+k_2.$$

It is easy to see that

$$|k_1|\leqslant d_{17}\gamma^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b],$$

$$|k_2|\leqslant d_{18}\, t_2^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b]\int_{t_2}^{t_1}\int_0^1\frac{\partial G}{\partial t}(t_1, x; 0, \xi)d\xi\, dt$$

$$\leqslant d_{19}\,\gamma^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b],$$

therefore

$$\left|\frac{\partial v}{\partial t}(t_1, x)-\frac{\partial v}{\partial t}(t_2, x)\right|\leqslant d_{20}\,\gamma^{\frac{\alpha}{2}}H_t^{\frac{\alpha}{2}}[b] \text{ for } t_1-2\gamma\geqslant 0. \tag{2.33}$$

If $t_1-2\gamma=t_2-\gamma<0$, noticing that

$$L_1(t_1, x)-L_1(t_2, x)$$

$$=\int_0^{t_1}\int_0^1\frac{\partial G}{\partial t}(t_1, x; \tau, \xi)(b(\tau, \xi)-b(t_1, \xi))d\xi\, d\tau$$

$$-\int_0^{t_2}\int_0^1\frac{\partial G}{\partial t}(t_2, x; \tau, \xi)(b(\tau, \xi)-b(t_2, \xi))d\xi\, d\tau,$$

$$L_2(t_1, x)-L_2(t_2, x)$$

$$=\int_0^1 G(t_1, x; 0, \xi)b(t_1, \xi)d\xi-\int_0^1 G(t_2, x; 0, \xi)b(t_2, \xi)d\xi$$

and using (2.18), it follows from Lemma 2.1 and (2.10)—(2.13) that (2.33) holds also for $t_1-2\gamma<0$.

Hence, we get

$$H_t^{\frac{\alpha}{2}}\left[\frac{\partial v}{\partial t}\right] \leqslant d_{21} H_t^{\frac{\alpha}{2}}[b] \quad \text{on } R(\delta).$$

Let $\gamma=(x_1-x_2)^2$.

If $t \geqslant \gamma$, then

$$\begin{aligned} & L_1(t, x_1)-L_1(t, x_2) \\ & =\int_{t-\gamma}^t \int_0^1 \frac{\partial G}{\partial t}(t, x_1; \tau, \xi)(b(\tau, \xi)-b(t, \xi)) d\xi\, d\tau \\ & \quad-\int_{t-\gamma}^t \int_0^1 \frac{\partial G}{\partial t}(t, x_2; \tau, \xi)(b(\tau, \xi)-b(t, \xi)) d\xi\, d\tau \\ & \quad+\int_0^{t-\gamma} \int_0^1\left(\frac{\partial G}{\partial t}(t_1, x_1; \tau, \xi)-\frac{\partial G}{\partial t}(t, x_2; \tau, \xi)\right)(b(\tau, \xi)-b(t, \xi)) d\xi\, d\tau \\ & =L_{11}+L_{12}+L_{13}. \end{aligned}$$

Noticing

$$L_{13}=\int_{x_1}^{x_2}\int_0^{t-\gamma} \int_0^1 \frac{\partial^2 G}{\partial t\, \partial x}(t, x; \tau, \xi)(b(\tau, \xi)-b(t, \xi)) d\xi\, d\tau\, dx,$$

it is easy to see that

$$|L_{11}|,\ |L_{12}|,\ |L_{13}| \leqslant d_{22} \gamma^{\frac{\alpha}{2}} H_t^{\frac{\alpha}{2}}[b],$$

that is

$$|L_1(t, x_1)-L_1(t, x_2)| \leqslant d_{23} \gamma^{\frac{\alpha}{2}} H_t^{\frac{\alpha}{2}}[b].$$

If $t<\gamma$, estimating $L_1(t, x_1)$ and $L_1(t, x_2)$ respectively, we can obtain the same result.

Moreover, for $t \geqslant \gamma$ we have

$$\begin{aligned} |L_2(t, x_1)-L_2(t, x_2)| & =\left|\int_{x_2}^{x_1} \int_0^1 \frac{\partial G}{\partial x}(t, x; 0, \xi) b(t, \xi) d\xi\, dx\right| \\ & \leqslant d_{24} H_t^{\frac{\alpha}{2}}[b] \gamma^{\frac{\alpha}{2}}; \end{aligned}$$

by means of estimating $L_2(t, x_1)$ and $L_2(t, x_2)$ respectively, it is easy to see that this estimation stays still valid for $t<\gamma$.

Hence, we get

$$H_x^\alpha\left[\frac{\partial v}{\partial t}\right] \leqslant d_{25} H_t^{\frac{\alpha}{2}}[b] \quad \text{on } R(\delta).$$

Using the fact that $v(t, x)$ satisfies equation (2.1), it follows immediately that

$$H^\alpha\left[\frac{\partial^2 v}{\partial x^2}\right] \leqslant d_{26} H^\alpha[b] \quad \text{on } R(\delta).$$

The proof of Lemma 2.6 is completed.

Now we turn to the following first initial-boundary value problem for heat equations

$$\begin{cases} \dfrac{\partial v}{\partial t}=\dfrac{\partial^2 v}{\partial x^2}+b(t, x), & (2.34) \\ t=0:\ v=0, & (2.35) \\ x=0:\ v=\varphi_1(t), & (2.36) \\ x=1:\ v=\varphi_2(t), & (2.37) \end{cases}$$

where we assume that $b(t, x)$ satisfies the same hypotheses as in Lemma 2.3, $\varphi_i\in C^{1+\frac{\alpha}{2}}$ and

$$\varphi_i(0)=0,\ \varphi_i'(0)=0 \quad (i=1, 2). \tag{2.38}$$

In this case, by means of a simple transformation of unknown function

$$\bar{v}=v-(x\varphi_2(t)+(1-x)\varphi_1(t)), \tag{2.39}$$

we can reduce problem (2.34)—(2.37) to problem (2.1)—(2.4), hence we have

Theorem 2.1 *Under the preceding assumptions, first initial-boundary value problem* (2.34)—(2.37) *admits a unique solution* $v=v(t, x)\in \bar{C}^{2+\alpha}$ *on* $R(\delta_0)$ *and the following a priori eseimations hold on* $R(\delta)$ $(0\leqslant\delta\leqslant\delta)$:

$$|v|\leqslant C_1\delta^{\frac{1}{2}}(\|b\|+\|\varphi\|_1), \tag{2.40}$$

$$|v|_1\leqslant C_2(\|b\|+\delta^{\frac{\alpha}{2}}H_x^{\alpha}[b]+\|\varphi\|_1), \tag{2.41}$$

$$|v|_2\leqslant C_3(\|b\|+H^{\alpha}[b]+\|\varphi\|_1+H_t^{\frac{\alpha}{2}}[\dot{\varphi}]), \tag{2.42}$$

where C_1, C_2 *and* C_3 *are constants depending only on* δ_0.

§ 3. Existence and uniqueness

By means of the a priori estimations established in § 2 for the solutions of first initial-boundary value problems for heat equations and the a priori estimaitons established in [2] for the solutions of mixed initial-boundary value problems for linear hyperbolic systems, we prove in this section the existence and the uniqueness of the solution for the first initial-boundary value problem for quasilinear hyperbolic-parabolic coupled systems.

Suppose that the coefficients of system (1.2), (1.3) and the boundary conditions satisfy the following conditions of smoothness on the domain under consideration:

(i) $\zeta_{ij}(t, x, u, v)$, $\zeta_l(t, x, u, v)\in C^{1+\frac{\alpha}{2}}$ with respect to all arguments;

(ii) $\lambda_l(t, x, u, v, r)$ $\left(r=\frac{\partial v}{\partial x}\right)$ and $\frac{\partial\lambda_l}{\partial x}$, $\frac{\partial\lambda_l}{\partial u_k}$, $\frac{\partial\lambda_l}{\partial v}$, $\frac{\partial\lambda_l}{\partial r}$ are continuous, λ_l is Hölder continuous with respect to t with the exponent $\frac{\alpha}{2}$, $\frac{\partial\lambda_l}{\partial x}$, $\frac{\partial\lambda_l}{\partial u_k}$, $\frac{\partial\lambda_l}{\partial v}$ are Hölder continuous with respect to t, x, u, v, r with the exponent $\frac{\alpha}{2}$, $\frac{\partial\lambda}{\partial r}$ is Hölder continuous with respect to (t, x, u, v) and r with the exponents $\frac{\alpha}{2}$ and $\frac{1}{2}$ respectively; the same hypotheses for μ_{l};

(iii) $a(t, x, u, v, r)$ and $\frac{\partial a}{\partial x}$, $\frac{\partial a}{\partial u}$, $\frac{\partial a}{\partial v}$, $\frac{\partial a}{\partial r}$ are continuous, a is Hölder continuous with respect to t with the exponent $\frac{\alpha}{2}$; the same hypotheses for b;

(iv) $G_{\bar{r}}(t, u)$ ($\bar{r}=1, \cdots, h$) and $\hat{G}_{\hat{s}}(t, u)$ ($\hat{s}=m+1, \cdots, n$) belong to $C^{1+\frac{\alpha}{2}}$ with respect to all arguments;

(v) $\varphi_i(t) \in C^{1+\frac{\alpha}{2}}$ $(i=1, 2)$.

We have the following

Theorem 3.1 (Existence and Uniqueness Theorem) *Suppose that the coefficients of the system and the boundary conditions satisfy the preceding conditions of smoothness. Suppose further that conditions of orientability* (1.13), (1.14), *conditions of compatibility* (1.15)—(1.19) *and conditions of solvability* (1.20)—(1.21) *hold. Then, there exists a positive number* $\delta_* \leqslant \delta_0$ *such that on* $R(\delta_*)$ *first initial-boundary value problem* (1.2)—(1.5), (1.9)—(1.12) *admits a unique solution* $u \in C^{1+\frac{\alpha}{2}}$, $v \in \bar{C}^{2+\alpha}$.

Proof As in [2], without loss of generality we may assume that the following contraction condition holds:

$$\theta = \max_{\substack{1\leqslant \bar{r}\leqslant h \\ m+1\leqslant \hat{s}\leqslant n}} \left(\sum_{j=1}^{n} \left| \frac{\partial G_{\bar{r}}}{\partial u_j}(0, 0) \right|, \sum_{j=1}^{n} \left| \frac{\partial \hat{G}_{\hat{s}}}{\partial u_j}(0, 0) \right| \right) < 1. \tag{3.1}$$

Moreover, according to conditions of orientability (1.13), (1.14) we have

$$d_0 = \min_{\substack{1\leqslant \bar{r}\leqslant h \\ m+1\leqslant \hat{s}\leqslant n}} \{-\lambda_{\bar{r}}(0, 1, 0, 0, 0), \lambda_{\hat{s}}(0, 0, 0, 0, 0)\} > 0. \tag{3.2}$$

Hence, we can choose $\varepsilon > 0$ suitably small such that

$$\theta_1 = (1 + d_0^{-1}\varepsilon)\theta < 1, \tag{3.3}$$

$$\theta_2 = (1 + 2d_0^{-1}\varepsilon + d_0^{-2}\varepsilon)\theta < 1. \tag{3.4}$$

Introduce the following sets of functions on $R(\delta)$:

$$\Sigma_*(\delta) = \left\{ (u, v) \,\middle|\, u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, v, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial^2 v}{\partial x^2} \in C^0, u(0, x) = v(0, x) = 0, \right\} \tag{3.5}$$

$$\Sigma_1(\delta) = \left\{ (u, v) \,\middle|\, \begin{array}{l} u \in C^{1+\frac{\alpha}{2}}, v \in \bar{C}^{2+\alpha}, u(0, x) = v(0, x) = 0 \\ \frac{\partial u_j}{\partial t}(0, x) = \mu_j(0, x, 0, 0, 0), \frac{\partial v}{\partial t}(0, x) = 0, \end{array} \right\} \tag{3.6}$$

$$\Sigma(\delta) = \left\{ (u, v) \,\middle|\, \begin{array}{l} (u, v) \in \Sigma_1(\delta), \|u\| \leqslant A_0, \|u_1\|^* \leqslant A_1, \|u\|^*_{1+\frac{\alpha}{2}} \leqslant A_2 \\ |v| \leqslant B_0, |v|_1 \leqslant B_1, |v|_2 \leqslant B_2, \end{array} \right\} \tag{3.7}$$

where A_i, B_i $(i=0, 1, 2)$ are positive constants to be chosen later on with $A_0 \leqslant A_1 \leqslant A_2$, $B_0 \leqslant B_1 \leqslant B_2$ and

$$\|u\|_1^* = \|u\| + \left\| \frac{\partial u}{\partial t} \right\| + \varepsilon \left\| \frac{\partial u}{\partial x} \right\|,$$

$$\|u\|^*_{1+\beta} = \|u\|_1^* + H_t^\beta\left[\frac{\partial u}{\partial t}\right] + \varepsilon \left(H_x^\beta\left[\frac{\partial u}{\partial t}\right] + H_*^\beta\left[\frac{\partial u}{\partial x}\right] \right),$$

in which

$$H_*^\beta[f] = H_x^\beta[f] + H_t^\beta[f].$$

For any $(\tilde{u}, \tilde{v}) \in \Sigma_1(\delta)$, setting

$$\begin{cases}\tilde{\zeta}_{lj}(t, x)=\zeta_{lj}(t, x, \tilde{u}(t, x), \tilde{v}(t, x)),\\ \tilde{\zeta}_l(t, x)=\zeta_l(t, x, \tilde{u}(t, x), \tilde{v}(t, x)),\\ \tilde{\lambda}_l(t, x)=\lambda_l\left(t, x, \tilde{u}(t, x), \tilde{v}(t, x), \frac{\partial\tilde{v}}{\partial x}(t, x)\right),\\ \tilde{\mu}_l(t, x)=\mu_l\left(t, x, \tilde{u}(t, x), \tilde{v}(t, x), \frac{\partial\tilde{v}}{\partial x}(t, x)\right),\\ \tilde{b}(t, x)=b\left(t, x, \tilde{u}(t, x), \tilde{v}(t, x), \frac{\partial\tilde{v}}{\partial x}(t, x)\right)-\left[a\left(t, x, \tilde{u}, \tilde{v}, \frac{\partial\tilde{v}}{\partial x}\right)-1\right]\frac{\partial^2\tilde{v}}{\partial x^2},\end{cases}\tag{3.8}$$

we solve on $R(\delta)$ the following linear problem:

$$\sum_{j=1}^{n}\tilde{\zeta}_{lj}(t, x)\left(\frac{\partial u_j}{\partial t}+\tilde{\lambda}_l(t, x)\frac{\partial u_j}{\partial x}\right)=\tilde{\zeta}_l(t, x)\left(\frac{\partial\tilde{v}}{\partial t}+\tilde{\lambda}_l(t, x)\frac{\partial\tilde{v}}{\partial x}\right)+\tilde{\mu}_l(t, x)\quad (l=1, \cdots, n),\tag{3.9}$$

$$\frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial x^2}=\tilde{b}(t, x),\tag{3.10}$$

$$t=0:\ u_j=0\ (j=1, \cdots, n),\tag{3.11}$$

$$v=0,\tag{3.12}$$

$$x=1:\ \sum_{j=1}^{n}\tilde{\zeta}_{\bar{r}j}(t, 1)u_j=G_{\bar{r}}(t, \tilde{u}(t, 1))+\sum_{j=1}^{n}(\tilde{\zeta}_{\bar{r}j}(t, 1)-\delta_{\bar{r}j})\tilde{u}_j(t, 1)\equiv\psi_{\bar{r}}(t)\quad (\bar{r}=1, \cdots, h),\tag{3.13}$$

$$v=\varphi_2(t),\tag{3.14}$$

$$x=0:\ \sum_{j=1}^{n}\tilde{\zeta}_{\hat{s}j}(t, 0)u_j=\hat{G}_{\hat{s}}(t, \tilde{u}(t, 0))+\sum_{j=1}^{n}(\tilde{\zeta}_{\hat{s}j}(t, 0)-\delta_{\hat{s}j})\tilde{u}_j(t, 0)\equiv\hat{\psi}_{\hat{s}}(t)\quad (\hat{s}=m+1, \cdots, n),\tag{3.15}$$

$$v=\varphi_1(t).\tag{3.16}$$

(3.10)、(3.12)、(3.14)、(3.16) is a first initial-boundary value problem for heat equations discussed in § 2, hence, by means of Theorem 2.1 it possesses a unique solution $v(t, x)\in\bar{C}^{2+\alpha}$ on $R(\delta)$ which satisfies estimations (2.40)—(2.42). On the other hand, (3.9)、(3.11)、(3.13)、(3.15) is a mixed initial-boundary value problem for linear hyperbolic systems handled in [2], hence, it admits a unique solution $u\in C^{1+\frac{\alpha}{2}}$ on $R(\delta)$ which satisfies the following estimations:

$$\|u\|\leqslant(1+\tilde{K}_1\delta)\|\psi\|+(\tilde{H}_0+\tilde{K}_1\delta)\|\tilde{v}\|+\tilde{K}_1\delta\|\tilde{\mu}\|,\tag{3.17}$$

$$\|u\|_1^*\leqslant(1+d_0^{-1}\varepsilon+\tilde{K}_2\delta^{\frac{\alpha}{2}})\|\dot{\psi}\|+(\tilde{K}_0+\tilde{K}_2\delta)(1+\|\tilde{v}\|_1),\tag{3.18}$$

$$\|u\|^*_{1+\frac{\alpha}{2}}\leqslant(1+2d_0^{-1}\varepsilon+d_0^{-2}\varepsilon+\tilde{K}_2\delta^{\frac{\alpha}{2}})H_t^{\frac{\alpha}{2}}[\dot{\psi}]+(\tilde{K}_2+\tilde{K}_3\delta)(1+\|\dot{\psi}\|+\|\tilde{v}\|_{1+\frac{\alpha}{2}}),\tag{3.19}$$

in which d_0 is defined by (3.2), $\tilde{H}_0=2\sup\limits_{\substack{l=1,\cdots,n\\(t,x)\in R(\delta_0)}}|\tilde{\zeta}_l(t, x)|$, Constants $\tilde{K}_0$ and $\tilde{K}_1$ depend only on the norms $\|\tilde{\Gamma}_0\|$ and $\|\tilde{\Gamma}_1\|$ on $R(\delta_0)$ respectively, constant $\tilde{K}_2$ depends only on the norm $\|\tilde{\Gamma}_2\|$ and $H_t^{\frac{\alpha}{2}}[\tilde{\Gamma}_0]$ on $R(\delta_0)$, $\tilde{K}_3$ depends only on $\|\tilde{\Gamma}_2\|$

and $H_*^{\frac{\alpha}{2}}[\tilde{\Gamma}_2]$ on $R(\delta_0)$, where

$$\tilde{\Gamma}_0=\left\{\tilde{\lambda}_l,\ \tilde{\zeta}_l,\ \tilde{\mu}_l,\ \frac{1}{\tilde{\lambda}_{\bar{r}}(t,\ 1)},\ \frac{1}{\tilde{\lambda}_{\hat{s}}(t,\ 0)}\right\},$$

$$\tilde{\Gamma}_1=\left\{\tilde{\zeta}_{lj},\ \frac{\partial\tilde{\zeta}_{lj}}{\partial t},\ \frac{\partial\tilde{\zeta}_{lj}}{\partial x},\ \tilde{\zeta}_l,\ \frac{\partial\tilde{\zeta}_l}{\partial t},\ \frac{\partial\tilde{\zeta}_l}{\partial x},\ \tilde{\lambda}_l,\ \frac{1}{\det|\tilde{\zeta}_{lj}|}\right\}, \tag{3.20}$$

$$\tilde{\Gamma}_2=\tilde{\Gamma}_1\cup\left\{\frac{\partial\tilde{\lambda}_l}{\partial x},\ \tilde{\mu}_l,\ \frac{\partial\tilde{\mu}_l}{\partial x},\ \frac{1}{\tilde{\lambda}_{\bar{r}}(t,\ 1)},\ \frac{1}{\hat{\lambda}_{\hat{s}}(t,\ 0)}\right\}.$$

By means of problem (3.9)—(3.16) we define an iterative operator T: $(u,\ v)=T(\tilde{u},\ \tilde{v})$. Obviously, T maps $\Sigma_*(\delta)$ to itself.

From the definition of $\tilde{b}\ (t,\ x)$ it follows that for any $(\tilde{u},\ \tilde{v})\in\Sigma(\delta)$,

$$\|\tilde{b}\|\leqslant D_1(A_0,\ B_0)+D_2(A_1,\ B_1)\delta^{\frac{\alpha}{2}}, \tag{3.21}$$

$$H^\alpha[\tilde{b}]\leqslant D_3(A_1,\ B_1)+D_4(A_1,\ B_2)\delta^{\frac{\alpha}{2}}, \tag{3.22}$$

where, for instance, $D_1(A_0,\ B_0)$ denotes a constant depending only on A_0 and B_0. Substituting (3.21), (3.22) into estimations (2.40)—(2.42), we get

$$|v|\leqslant D_5(A_1,\ B_1)\delta^{\frac{1}{2}}, \tag{3.23}$$

$$|v|_1\leqslant D_6(A_0,\ B_0)+D_7(A_1,\ B_2)\delta^{\frac{\alpha}{2}}, \tag{3.24}$$

$$|v|_2\leqslant D_8(A_1,\ B_1)+D_9(A_1,\ B_2)\delta^{\frac{\alpha}{2}}. \tag{3.25}$$

It follows from (3.20) that

$$\begin{cases}\|\tilde{\Gamma}_0\|\leqslant D_{10}(A_0,\ B_0),\\ \|\tilde{\Gamma}_1\|,\ \|\tilde{\Gamma}_2\|,\ H_t^{\frac{\alpha}{2}}[\tilde{\Gamma}_0]\leqslant D_{11}(A_1,\ B_1),\\ H_*^{\frac{\alpha}{2}}[\tilde{\Gamma}_2]\leqslant D_{12}(A_2,\ B_2).\end{cases} \tag{3.26}$$

Besides,

$$\|\dot{\psi}_{\bar{r}}(t)\|\leqslant\theta A_1+D_{13}(A_0)+D_{14}(A_1,\ B_1)\delta^{\frac{\alpha}{2}}, \tag{3.27}$$

$$H_t^{\frac{\alpha}{2}}[\dot{\psi}_{\bar{r}}(t)]\leqslant\theta A_2+D_{15}(A_1,\ B_1)+D_{16}(A_2,\ B_2)\delta^{\frac{\alpha}{2}} \tag{3.28}$$

and the same estimations for $\dot{\hat{\psi}}_{\hat{s}}(t)$. Hence, estimations (3.18) and (3.19) give

$$\|u\|_1^*\leqslant\theta_1A_1+D_{17}(A_0,\ B_1)+D_{18}(A_1,\ B_1)\delta^{\frac{\alpha}{2}}, \tag{3.29}$$

$$\|u\|_{1+\frac{\alpha}{2}}^*\leqslant\theta_2A_2+D_{19}(A_1,\ B_2)+D_{20}(A_2,\ B_2)\delta^{\frac{\alpha}{2}} \tag{3.30}$$

and from (3.29) and (3.11) we have

$$\|u\|\leqslant D_{21}(A_1,\ B_1)\delta. \tag{3.31}$$

By means of the preceding estimations (3.23)—(3.25), (3.29)—(3.31) we can choose as in [2] constants A_0, A_1, A_2, B_0, B_1, B_2 and δ_* such that the operator T maps $\Sigma(\delta_*)$ into itself. Then, according to Leray Schauder fixed point theorem (cf. [5, 6]) we get as in [1] and [2] the existence of the solution for the first initial-boundary value problem for quasilinear hyperbolic-parabolic coupled systems.

The uniqueness of the solution can be obtained in a similar way as in [2], we omit the detail here.

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