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INITIAL-BOUNDARY VALUE PROBLEMS WITH INTERFACE FOR QUASILINEAR HYPERBOLIC-PARABOLIC COUPLED SYSTEMS

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Abstract

The local time existence and uniqueness theorem is proved for mixed initial-boundary value problems with interface for quasilinear hyperbolic-parabolic coupled systems in two independent variables.

§1. Introduction

In [1, 2, 3] we have proved the local time existence and uniqueness theorems respectively for initial value problems, first initial-boundary value problems and second initial-boundary value problems for quasilinear hyperbolic-parabolic coupled systems. In the study of discontinuous solutions, we may meet initial-boundary value problems with interface or free interface. In this paper we deal with the initial-boundary value problems with interface for quasilinear hyperbolic-parabolic coupled system. We shall use the same notations as in [1, 2, 3].

For the convenience of statement, we shall only pay our attention to the first initial-boundary value problem with interface in what follows. The second initial-boundary value problem with interface can be discussed in a similar way and the corresponding results will be shown at the end of this paper.

Set

$$R(\delta)=\{0\leqslant t\leqslant\delta,\ -1\leqslant x\leqslant 1\},\tag{1.1}$$

$$R^+(\delta)=\{0\leqslant t\leqslant\delta,\ 0\leqslant x\leqslant 1\},\tag{1.2}$$

$$R^-(\delta)=\{0\leqslant t\leqslant\delta,\ -1\leqslant x\leqslant 0\}.\tag{1.3}$$

We consider the following systems on the domains $R^+(\delta)$ and $R^-(\delta)$ respectively.

$$\begin{aligned}\sum_{j=1}^{n}\zeta_{lj}^{\pm}(t,x,u^{\pm},v^{\pm})\left(\frac{\partial u_j^{\pm}}{\partial t}+\lambda_l^{\pm}(t,x,u^{\pm},v^{\pm},v_x^{\pm})\frac{\partial u_j^{\pm}}{\partial x}\right)\\ =\zeta_l^{\pm}(t,x,u^{\pm},v^{\pm})\left(\frac{\partial v^{\pm}}{\partial t}+\lambda_l^{\pm}(t,x,u^{\pm},v^{\pm},v_x^{\pm})\frac{\partial v^{\pm}}{\partial x}\right)\\ +\mu_l^{\pm}(t,x,u,v^{\pm},v_x^{\pm})\qquad(l=1,\cdots,n),\end{aligned}\tag{1.4}$$

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$$\frac{\partial v^\pm}{\partial t}-a^\pm(t,\ x,\ u^\pm,\ v^\pm,\ v_x^\pm)\frac{\partial^2 v^\pm}{\partial x^2}=b^\pm(t,\ x,\ u^\pm,\ v^\pm,\ v_x^\pm), \tag{1.5}$$

where $(u^+,\ v^+)$ and $(u^-,\ v^-)$ are the unknown functions defined on R_δ^+ and R_δ^- respectively, and $\zeta_{ij}^\pm(t,\ x,\ u^\pm,\ v^\pm)$ denote $\zeta_{ij}^+(t,\ x,\ u^+,\ v^+)$ and $\zeta_{ij}^-(t,\ x,\ u^-,\ v^-)$ on R_δ^+ and on R_δ^- respectively, etc.

Without loss of generality, the initial condition can be written as

$$t=0:\ u_i^\pm=0 \qquad (i=1,\ \cdots,\ n), \tag{1.6}$$

$$v^\pm=0, \tag{1.7}$$

and we may assume that

$$a^\pm(0,\ x,\ 0,\ 0,\ 0)=1, \tag{1.8}$$

$$b^\pm(0,\ x,\ 0,\ 0,\ 0)=0, \tag{1.9}$$

$$\zeta_{ij}^\pm(0,\ x,\ 0,\ 0)=\delta_{ij}. \tag{1.10}$$

The boundary conditions are as follows:

$$x=1:\ u_{\bar r}^+=G_{\bar r}(t,\ u^+) \qquad (\bar r=1,\ \cdots,\ R;\ R\leqslant n), \tag{1.11}$$

$$v^+=\varphi^+(t); \tag{1.12}$$

$$x=-1:\ u_{\hat s}^-=\hat G_{\hat s}(t,\ u^-) \qquad (\hat s=S+1,\ \cdots,\ n;\ S\geqslant 0), \tag{1.13}$$

$$v^-=\varphi^-(t). \tag{1.14}$$

Moreover, we assume that the following conditions must be satisfied on the interface $x=0$:

$$u_{\hat q}^+=\hat H_{\hat q}(t,\ u^+,\ u^-,\ v^+,\ v^-) \qquad (\hat q=Q+1,\ \cdots,\ n;\ Q\geqslant 0), \tag{1.15}$$

$$u_{\bar p}^-=H_{\bar p}(t,\ u^+,\ u^-,\ v^+,\ v^-) \qquad (\bar p=1,\ \cdots,\ P;\ P\leqslant n), \tag{1.16}$$

$$v^-=v^++f(t), \tag{1.17}$$

$$\frac{\partial v^-}{\partial x}=\lambda(t,\ u^+,\ u^-,\ v^+,\ v^-)\frac{\partial v^+}{\partial x}+g(t,\ u^+,\ u^-,\ v^+,\ v^-), \tag{1.18}$$

where $\lambda>0$.

As usual, the number of the boundary conditions (1.11), (1.13) or the interface conditions (1.15), (1.16) should be equal to the number of the departing characteristic directions for the hyperbolic systems, i,e., the distribution of characteristic directions should satisfy the following conditions of orientability:

$$\lambda_{\bar r}^+(0,\ 1,\ 0,\ 0,\ 0)<0,\ \lambda_{\bar s}^+(0,\ 1,\ 0,\ 0,\ 0)>0$$
$$(\bar r=1,\ \cdots,\ R;\ \bar s=R+1,\ \cdots,\ n), \tag{1.19}$$

$$\lambda_{\hat r}^-(0,\ -1,\ 0,\ 0,\ 0)<0,\ \lambda_{\hat s}^-(0,\ -1,\ 0,\ 0,\ 0)>0$$
$$(\hat r=1,\ \cdots,\ S;\ \hat s=S+1,\ \cdots,\ n), \tag{1.20}$$

$$\lambda_{\hat p}^+(0,\ 0,\ 0,\ 0,\ 0)<0,\ \lambda_{\hat q}^+(0,\ 0,\ 0,\ 0,\ 0)>0$$
$$(\hat p=1,\ \cdots,\ Q;\ \hat q=Q+1,\ \cdots,\ n), \tag{1.21}$$

$$\lambda_{\bar p}^-(0,\ 0,\ 0,\ 0,\ 0)<0,\ \lambda_{\bar q}^-(0,\ 0,\ 0,\ 0,\ 0)>0$$
$$(\bar p=1,\ \cdots,\ P;\ \bar q=P+1,\ \cdots,\ n). \tag{1.22}$$

In addition, we assume that the following conditions of compatibility are satisfied:

$$
\begin{cases}
G_{\bar r}(0,0)=0 & (\bar r=1,\cdots,R),\\
\hat G_{\hat s}(0,0)=0 & (\hat s=S+1,\cdots,n),\\
\hat H_{\hat q}(0,0,0,0,0)=0 & (\hat q=Q+1,\cdots,n),\\
H_{\bar p}(0,0,0,0,0)=0 & (\bar p=1,\cdots,P),\\
f(0)=0,\\
g(0,0,0,0,0)=0,\\
\varphi^\pm(0)=0;
\end{cases}
\tag{1.23}
$$

$$
\begin{cases}
\mu_{\bar r}^+(0,1,0,0,0)=\dfrac{\partial G_{\bar r}}{\partial t}(0,0)+\sum_{j=1}^n \dfrac{\partial G_{\bar r}}{\partial u_j}(0,0)\mu_j^+(0,1,0,0,0)\\
\qquad (\bar r=1,\cdots,R),\\
\mu_{\hat s}^-(0,-1,0,0,0)=\dfrac{\partial \hat G_{\hat s}}{\partial t}(0,0)+\sum_{j=1}^n \dfrac{\partial \hat G_{\hat s}}{\partial u_j}(0,0)\mu_j^-(0,-1,0,0,0)\\
\qquad (\hat s=S+1,\cdots,n),\\
\mu_{\hat q}^+(0,0,0,0,0)=\dfrac{\partial \hat H_{\hat q}}{\partial t}(0,0,0,0,0)+\sum_{\pm}\sum_{j=1}^n \dfrac{\partial \hat H_{\hat q}}{\partial u_j^\pm}(0,0,0,0,0)\mu_j^\pm(0,0,0,0,0)\\
\qquad (\hat q=Q+1,\cdots,n),\\
\mu_{\bar p}^-(0,0,0,0,0)=\dfrac{\partial H_{\bar p}}{\partial t}(0,0,0,0,0)+\sum_{\pm}\sum_{j=1}^n \dfrac{\partial H_{\bar p}}{\partial u_j^\pm}(0,0,0,0,0)\mu_j^\pm(0,0,0,0,0)\\
\qquad (\bar p=1,\cdots,P),\\
\dot\varphi^\pm(0)=0,\\
\dot f(0)=0.
\end{cases}
\tag{1.24}
$$

Finally, we assume that the following conditions of solvability hold i.e., the boundary conditions (1.11), (1.13) and the interface conditions (1.15), (1.16) may be rewritten as the following forms:

$$x=1:\quad u_{\bar r}^+=I_{\bar r}(t,u_{\bar s}^+)\qquad (\bar r=1,\cdots,R,\ \bar s=R+1,\cdots,n),\tag{1.25}$$

$$x=-1:\ u_{\hat s}^-=\hat I_{\hat s}(t,u_{\hat r}^-)\qquad (\hat r=1,\cdots,S,\ \hat s=S+1,\cdots,n)\tag{1.26}$$

and

$$x=0:\ u_{\hat q}^+=\hat J_{\hat q}(t,u_{\hat p}^+,u_{\bar q}^-,v^+,v^-)\qquad (\hat p=1,\cdots,Q,\ \hat q=Q+1,\cdots,n),\tag{1.27}$$

$$u_{\bar p}^-=J_{\bar p}(t,u_{\hat p}^+,u_{\bar q}^-,v^+,v^-)\qquad (\bar p=1,\cdots,P,\ \bar q=P+1,\cdots,n).\tag{1.28}$$

In this case, in the proof we can assume, without loss of generality (refer to [2]), that the following conditions are satisfied:

$$\sum_{j=1}^n\left|\frac{\partial G_{\bar r}}{\partial u_j^+}(0,0)\right|<1,\quad \sum_{j=1}^n\left|\frac{\partial \hat G_{\hat s}}{\partial u_j^-}(0,0)\right|<1,\tag{1.29}$$

$$
\begin{aligned}
&\sum_{\pm}\sum_{j=1}^n\left|\frac{\partial \hat H_{\hat q}}{\partial u_j^\pm}(0,0,0,0,0)\right|<1,\\
&\sum_{\pm}\sum_{j=1}^n\left|\frac{\partial H_{\bar p}}{\partial u_j^\pm}(0,0,0,0,0)\right|<1.
\end{aligned}
\tag{1.30}
$$

Here and in (1.24) the meaning of the notation $\sum_{\pm}$ is as follows

$$\sum_{\pm}h^\pm=h^++h^-.$$

§ 2. Some estimates for the solutions of the first initial-boundary value problem with interface for heat equations

In this section we discuss the first initial-boundary value problem with interface for heat equations and establish some à priori estimates for its solutions.

On $R(\delta_0)$ we consider the following problem:

$$\begin{cases} \dfrac{\partial v^+}{\partial t}-\dfrac{\partial^2 v^+}{\partial x^2}=0 & (t>0,\ 0\leqslant x\leqslant 1), & (2.1)\\ \dfrac{\partial v^-}{\partial t}-\dfrac{\partial^2 v^-}{\partial x^2}=0 & (t>0,\ -1\leqslant x\leqslant 0), & (2.2)\\ t=0:\quad v^+=0 & (0\leqslant x\leqslant 1), & (2.3)\\ \qquad\quad v^-=0 & (-1\leqslant x\leqslant 0), & (2.4)\\ x=-1:\quad v^-=\varphi^-(t), & & (2.5)\\ x=1:\quad v^+=\varphi^+(t), & & (2.6)\\ x=0:\quad v^-=v^++f(t), & & (2.7)\\ \qquad\quad \dfrac{\partial v^-}{\partial x}=\lambda(t)\dfrac{\partial v^+}{\partial x}+g(t), & & (2.8)\end{cases}$$

where $\lambda_1\geqslant\lambda(t)\geqslant\lambda_0>0$ (λ_0, λ_1 are constants), $x=0$ is the interface of the solutions v^+ and v^-, and $\varphi^\pm(t)$, $f(t)$, $g(t)$ are given functions.

We first give some lemmas, the proof of which can be found in [6].

Lemma 2.1. *If* $f(t)\in \mathrm{Lip}\,\dfrac{1+\alpha}{2}$ $(0<\alpha<1)$ *and* $f(a)=0$, *then set*

$$F(t)=\int_a^t (t-s)^{-\frac{1}{2}}f(s)\,ds, \tag{2.9}$$

we heve

$$\dot F(t)=f(t)(t-a)^{-\frac{1}{2}}-\frac{1}{2}\int_a^t (t-s)^{-\frac{3}{2}}[f(s)-f(t)]\,ds. \tag{2.10}$$

Lemma 2.2. *If* $f(t)\in\mathrm{Lip}\,\dfrac{1+\alpha}{2}$ $(0<\alpha<1)$, *then set*

$$F(t)=\int_\tau^t (t-s)^{-\frac{1}{2}}(s-\tau)^{-\frac{1}{2}}f(s)\,ds, \tag{2.11}$$

we have

$$\dot F(t)=\frac{1}{2}\int_\tau^t (t-s)^{-\frac{3}{2}}(s-\tau)^{-\frac{1}{2}}(f(t)-f(s))\,ds. \tag{2.12}$$

Lemma 2.3: *If* $\varphi^\pm(t)$, $f(t)\in\mathrm{Lip}\,\dfrac{1+\alpha}{2}$ $(0<\alpha<1)$, $\lambda(t)$, $g(t)\in C^0$ *and*

$$\varphi^\pm(0)=f(0)=g(0)=0, \tag{2.13}$$

then problem (2.1) – (2.8) *admits a unique solution* $v^\pm(t,x)$ *on* $R^\pm(\delta_0)$, *this solution together with its first order derivative with respect to* x *are continuous.*

It follows from [6] that the solution of problem (2.1)—(2.8) can be expressed as follows:

$$\begin{cases} v^-(t, x) = \int_0^t U(t, x; \tau, -1)\varphi_1(\tau)d\tau + \int_0^t U(t, x; \tau, 0)\varphi_2(\tau)d\tau, \\ v^+(t, x) = \int_0^t U(t, x; \tau, 0)\varphi_3(\tau)d\tau + \int_0^t U(t, x; \tau, 1)\varphi_4(\tau)d\tau. \end{cases} \tag{2.14}$$

Moreover

$$\begin{cases} \frac{\partial v^-}{\partial x} = -\frac{1}{2}\int_0^t V(t, x; \tau, -1)\varphi_1(\tau)d\tau - \frac{1}{2}\int_0^t V(t, x; \tau, 0)\varphi_2(\tau)d\tau, \\ \frac{\partial v^+}{\partial x} = -\frac{1}{2}\int_0^t V(t, x; \tau, 0)\varphi_3(\tau)d\tau - \frac{1}{2}\int_0^t V(t, x; \tau, 1)\varphi_4(\tau)d\tau, \end{cases} \tag{2.15}$$

where $U(t, x; \tau, \xi)$ is the fundamental solution of the heat equation:

$$U(t, x; \tau, \xi) = \frac{1}{\sqrt{2\pi(t-\tau)}} e^{-\frac{(x-\xi)^2}{4(t-z)}}, \ (t \geqslant \tau), \tag{2.16}$$

$$V(t, x; \tau, \xi) = \frac{(x-\xi)}{2\sqrt{\pi}\ (t-\tau)^{3/2}} e^{-\frac{(x-\xi)^2}{4(t-\tau)}}, \tag{2.17}$$

and $\varphi_i(t)\ (i=1, \cdots, 4)$ is the solution of the following system of integral equations:

$$\varphi_1(t) = \frac{2}{\sqrt{\pi}}\left[t^{-\frac{1}{2}}\varphi^-(t) - \frac{1}{2}\int_0^t (t-\tau)^{-\frac{3}{2}}(\varphi^-(\tau) - \varphi^-(t))d\tau - \int_0^t \frac{\partial U_1(\tau, t)}{\partial t}\varphi_2(\tau)d\tau, \right] \tag{2.18}$$

$$\begin{aligned} \varphi_2(t) = \frac{2}{1+\lambda(t)}\Big\{ & g(t) + \frac{1}{2}\int_0^t V(t, 0; \tau, -1)\varphi_1(\tau)d\tau \\ & - \frac{\lambda(t)}{2}\int_0^t V(t, 0; \tau, 1)\varphi_4(\tau)d\tau + \\ & + \frac{\lambda(t)}{\sqrt{\pi}}\Big[t^{-\frac{1}{2}}f(t) - \frac{1}{2}\int_0^t (t-\tau)^{-\frac{3}{2}}(f(\tau) - f(t))d\tau \\ & - \int_0^t \frac{\partial U_1}{\partial t}(\tau, t)\varphi_1(\tau)d\tau + \int_0^t \frac{\partial U_2}{\partial t}(\tau, t)\varphi_4(\tau)d\tau\Big]\Big\}, \end{aligned} \tag{2.19}$$

$$\begin{aligned} \varphi_3(t) = \frac{2}{1+\lambda(t)}\Big\{ & g(t) + \frac{1}{2}\int_0^t V(t, 0; \tau, -1)\varphi_1(\tau)d\tau \\ & - \frac{\lambda(t)}{2}\int_0^t V(t, 0; \tau, 1)\varphi_4(\tau)d\tau \\ & - \frac{1}{\sqrt{\pi}}\Big[t^{-\frac{1}{2}}f(t) - \frac{1}{2}\int_0^t (t-\tau)^{-\frac{3}{2}}(f(\tau) - f(t))d\tau \\ & - \int_0^t \frac{\partial U_1}{\partial t}(\tau, t)\varphi_1(\tau)d\tau + \int_0^t \frac{\partial U_2}{\partial t}\varphi_4(\tau)d\tau \Big]\Big\}, \end{aligned} \tag{2.20}$$

$$\begin{aligned} \varphi_4(t) = \frac{2}{\sqrt{\pi}}\Big[& t^{-\frac{1}{2}}\varphi^+(t) - \frac{1}{2}\int_0^t (t-\tau)^{-\frac{3}{2}}(\varphi^+(\tau) - \varphi^-(t))d\tau \\ & - \int_0^t \frac{\partial U_2}{\partial t}(\tau, t)\varphi_3(\tau)d\tau\Big], \end{aligned} \tag{2.21}$$

in which

$$U_1(\tau, Z) = \int_\tau^Z (Z-t)^{-\frac{1}{2}}U(t, 0; \tau, -1)dt, \tag{2.22}$$

$$U_2(\tau, Z) = \int_\tau^Z (Z-t)^{-\frac{1}{2}}U(t, 0; \tau, 1)dt. \tag{2.23}$$

For the smoothness of the solution (2.14), we prove tne following:

Lemma 2.4. *If* $\dot{\varphi}^{\pm}(t), \dot{f}(t)\in \mathrm{Lip}\frac{\alpha}{2}$, $\lambda(t), g(t)\in \mathrm{Lip}\,\frac{1+\alpha}{2}$ $(0<\alpha<1)$ *and* $\varphi^{\pm}(0)=f(0)=g(0)=0$, $\dot{\varphi}^{\pm}(0)=\dot{f}(0)=0$, *then the solution* $v^{\pm}(t, x)$ *given by* (2.14) *possesses the second order continuous derivative* $\frac{\partial^2 v^{\pm}}{\partial x^2}$ *with respect to* x *on the closed domains* $R^{\pm}(\delta)$ *respectively. Moreover*

$$\frac{\partial^2 v^-}{\partial x^2}=-\frac{1}{2}\int_0^t \frac{\partial V}{\partial x}(t, x; \tau, -1)(\varphi_1(\tau)-\varphi_1(t))d\tau-\frac{1}{2}\int_0^t \frac{\partial V}{\partial x}(t, x; \tau, 0)$$
$$(\varphi_2(\tau)-\varphi_2(t))d\tau+U(t, x; 0, -1)\varphi_1(t)+U(t, x; 0, 0)\varphi_2(t), \tag{2.24}$$
$$\frac{\partial^2 v^+}{\partial x^2}=-\frac{1}{2}\int_0^t \frac{\partial V}{\partial x}(t, x; \tau, 0)(\varphi_3(\tau)-\varphi_3(t))d\tau-\frac{1}{2}\int_0^t \frac{\partial V}{\partial x}(t, x; \tau, 1).$$
$$(\varphi_4(\tau)-\varphi_4(t))d\tau+U(t, x; 0, 0)\varphi_3(t)+U(t, x; 0, 1)\varphi_4(t). \tag{2.25}$$

Proof The crucial point is to prove that $\varphi_i(t)$ $(i=1, \cdots, 4)$ belongs to $\mathrm{Lip}\,\frac{1+\alpha}{2}$.

We first prove that all the inhomogeneous terms in the system of integral equations (2.18)—(2.21) belong to $\mathrm{Lip}\,\frac{1+\alpha}{2}$. These inhomogeneous terms are composed of two parts, without considering a constant factor, one is $\frac{g(t)}{1+\lambda(t)}$ which belongs to $\mathrm{Lip}\,\frac{1+\alpha}{2}$ obviously, and another one is of the following form

$$t^{-\frac{1}{2}}\varphi^-(t)-\frac{1}{2}\int_0^t (t-\tau)^{-\frac{3}{2}}(\varphi^-(\tau)-\varphi^-(t))d\tau\equiv J(t) \tag{2.26}$$

$\left(\text{or with a factor } \frac{1}{1+\lambda(t)}\right)$, hence we only need to prove that $J(t)$ defined by (2.26) belongs to $\mathrm{Lip}\frac{1+\alpha}{2}$. Since $\dot{\varphi}^-(t)\in\mathrm{Lip}\frac{\alpha}{2}$ and $\dot{\varphi}^-(0)=0$, we get

$$R(t)=\int_0^t (t-\tau)^{-\frac{1}{2}}\varphi^-(\tau)d\tau=2\int_0^t (t-\tau)^{\frac{1}{2}}\,\dot{\varphi}^-(\tau)d\tau,$$

thus by Lemma 2.1

$$J(t)=\dot{R}(t)=\int_0^t (t-\tau)^{-\frac{1}{2}}\,\dot{\varphi}^-(\tau)d\tau$$
$$=\int_0^t (t-\tau)^{-\frac{1}{2}}(\dot{\varphi}^-(\tau)-\dot{\varphi}^-(t))d\tau+\int_0^t (t-\tau)^{-\frac{1}{2}}\,\dot{\varphi}^-(t)d\tau=J_1+J_2.$$

Assuming $t_1>t_2>0$, $\gamma=t_1-t_2\leqslant t_2$, we have

$$J_1(t_1)-J_1(t_2)=\int_{t_1-2\gamma}^{t_1}(t_1-\tau)^{-\frac{1}{2}}(\dot{\varphi}^-(\tau)-\dot{\varphi}^-(t_1))d\tau$$
$$-\int_{t_2-\gamma}^{t_2}(t_2-\tau)^{-\frac{1}{2}}(\dot{\varphi}^-(\tau)-\dot{\varphi}^-(t_2))d\tau$$
$$+\int_0^{t_2-\gamma}[(t_1-\tau)^{-\frac{1}{2}}-(t_2-\tau)^{-\frac{1}{2}}](\dot{\varphi}^-(\tau)-\dot{\varphi}^-(t_2)]d\tau$$
$$+\int_0^{t_2-\gamma}(t_1-\tau)^{-\frac{1}{2}}(\dot{\varphi}^-(t_2)-\dot{\varphi}^-(t_1))d\tau$$
$$=J_{11}+J_{12}+J_{13}+J_{14}.$$

It is easy to see that

$$|J_{11}|\leqslant L_1\int_{t_1-2\gamma}^{t_1}(t_1-\tau)^{-\frac{1}{2}+\frac{\alpha}{2}}d\tau\leqslant L_2\gamma^{\frac{1+\alpha}{2}}.$$

Here and hereafter, $L_i(=1, 2, \cdots)$ denote constants dependent only on $\|\dot{\varphi}^-\|+H^{\frac{\alpha}{2}}[\dot{\varphi}^-]$.

In a similar way, we get

$$|J_{12}|\leqslant L_3\cdot\gamma^{\frac{1+\alpha}{2}},$$

$$|J_{13}|\leqslant L_4\int_0^{t_2-\gamma}\int_{t_2}^{t_1}(t-\tau)^{-\frac{3}{2}}(t_2-\tau)^{\frac{\alpha}{2}}dt\,d\tau\leqslant L_5\gamma^{\frac{1+\alpha}{2}}.$$

Moreover

$$\begin{aligned}J_{14}+J_2(t_1)-J_2(t_2)&=\int_{t_1-2\gamma}^{t_1}(t_1-\tau)^{-\frac{1}{2}}\dot{\varphi}^-(t_1)d\tau+\int_0^{t_2-\gamma}(t_1-\tau)^{-\frac{1}{2}}\dot{\varphi}^-(t_2)d\tau\\&\quad-\int_0^{t_2}(t_2-\tau)^{-\frac{1}{2}}\dot{\varphi}^-(t_2)d\tau\\&=\int_{t_2-2\gamma}^{t_2}(t_2-\tau)^{-\frac{1}{2}}(\dot{\varphi}^-(t_1)-\dot{\varphi}^-(t_2))d\tau+\int_{-\gamma}^{0}(t_2-\tau)^{-\frac{1}{2}}\dot{\varphi}^-(t_2)d\tau\\&=S_1+S_2,\end{aligned}$$

where

$$|S_1|\leqslant L_6\int_{t_2-2\gamma}^{t_2}(t_2-\tau)^{-\frac{1}{2}}\gamma^{\frac{\alpha}{2}}d\tau\leqslant L_7\gamma^{\frac{1+\alpha}{2}},$$

$$|S_2|\leqslant L_8t_2^{\frac{\alpha}{2}}\int_{-\gamma}^{0}(t_2-\tau)^{-\frac{1}{2}}d\tau\leqslant L_9\gamma^{\frac{1+\alpha}{2}}.$$

In the case $t_2<\gamma$, we can obtain the same results, too.

The combination of the preceding estimates gives that the inhomogeneous term in system (2.18)—(2.21) denoted by $\varphi_0(t)$) belongs to Lip $\frac{1+\alpha}{2}$ and satisfies

$$H^{\frac{1+\alpha}{2}}[\varphi_0(t)]\leqslant C_1\|\omega\|, \tag{2.27}$$

where constant C_1 depends only on δ_0, and

$$\|\omega\|=\sum_{\pm}(\|\varphi^\pm\|_1+H^{\frac{\alpha}{2}}[\dot{\varphi}^\pm]+\|f\|_1+H^{\frac{\alpha}{2}}[\dot{f}]+\|g\|+H^{\frac{1+\alpha}{2}}[g]+H^{\frac{1+\alpha}{2}}[\lambda]). \tag{2.28}$$

Noticing that the integral kernels in system (2.18)—(2.21), i. e. $V(t,0;\tau,-1)$, $V(t, 0;\tau, 1)$ are C^∞ functions, and it is easy to prove that $\frac{\partial U_1}{\partial t}$, $\frac{\partial U_2}{\partial t}$ are continuously differentiable with respect to t. Thus the solution $\varphi_i(t)$ $(i=1, \cdots, 4)$ of the system of integral equations belongs to Lip $\frac{1+\alpha}{2}$, then $\frac{\partial^2 v^\pm}{\partial x^2}$ exists and (2.24), (2.25) hold. The proof of Lemma 2.4 is completed.

In what follows we need som à priori estimates for the solution $\varphi_i(t)$ $(i=1, \cdots, 4)$ of system (2.18)—(2.21). This system can be rewritten in the following form

$$\varphi(t)=\varphi(t)+\int_0^t N(\tau, t)\varphi(\tau)d\tau, \tag{2.29}$$

where $\varphi(t)$, $\varphi_0(t)$ are 4-dimensional vectors, and N is a 4×4 matrix. By the expression of $\varphi_0(t)$, we have

$$\|\varphi_0\| = \max_{1\leqslant i\leqslant 4, 0\leqslant t\leqslant\delta} \|\varphi_{0i}(t)\| \leqslant K_1(\|g\| + \delta^{\frac{1}{2}}(\sum_{\pm}\|\varphi^{\pm}\|_1 + \|f\|_1), \tag{2.30}$$

here and hereafter $K_i(i=1, 2, \cdots)$ denote constants depending only on δ_0. Then it is easy to get

$$\|\varphi\| = \max_{1\leqslant i\leqslant 4, 0\leqslant t\leqslant\delta} |\varphi_i(t)| \leqslant K_2\|\varphi_0\| \leqslant K_3\delta^{\frac{1+\alpha}{2}}\|\omega\|, \tag{2.31}$$

where $\|\omega\|$ is defined by (2.28).

By (2.29) we have

$$H^{\frac{1+\alpha}{2}}[\varphi] = \max_{1\leqslant i\leqslant 4} H^{\frac{1+\alpha}{2}}[\varphi_i] \leqslant H^{\frac{1+\alpha}{2}}[\varphi] + K_4\|\varphi\|,$$

then from (2.27) and (2.31) it follows that

$$H^{\frac{1+\alpha}{2}}[\varphi] \leqslant K_5\|\omega\|. \tag{2.32}$$

We now establish some à priori estimates for the solution of problem (2.1)—(2.8).

Lemma 2.5. *Under the same assumptions as in Lemma 2.4, for any $\delta(0\leqslant\delta\leqslant\delta_0)$ we have on $R(\delta)$ that*

$$\|v\| \leqslant A_1\delta^{\frac{1}{2}}(\|g\| + \delta^{\frac{1}{2}}\sum_{\pm}\|\varphi^{\pm}\|_1 + \delta^{\frac{1}{2}}\|f\|_1), \tag{2.33}$$

$$\left\|\frac{\partial v}{\partial x}\right\| \leqslant A_2(\|g\| + \delta^{\frac{1}{2}}\|f\|_1 + \delta^{\frac{1}{2}}\sum_{\pm}\|\varphi^{\pm}\|_1) \leqslant A_3\delta^{\frac{1+\alpha}{2}}\|\omega\| \tag{2.34}$$

and

$$\left\|\frac{\partial v}{\partial t}\right\| = \left\|\frac{\partial^2 v}{\partial x^2}\right\| \leqslant A_4\delta^{\frac{\alpha}{2}}\|\omega\|. \tag{2.35}$$

Here and hereafter A_i $(i=1, 2, \cdots)$ are constants depending only on δ_0, and $\|v\| = \sum_{\pm}\|v^{\pm}\|$, etc.

Proof By means of (2.14), (2.15) and noticing the properties of the double layer heat potential, (2.33), (2.34) follow from (2.30), (2.31).

Furthermore, for the nonintegral terms in (2.24) we have, for instance,

$$|U(t, x; 0, -1)\varphi_1(t)| \leqslant K_6\frac{1}{\sqrt{t}}|\varphi_1(t) - \varphi_1(0)| \leqslant K_7 t^{\frac{\alpha}{2}} H^{\frac{1+\alpha}{2}}[\varphi_1].$$

On the other hand, for the integral terms in (2.24) we have, for instance,

$$\left|\int_0^t \frac{\partial V}{\partial x}(t, x; \tau, -1)(\varphi_1(\tau) - \varphi_1(t))d\tau \leqslant K_8 H^{\frac{1+\alpha}{2}}[\varphi_1]\cdot t^{\frac{\alpha}{2}}.\right.$$

Thus, using (2.32) we get (2.35).

Lemma 2.6. *Under the same assumptions as in Lemma 2.4, we have*

$$H_t^{\frac{1+\alpha}{2}}\left[\frac{\partial v}{\partial x}\right] \leqslant A_5\|\omega\|, \tag{2.36}$$

$$H^{\alpha}\left[\frac{\partial v}{\partial t}\right], H^{\alpha}\left[\frac{\partial^2 v}{\partial x^2}\right] \leqslant A_6\|\omega\|. \tag{2.37}$$

Proof Using the inequality

$$\left|\frac{\partial^{i+j}U(t,\ x;\tau,\ \xi)}{\partial x^i\partial t^j}\right|\leqslant P_{ij}(t-\tau)^{-\frac{i+2j+1}{2}}\qquad (P_{ij}\text{ are constants}),\tag{2.38}$$

in a similar way as estimating $H^{\frac{1+\alpha}{2}}[J(t)]$ in Lemma 2.4, we get (2.36).

We now derive inequality (2.37). In (2.24), set

$$I(t,\ x)=\int_0^t\frac{\partial V}{\partial x}(t,\ x;\tau,\ -1)(\varphi_1(\tau)-\varphi_1(t))d\tau.$$

If $t\geqslant\gamma^2=(x_1-x_2)^2$, then

$$\begin{aligned}I(t,\ x_1)-I(t,\ x_2)&=\int_0^{t-\gamma^2}\Big(\frac{\partial V}{\partial x}(t,\ x_1;\tau,\ -1)-\frac{\partial V}{\partial x}(t,\ x_2;\tau,\ -1)\Big)(\varphi_1(\tau)-\varphi_1(t))d\tau\\&+\int_{t-\gamma^2}^t\frac{\partial V}{\partial x}(t,\ x_1;\tau,\ -1)(\varphi_1(\tau)-\varphi_1(t))d\tau\\&-\int_{t-\gamma^2}^t\frac{\partial V}{\partial x}(t,\ x_2;\tau,\ -1)(\varphi_1(\tau)-\varphi_1(t))d\tau\\&=I_1+I_2+I_3,\end{aligned}$$

where

$$\begin{aligned}|I_1|&\leqslant H_t^{\frac{1+\alpha}{2}}[\varphi_1]\int_0^{t-\gamma^2}\int_{x_2}^{x_1}\frac{\partial^2V}{\partial x^2}(t,\ x;\tau,\ -1)(t-\tau)^{\frac{1+\alpha}{2}}dx\,d\tau\\&\leqslant K_9H_t^{\frac{1+\alpha}{2}}[\varphi_1]\cdot\gamma\cdot\int_0^{t-\gamma^2}(t-\tau)^{-2}(t-\tau)^{\frac{1+\alpha}{2}}d\tau\leqslant K_{10}\gamma^\alpha H_t^{\frac{1+\alpha}{2}}[\varphi_1],\\|I_2|&\leqslant K_{11}H_t^{\frac{1+\alpha}{2}}[\varphi_1]\int_{t-\gamma^2}^t(t-\tau)^{-\frac{3}{2}}(t-\tau)^{\frac{1+\alpha}{2}}d\tau\leqslant K_{12}\gamma^\alpha H_t^{\frac{1+\alpha}{2}}[\varphi_1],\end{aligned}$$

and we can also get the same estimate for I_3. Thus

$$|I(t,\ x_1)-I(t,\ x_2)|\leqslant K_{13}\gamma^\alpha H_t^{\frac{1+\alpha}{2}}[\varphi_1].\tag{2.39}$$

In the case $t<\gamma^2$, the same estimates can be obtained immediately.

For the nonintegral terms in (2.24), for instance, we have

$$\begin{aligned}&|U(t,\ x_1;0,\ 0)\varphi_2(t)-U(t,\ x_2;0,\ 0)\varphi_2(t)|\\&=\frac{\varphi_2(t)}{\sqrt{t}}\big(e^{-\frac{x_1^2}{4t}}-e^{-\frac{x_2^2}{4t}}\big)=-\frac{1}{2}\frac{\varphi_2(t)}{\sqrt{t}}\int_0^1\frac{1}{t}(x_1-x_2)(x_2+\tau(x_1-x_2))e^{-\frac{(x_2+\tau(x_1-x_2))^2}{4t}}d\tau\\&\leqslant K_{14}(\|\varphi_2\|+H^{\frac{1+\alpha}{2}}[\varphi_2])\cdot\int_0^1t^{\frac{\alpha}{2}-1}(x_1-x_2)(x_2+\tau(x_1-x_2))e^{-\frac{(x_2+\tau(x_1-x_2))^2}{4t}}d\tau\\&\leqslant K_{15}(\|\varphi_2\|+H^{\frac{1+\alpha}{2}}[\varphi_2])\cdot\int_0^1(x_2+\tau(x_1-x_2))^{\alpha-1}(x_1-x_2)d\tau\\&\leqslant K_{16}(\|\varphi_2\|+H^{\frac{1+\alpha}{2}}[\varphi_2])\cdot\gamma^\alpha,\end{aligned}\tag{2.40}$$

where $\gamma^2=(x_1-x_2)^2$. Using (2.39) and (2.40) it follows from (2.24) and (2.25) that

$$H_x^\alpha\Big[\frac{\partial^2v}{\partial x^2}\Big]\leqslant K_{17}(\|\boldsymbol{\varphi}\|+H^{\frac{1+\alpha}{2}}[\boldsymbol{\varphi}]).$$

In a similar way, we have

$$H_t^{\frac{\alpha}{2}}\Big[\frac{\partial^2v}{\partial x^2}\Big]\leqslant K_{18}(\|\boldsymbol{\varphi}\|+H^{\frac{1+\alpha}{2}}[\boldsymbol{\varphi}]).$$

Hence, using (2.31), (2.32) we get estimate (2.37).

We now turn to the case of inhomogeneous equations.

Consider the solution of the inhomogeneous equations

$$\begin{cases} \dfrac{\partial v^+}{\partial t}-\dfrac{\partial^2 v^+}{\partial x^2}=b^+(t, x) \qquad (t>0,\ 0\leqslant x\leqslant 1), & (2.41)\\ \dfrac{\partial v^-}{\partial t}-\dfrac{\partial^2 v^-}{\partial x^2}=b^-(t, x) \qquad (t>0,\ -1\leqslant x\leqslant 0) & (2.42)\end{cases}$$

with conditions (2.3)—(2.8).

Let

$$w^+(t, x)=\int_0^t\int_0^1 G(t, x; \tau, \xi)b^-(\xi, \tau)d\xi\, d\tau, \tag{2.43}$$

$$w^-(t, x)=\int_0^t\int_{-1}^0 G(t, x; \tau, \xi)b^+(\xi, \tau)d\xi\, d\tau, \tag{2.44}$$

where $G(t, x; \tau, \xi)$ is the Green function for the first initial-boundary value problem of the heat equation. Obviously $w^\pm(t, x)$ are the solutions of (2.41) and (2.42)[3] respectively, which satisfies the homogeneous boundary condition $w^\pm=0$ on both $x=0$ and $x=1$. Furthermove, we have

Lemma 2.7. *If $b^\pm(0, x)=0$, and $b^\pm(t, x)$ belong to* Lip α *and* Lip$\frac{\alpha}{2}$ $(0<\alpha<1)$ *with respect to x and t on $R^\pm(\delta_0)$ respectively, then $w^\pm(t, x)$ and thier derivatives $\frac{\partial w^\pm}{\partial t}$, $\frac{\partial^2 w^\pm}{\partial x^2}$ are continuous on the closed domain $R^\pm(\delta_0)$ respectively, $\frac{\partial w^\pm}{\partial x}$ belong to* Lip $\frac{1+\alpha}{2}$ *with respect to t, $\frac{\partial^2 w^\pm}{\partial x^2}$, $\frac{\partial w^\pm}{\partial t}$ belong to* Lip $\frac{\alpha}{2}$ *and* Lip α *with respect to t and x respectieely. Moreover*

$$w^\pm(0, x)=\frac{\partial w^\pm}{\partial x}(0, x)=\frac{\partial w^\pm}{\partial t}(0, x)=\frac{\partial^2 w^\pm}{\partial x^2}(0, x)=0, \tag{2.45}$$

and the following estimates hold on $R(\delta)$ (the proof can be found in [3])

$$\begin{cases}\|w^\pm\|\leqslant K_{19}\delta\|b^\pm\|,\\ \left\|\dfrac{\partial w^\pm}{\partial x}\right\|\leqslant K_{20}\delta^{\frac{1}{2}}\|b^\pm\|,\end{cases} \tag{2.46}$$

$$\begin{cases}\left\|\dfrac{\partial w^\pm}{\partial t}\right\|+\left\|\dfrac{\partial^2 w^\pm}{\partial x^2}\right\|\leqslant K_{21}(\delta^{\frac{\alpha}{2}}H_x^{\frac{\alpha}{2}}[b^\pm]+\|b^\pm\|),\\ H_t^{\frac{1}{2}}\left[\dfrac{\partial w^\pm}{\partial x}\right]\leqslant K_{22}\|b^\pm\|,\end{cases} \tag{2.47}$$

$$\begin{cases}H^\alpha\left[\dfrac{\partial w^\pm}{\partial t}\right],\ H^\alpha\left[\dfrac{\partial^2 w^\pm}{\partial x^2}\right]\leqslant K_{23}(\|b^\pm\|+H^\alpha[b^\pm]),\\ H_t^{\frac{1+\alpha}{2}}\left[\dfrac{\partial w^\pm}{\partial x}\right]\leqslant K_{24}H_t^{\frac{\alpha}{2}}[b^\pm],\end{cases} \tag{2.48}$$

Set

$$\bar{v}^\pm=v^\pm-w^\pm, \tag{2.49}$$

problem (2.41), (2.42), (2.3)—(2.8) can be reduced to

$$\frac{\partial \bar{v}^-}{\partial t}-\frac{\partial^2 \bar{v}^-}{\partial x^2}=0 \qquad (t>0,\ -1\leqslant x\leqslant 0), \tag{2.50}$$

$$\frac{\partial \bar{v}^+}{\partial t}-\frac{\partial^2 \bar{v}^+}{\partial x^2}=0 \qquad (t>0,\ 0\leqslant x\leqslant 1), \tag{2.51}$$

$$t=0:\quad \bar{v}^\pm=0, \tag{2.52}$$

$$x=-1: \quad \bar{v}^- = \varphi^-(t), \tag{2.53}$$

$$x=1: \quad \bar{v}^+ = \varphi^+(t), \tag{2.54}$$

$$x=0: \quad \bar{v}^- = \bar{v}^+ + f(t), \tag{2.55}$$

$$\frac{\partial \bar{v}^-}{\partial x} = \lambda(t)\frac{\partial \bar{v}^+}{\partial x} + g(t) - \frac{\partial w^-}{\partial x}(0, t) + \lambda(t)\frac{\partial w^+}{\partial x}(0, t). \tag{2.56}$$

The problem (2.50)—(2.56) is of the first initial-boundary value problem with interface, which we have discussed above. Hence, by means of Lemma 2.3—2.7, we have

Lemma 2.8: *If the functions $b^\pm(t, x)$, $\varphi^\pm(t)$, $f(t)$, $\lambda(t)$, and $g(t)$ satisfy all the assumptions in Lemma 2.7 and Lemma 2.4, then the problem (2.41), (2.42), (2.3)—(2.8) of inhomogeneous equations admits a unique solution $v^\pm(t, x)$ on $R(\delta_0)$. Moreover, the following à priori estimates hold on $R(\delta)$ $(0<\delta\leqslant\delta_0)$:*

$$\|v\| \leqslant K_{25}\delta^{\frac{1}{2}}(\|g\| + \delta^{\frac{1}{2}}\sum_{\pm}(\|\varphi^\pm\|_1 + \|f\|_1 + \|b^\pm\|)), \tag{2.57}$$

$$\left\|\frac{\partial v}{\partial x}\right\| \leqslant K_{26}(\|g\| + \delta^{\frac{1}{2}}\sum_{\pm}(\|\varphi^\pm\|_1 + \|f\|_1 + \|b^\pm\|))$$

$$\leqslant K_{27}\delta^{\frac{1+\alpha}{2}}(\|\omega\| + \sum_{\pm} H_t^{\frac{\alpha}{2}}[b^\pm]), \tag{2.58}$$

$$\left\|\frac{\partial v}{\partial t}\right\|, \left\|\frac{\partial^2 v}{\partial x^2}\right\| \leqslant K_{28}\delta^{\frac{\alpha}{2}}(\|\omega\| + \sum_{\pm} H^\alpha[b^\pm]), \tag{2.59}$$

$$H_t^{\frac{1}{2}}\left[\frac{\partial v}{\partial x}\right] \leqslant K_{29}(\|b\| + \delta^{\frac{\alpha}{2}}(\|\omega\| + \sum_{\pm} H_t^{\frac{\alpha}{2}}[b^\pm])), \tag{2.60}$$

$$H_t^{\frac{1+\alpha}{2}}\left[\frac{\partial v}{\partial x}\right] \leqslant K_{30}(\|\omega\| + \sum_{\pm} H_t^{\frac{\alpha}{2}}[b^\pm]), \tag{2.61}$$

$$H^\alpha\left[\frac{\partial v}{\partial t}\right], \ H^\alpha\left[\frac{\partial^2 v}{\partial x^2}\right] \leqslant K_{31}(\|\omega\| + \sum_{\pm} H^\alpha[b^\pm]). \tag{2.62}$$

§ 3. Existence and uniqueness of the solution for the first initial-boundary value problem with interface

By means of the à priori estimates established in § 2 for the solutions of the first initial-boundary value problem with interface for heat equations and the à priori estimates obtained in [2] for the solutions of the mixed initial-boundary value problem for linear hyperbolic systems, we are, in this section, going to prove the existence and uniqueness of the solution of the first initial-boundary value problem with interface for quasilinear hyperbolic-parabolic coupled systems.

Suppose that the coefficients of system (1.4), (1.5) and the given functions in the boundary conditions satisfy the fallowing conditions of smoothness on the domain under consideration:

(i) $\zeta_{ij}^\pm(t, x, u^\pm, v^\pm)$, $\zeta_i^\pm(t, x, u^\pm, v^\pm) \in C^{1+\frac{\alpha}{2}}$ with respect to all of the arguments.

(ii) $\lambda_i^\pm(t, x, u^\pm, v^\pm, r^\pm)$ $\left(r=\frac{\partial v}{\partial x}\right)$ and $\frac{\partial\lambda_i^\pm}{\partial x}$, $\frac{\partial\lambda_i^\pm}{\partial u_k}$, $\frac{\partial\lambda_i^\pm}{\partial v}$, $\frac{\partial\lambda_i^\pm}{\partial r}$ are continuous, $\lambda_i^\pm$ is Hölder continuons with respect to t with the exponent $\frac{\alpha}{2}$, $\frac{\partial\lambda_i^\pm}{\partial x}$, $\frac{\partial\lambda_i^\pm}{\partial u_k}$, $\frac{\partial\lambda_i^\pm}{\partial v}$ are Hölder continuous with respect to $(t, x, u^\pm, v^\pm, r^\pm)$ with the exponent $\frac{\alpha}{2}$, $\frac{\partial\lambda_i^\pm}{\partial r}$ is Hölder continuous with respect to $(t, x, u^\pm, v^\pm)$ and $r^\pm$ with the exponents $\frac{\alpha}{2}$ and $\frac{1}{2}$ respectively; the same hypotheses for μ_l.

(iii) $a^\pm$ $(t, x, , u^\pm, v^\pm, r^\pm)$ is Hölder continuous with respect to t, x and $(u^\pm, v^\pm, r^\pm)$ with the exponents $\frac{\alpha}{2}$, α and 1 respectively; the same hypotheses for b.

(iv) $G_{\bar r}(t, u^+)$ $(\bar r=1, \cdots, R)$, $\hat G_{\hat s}(t, u^-)$ $(\hat s=S+1, \cdots, n)$, $H_{\bar p}(t, u^\pm, v^\pm)$ $(\bar p=1, \cdots, P)$, $\hat H_{\hat q}(t, u^\pm, v^\pm)$ $(\hat q=Q+1, \cdots, n)$ belong to $C^{1+\frac{\alpha}{2}}$ with respect to all of the arguments.

(v) $\varphi^\pm(t)\in C^{1+\frac{\alpha}{2}}$.

(vi) $f(t)\in C^{1+\frac{\alpha}{2}}$, $g(t, u^+, u^-, v^+, v^-)$ is Hölder continuous with respect to t and (u^+, u^-, v^+, v^-) with the exponents $\frac{1+\alpha}{2}$ and 1 respectively; the same hypotheses for $\lambda(t, u^+, u^-, v^+, v^-)$ (>0).

We have the following

Theorem 3.1 (Existence and uniqueness theoreim): *Suppose that the coefficients of the system and the given functions in the boundary conditions satisfy the preceding conditions of smoothness and that* (1.8)—(1.10) *hold. Suppose further that conditions of orientability* (1.9)—(1.22), *conditions of compatibility* (1.23), (1.24) *and conditions of solvability* (*or conditions* (1.29), (1.30)) *hold. Then there exists a positive number* $\delta_*(\leqslant\delta_0)$ *such that on* $R(\delta_*)$ *the first initial-boundary value problem with interface* (1.4)—(1.7), (1.11)—(1.18) *admits a unique solution* $u^\pm\in C^{1+\frac{\alpha}{2}}(R^\pm(\delta_+))$, $v^\pm\in\bar C^{2+\alpha}(R^\pm(\delta_*))$.

Proof It follows from conditions (1.29), (1.30) and conditions of orientibility (1.19)—(1.22) that

$$\theta=\max\left(\sum_{j=1}^n\left|\frac{\partial G_{\bar r}}{\partial u_j}(0, 0)\right|, \sum_{j=1}^n\left|\frac{\partial\hat G_{\hat s}}{\partial u_j}(0, 0)\right|, \sum_\pm\sum_{j=1}^n\left|\frac{\partial\hat H_{\hat q}}{\partial u_j^\pm}(0, 0, 0, 0, 0)\right|, \sum_\pm\sum_{j=1}^n\left|\frac{\partial H_{\bar p}}{\partial u_j^\pm}(0, 0, 0, 0, 0)\right|\right)<1, \tag{3.1}$$

$$d_0=\min\{-\lambda_{\bar r}^+(0, 1, 0, 0, 0)\lambda_{\hat s}^-(0, -1, 0, 0, 0), \lambda_{\hat q}^+(0, 0, 0, 0, 0), -\lambda_{\bar p}^-(0, 0, 0, 0, 0)\}>0, \tag{3.2}$$

where Max and Min are taken for $1\leqslant\bar r\leqslant R$, $S+1\leqslant\hat s\leqslant n$, $P+1\leqslant\hat q\leqslant n$ and $1\leqslant\bar p\leqslant P$. Hence, we can choose $\varepsilon>0$ suitably small such that

$$\theta_1=(1+d_0^{-1}\varepsilon)\theta<1, \tag{3.3}$$

$$\theta_2=(1+2d_0^{-1}\varepsilon+d_0^{-2}\varepsilon)\theta<1. \tag{3.4}$$

Introduce the following sets of functions on $R(\delta)$:

$$\Sigma_*(\delta)=\left\{(u,\ v)=(u^\pm,\ v^\pm)\left|\begin{array}{l} u^\pm,\ \dfrac{\partial u^\pm}{\partial t},\ \dfrac{\partial u^\pm}{\partial x},\ v^\pm,\ \dfrac{\partial v^\pm}{\partial x},\ \dfrac{\partial v^\pm}{\partial t},\ \dfrac{\partial^2 v^\pm}{\partial x^2}\in C^0(R^\pm(\delta)) \\ u^\pm(0,\ x)=v^\pm(0,\ x)=0,\end{array}\right.\right\}$$

$$\Sigma_1(\delta)=\left\{(u,\ v)\left|\begin{array}{l} u^\pm\in C^{1+\frac{\alpha}{2}}(R^\pm(\delta)),\ v^\pm\in \bar{C}^{2+\alpha}(R^\pm(\delta)),\ u^\pm(0,\ x)=v^\pm(0,\ x)=0 \\ \dfrac{\partial u_j^\pm}{\partial t}(0,\ x)=\mu_j^\pm(0,\ x,\ 0,\ 0,\ 0),\ \dfrac{\partial v^\pm}{\partial t}(0,\ x)=0\end{array}\right.\right\},$$

$$\Sigma(\delta)=\left\{(u,\ v)\left|\begin{array}{l}(u,\ v)\in\Sigma_1(\delta),\ \|u\|\leqslant A_0,\ \|u\|_1^*\leqslant A_1,\ \|u\|^*_{1+\frac{\alpha}{2}}\leqslant A_2 \\ |v|\leqslant B_0,\ |v|_1\leqslant B_1,\ |v|_2\leqslant B_2\end{array}\right.\right\},$$

where A_i, B_i $(i=0,\ 1,\ 2)$ are positive constants to be chosen later on with $A_0\leqslant A_1\leqslant A_2$, $B_0\leqslant B_1\leqslant B_2$.

First, noticing (1.8), (1.10), we can choose $A_0>0$, $B_0>0$ and $\delta_1>0$ suitably small such that for any element $(u,\ v)$ in $\Sigma_1(\delta)$ with $\|u\|\leqslant A_0$, $|v|\leqslant B_0$, we have $a^\pm(t,\ x,\ u^\pm,\ v^\pm,\ v_x^\pm)\geqslant a_0>0$, $\det|\zeta_{ij}^\pm|\geqslant D_0>0$, $\lambda_1\geqslant\lambda(t,\ u^\pm,\ v^\pm)\geqslant\lambda_0>0$ (a_0, D_0, λ_0 and λ_1 are constants).

For any $(\tilde{u},\ \tilde{v})\in\Sigma_1(\delta)$ with $\|\tilde{u}\|\leqslant A_0$, $|\tilde{v}|\leqslant B_0$, set

$$\tilde{\zeta}_{ij}^\pm(t,\ x)=\zeta_{ij}^\pm(t,\ x,\ \tilde{u}^\pm(t,\ x),\ \tilde{v}^\pm(t,\ x)), \tag{3.5}$$

with $\tilde{\zeta}_l^\pm$, $\tilde{\lambda}_l^\pm$, $\tilde{\mu}_l^\pm$ defined in a similar way, and

$$\tilde{b}^\pm(t,\ x)=b^\pm\left(t,\ x,\ \tilde{u}^\pm,\ \tilde{v}^\pm,\ \frac{\partial\tilde{v}^\pm}{\partial x}\right)+\left[a^\pm\left(t,\ x,\ \tilde{u}^\pm,\ \tilde{v}^\pm,\ \frac{\partial\tilde{v}^\pm}{\partial x}\right)-1\right]\frac{\partial^2\tilde{v}^\pm}{\partial x^2}, \tag{3.6}$$

We solve the following linear problem on $R^\pm(\delta)$:

$$\sum_{j=1}^n\tilde{\zeta}_{lj}^\pm(t,\ x)\left(\frac{\partial u_j^\pm}{\partial t}+\tilde{\lambda}_l(t,\ x)\frac{\partial u_j^\pm}{\partial x}\right)$$
$$=\tilde{\zeta}_l^\pm(t,\ x)\left(\frac{\partial\tilde{v}^\pm}{\partial t}+\tilde{\lambda}_l^\pm(t,\ x)\frac{\partial\tilde{v}^\pm}{\partial x}\right)+\tilde{\mu}_l^\pm(t,\ x)\quad(l=1,\ \cdots,\ n), \tag{3.7}$$

$$\frac{\partial v^\pm}{\partial t}-\frac{\partial^2 v^\pm}{\partial x^2}=\tilde{b}^\pm(t,\ x), \tag{3.8}$$

$$t=0:\quad u_j^\pm=0,\qquad(j=1,\ \cdots,\ n), \tag{3.9}$$

$$v^\pm=0, \tag{3.10}$$

$$x=1:\quad\sum_{j=1}^n\tilde{\zeta}_{\bar{r}j}^+(t,\ 1)u_j^+=G_{\bar{r}}(t,\ \tilde{u}^+(t,\ 1))+\sum_{j=1}^n(\tilde{\zeta}_{\bar{r}j}^+(t,\ 1)-\delta_{\bar{r}j})\tilde{u}_j^+(t,\ 1)$$
$$\equiv\psi_{\bar{r}}(t)\qquad \bar{r}=1,\ \cdots,\ R), \tag{3.11}$$

$$v^+=\varphi^+(t), \tag{3.12}$$

$$x=-1:\quad\sum_{j=1}^n\tilde{\zeta}_{\hat{s}j}^-(t,\ -1)u_j^-=\hat{G}_{\hat{s}}(t,\ \tilde{u}^-(t,\ -1))+\sum_{j=1}^n(\tilde{\zeta}_{\hat{s}j}^-(t,\ -1)-\delta_{\hat{s}j})\tilde{u}_j^-(t,\ -1)$$
$$\equiv\hat{\psi}_{\hat{s}}(t)\qquad(\hat{s}=S+1,\ \cdots,\ n;\ S\geqslant0), \tag{3.13}$$

$$v^-=\varphi^-(t), \tag{3.14}$$

$$x=0:\quad \sum_{j=1}^{n}\zeta_{\bar{p}j}^{-}(t,\ 0)u_j^{-}=H_{\bar{p}}(t,\ \tilde{u}^{+}(t,\ 0),\ \tilde{u}^{-}(t,\ 0),\ \tilde{v}^{+}(t,\ 0),\ \tilde{v}^{-}(t,\ 0))$$

$$+\sum_{j=1}^{n}(\tilde{\zeta}_{\bar{p}j}^{-}(t,\ 0)-\delta_{\bar{p}j})\tilde{u}_j^{-}(t,\ 0)\equiv\chi_{\bar{p}}(t)\qquad(\bar{p}=1,\ \cdots,\ P),\tag{3.15}$$

$$\sum_{j=1}^{n}\tilde{\zeta}_{\hat{q}j}^{+}(t,\ 0)u_j^{+}=\hat{H}_{\hat{q}}(t,\ \tilde{u}^{+}(t,\ 0),\ \tilde{u}^{-}(t,\ 0),\ \tilde{v}^{+}(t,\ 0),\ \tilde{v}^{-}(t,\ 0))$$

$$+\sum_{j=1}^{n}(\tilde{\zeta}_{\hat{q}j}^{+}(t,\ 0)-\delta_{\hat{q}j})\tilde{u}_j^{+}(t,\ 0)\equiv\hat{\chi}_{\hat{q}}(t)\qquad(\hat{q}=Q+1,\ \cdots,\ n),\tag{3.16}$$

$$v^{-}=v^{+}+f(t),\tag{3.17}$$

$$\frac{\partial v^{-}}{\partial x}=\lambda(t,\ \tilde{u}^{+},\ \tilde{u}^{-},\ \tilde{v}^{+},\ \tilde{v}^{-})\frac{\partial v^{+}}{\partial x}+g(t,\ \tilde{u}^{+},\ \tilde{u}^{-},\ \tilde{v}^{+},\ \tilde{v}^{-})$$

$$=\tilde{\lambda}(t)\ \frac{\partial v^{+}}{\partial x}+\tilde{g}(t).\tag{3.18}$$

It is easy to see that (3.8), (3.10), (3.12), (3.14), (3.17), (3.18) is a first initial-boundary value problem with interface for heat equations, which satisfies all the assumptions in Lemma 2.4 and Lemma 2.7, hence, by means of Lemma 2.8, this problem possesses a unique solution $v^{\pm}(t,\ x)$ with estimates (2.57)—(2.62).

For problem (3.7), (3.9), (3.11), (3.13), (3.15), (3.16), if we introduce the new unknown functions

$$w_j(t,\ x)=u_j(t,\ x)\quad x>0\qquad(j=1,\ \cdots,\ n),\tag{3.19}$$

$$w_{n+j}(t,\ x)=u_j(t,\ -x)\quad x>0\qquad(j=1,\ \cdots,\ n)\tag{3.20}$$

and set

$$\begin{cases}\zeta_{lj}(t,\ x)=\tilde{\zeta}_{lj}(t,\ x), & x>0,\\ \tilde{\tilde{\zeta}}_{n+l,n+j}(t,\ x)=\tilde{\zeta}_{lj}(t,\ -x), & x>0,\qquad(l,\ j=1,\ \cdots,\ n),\\ \tilde{\tilde{\zeta}}_{n+l,j}=\tilde{\tilde{\zeta}}_{l,n+j}(t,\ x)=0\end{cases}\tag{3.21}$$

and

$$\begin{cases}\lambda_l(t,\ x)=\lambda_l(t,\ x), & x>0,\\ \tilde{\tilde{\lambda}}_{n+l}(t,\ x)=-\lambda_l(t,\ -x), & x>0,\end{cases}\quad\text{etc.},$$

then this problem can be reduced to a mixed initial-boundary value problem for linear hyperbalic systems on $R^{+}(\delta)$ with $2n$ unknown functions $w_j(t,\ x)$ $(j=1,\ \cdots,\ 2n)$. It is easy to prove that the latter satisfies all the assumptions in Lemmas 3.1—3.5 in [2], so on $R^{+}(\delta)$ it admits a unique solution $w\in C^{1+\frac{\alpha}{2}}$ $(R^{+}(\delta))$ which satisfies the corresponding estimates. Finally, we can get

$$\|u\|\leqslant(1+K_1\delta)\max_{\bar{r},\hat{s},\bar{p},\hat{q}}(\|\psi_{\bar{r}}\|,\ \|\hat{\psi}_{\hat{s}}\|,\ \|\chi_{\bar{p}}\|,\ \|\hat{\chi}_{\hat{q}}\|)+(H_0+K_1\delta)\|\tilde{v}\|+K_1\delta\|\tilde{\mu}\|,\tag{3.22}$$

$$\|u\|_1^{*}=\|u\|+\left\|\frac{\partial u}{\partial t}\right\|+\varepsilon\left\|\frac{\partial u}{\partial x}\right\|$$

$$\leqslant(1+d_0^{-1}\varepsilon+K_2\delta^{\frac{\alpha}{2}})\max_{\bar{r},\hat{s},\bar{p},\hat{q}}(\|\dot{\psi}_{\bar{r}}\|,\ \|\dot{\hat{\psi}}_{\hat{s}}\|,\ \|\dot{\chi}_{\bar{p}}\|,\ \|\dot{\hat{\chi}}_{\hat{q}}\|)$$

$$+(K_0+K_2\delta)(1+\|\tilde{v}\|_1),\tag{3.23}$$

$$\|u\|^*_{1+\frac{\alpha}{2}} = \|u\|^*_1 + H_t^{\frac{\alpha}{2}}\left[\frac{\partial u}{\partial t}\right] + \varepsilon\left(H_x^{\frac{\alpha}{2}}\left[\frac{\partial u}{\partial t}\right] + H_*^{\frac{\alpha}{2}}\left[\frac{\partial u}{\partial x}\right]\right)$$

$$\leqslant (1+2d_0^{-1}\varepsilon + d_0^{-2}\varepsilon + K_2\delta^{\frac{\alpha}{2}})\max(H_t^{\frac{\alpha}{2}}[\dot{\psi}_{\bar r}],\ H_t^{\frac{\alpha}{2}}[\dot{\psi}_{\hat s}],\ H_t^{\frac{\alpha}{2}}\dot{\chi}_{\bar p}],\ H_t^{\frac{\alpha}{2}}[\dot{\hat{x}}_{\hat q}])$$
$$+ (K_2 + K_3\delta)(1+\max_{\bar r, \hat s, \bar p, \hat q}(\|\dot{\psi}_{\bar r}\|,\ \|\psi_{\hat s}\|,\ \|\dot{\hat{x}}_{\bar p}\|,\ \|\dot{\hat{x}}_{\hat q}\|) + \|\tilde v\|_{1+\frac{\alpha}{2}}), \tag{3.24}$$

where K_0, K_1, K_2, K_3, H_0 are constants depending on $\tilde{\tilde{\zeta}}_{ij}$, $\tilde{\tilde{f}}_l$ etc.

The preceding problem (3.7)—(3.18) defines an iterative operator T: $(u, v) = T(\tilde u, \tilde v)$. Obviously, $T(\tilde u, \tilde v) = (u, v) \in \Sigma_1(\delta)$. We now prove that we can choose constants A_1, A_2, B_1, B_2 and δ_* such that T is a map from $\Sigma(\delta_*)$ to itself.

It follows from the definition of $\tilde b(t, x)$ that for any $(\tilde u, \tilde v) \in \Sigma(\delta)$,

$$\|\tilde b\| \leqslant D_1(A_0, B_0) + D_2(A_1, B_1)\delta^{\frac{\alpha}{2}} \tag{3.25}$$

$$H^\alpha[b] \leqslant D_3(A_1, B_1) + D_4(A_1, B_2)\delta^{\frac{\alpha}{2}}, \tag{3.26}$$

where $D_1(A_0, B_0)$ denotes a constant depending only on A_0, B_0, and so on.

By (3.18)

$$\|\tilde g\| \leqslant D_5(A_0, B_0), \tag{3.27}$$

$$H_t^{\frac{1+\alpha}{2}}[\tilde g],\ H_t^{\frac{1+\alpha}{2}}[\tilde\lambda] \leqslant D_6(A_0, B_0) + D_7(A_1, B_1)\delta^{\frac{1-\alpha}{2}}. \tag{3.28}$$

Hence it follows from (2.28) that

$$\|\omega\| \leqslant D_8(A_0, B_0) + D_9(A_1, B_1)\delta^{\frac{1-\alpha}{2}}. \tag{3.29}$$

Substituting (3.29) into (2.57)—(2.62), we get

$$|v| \leqslant D_{10}(A_1, B_2)\delta^{\frac{1+\alpha}{2}}, \tag{3.30}$$

$$|v|_1 \leqslant D_{11}(A_0, B_0) + D_{12}(A_1, B_2)\delta^{\frac{\alpha}{2}}, \tag{3.31}$$

$$|v|_2 \leqslant D_{13}(A_1, B_1) + D_{14}(A_1, B_2)\delta^{\beta}, \tag{3.32}$$

where $\beta = \min\left(\frac{\alpha}{2}, \frac{1-\alpha}{2}\right)$.

For the mixed problem for linear hyperbolic systems, by means of the method in [2] it follows from (3.22)—(3.24) that

$$\|u\| \leqslant D_{15}(A_1, B_1)\delta, \tag{3.33}$$

$$\|u\|^*_1 \leqslant \theta_1 A_1 + D_{16}(A_0, B_1) + D_{17}(A_1, B_2)\delta^{\frac{\alpha}{2}}, \tag{3.34}$$

$$\|u\|^*_{1+\frac{\alpha}{2}} \leqslant \theta_2 A_2 + D_{18}(A_1, B_2) + D_{19}(A_2, B_2)\delta^{\frac{\alpha}{2}}. \tag{3.35}$$

Taking

$$\begin{cases} B_1 = D_{11}(A_0, B_0) + 1, \\ A_1 = (D_{16}(A_0, B_1) + 1)/(1-\theta_1), \\ B_2 = D_{13}(A_1, B_1) + 1, \\ A_2 = (D_{18}(A_1, B_2) + 1)/(1-\theta_2), \end{cases} \tag{3.36}$$

by means of the preceding estimates (3.30)—(3.35) we can choose $\delta_*>0$ suitably small such that the operator T maps $\sum(\delta_*)$ into itself. Hence as in [1], Leray-sSchauder fixed point theorem[4,5] shows that there exits $(u(t,x), v(t,x))\in\sum(\delta_*)$ such that $(u,v)=T(u,v)$, then we get the existence of the solution for the first initial-boundary value problem with interface for quasilinear hyperbolic-parabolie coupled systems.

The uniqueness of the solution can be proved in a similar way as in [2] (with the aid of some conclusions in [6]). We omit the detail here.

§ 4. Existence and uniqueness of the solution for the second initial boundary value problem with interface

For the second initial bouudary value problem with interface for quasilinear hyperbolic-parabolic coupled systems, we can, similarly to § 3, prove the existence and uniqueness of the solution. Here we only give the corresponding assumptions and the results.

Consider the following second initial-boundary value problem with interface for system (1.4), (1.5):

$$t=0:\quad u_i^\pm=v^\pm=0 \quad (i=1,\cdots,n), \tag{4.1}$$

$$x=1:\quad u_{\bar r}^+=G_{\bar r}(t,u^+,v^+) \quad (\bar r=1,\cdots,R;\ R\leqslant n), \tag{4.2}$$

$$\frac{\partial v^+}{\partial x}=F^+(t,u^+,v^+), \tag{4.3}$$

$$x=-1:\quad u_{\hat s}^-=\hat G_{\hat s}(t,u^-,v^-) \quad (\hat s=S+1,\cdots,n;\ S\geqslant 0), \tag{4.4}$$

$$\frac{\partial v^-}{\partial x}=F^-(t,u^-,v^-) \tag{4.5}$$

and (1.15)—(1.18) hold on the interface $x=0$. We assume that conditions of orientability (1.19)—(1.22) and the following conditions of compatibility are satisfied:

$$\begin{cases} G_{\bar r}(0,0,0)=0 & (\bar r=1,\cdots,R),\\ \hat G_{\hat s}(0,0,0)=0 & (\hat s=S+1,\cdots,n),\\ \hat H_{\hat q}(0,0,0,0,0)=0 & (\hat q=Q+1,\cdots,n),\\ H_{\bar p}(0,0,0,0,0)=0 & (\bar p=1,\cdots,P),\\ f(0)=0,\\ g(0,0,0,0,0)=0,\\ F^\pm(0,0,0)=0 \end{cases} \tag{4.6}$$

and

$$\begin{cases} \mu_{\bar r}^{+}(0,\ 1,\ 0,\ 0,\ 0)=\dfrac{\partial G_{\bar r}}{\partial t}(0,\ 0,\ 0)+\sum_{j=1}^{n}\dfrac{\partial G_{\bar r}}{\partial u_j}(0,\ 0,\ 0)\mu_j^{+}(0,\ 1,\ 0,\ 0,\ 0) \\ \qquad (\bar r=1,\ \cdots,\ R), \\ \mu_{\hat s}^{-}(0,\ -1,\ 0,\ 0,\ 0)=\dfrac{\partial \hat G_{\hat s}}{\partial t}(0,\ 0,\ 0)+\sum_{j=1}^{n}\dfrac{\partial \hat G_{\hat s}}{\partial u_j}(0,\ 0,\ 0)\mu_j^{-}(0,\ -1,\ 0,\ 0,\ 0) \\ \qquad (\hat s=S+1,\ \cdots,\ n), \\ \mu_{\hat q}^{+}(0,\ 0,\ 0,\ 0,\ 0)=\dfrac{\partial \hat H_{\hat q}}{\partial t}(0,0,0,0,0)+\sum_{\pm}\sum_{j=1}^{n}\dfrac{\partial \hat H_{\hat q}}{\partial u_j^{\pm}}(0,0,0,0,0)\mu_j^{\pm}(0,0,0,0,0), \\ \qquad (\hat q=Q+1,\ \cdots,\ n), \\ \mu_{\bar p}^{-}(0,\ 0,\ 0,\ 0,\ 0)=\dfrac{\partial H_{\bar p}}{\partial t}(0,\ 0,\ 0,\ 0,\ 0)+\sum_{\pm}\sum_{j=1}^{n}\dfrac{\partial H_{\bar p}}{\partial u_j^{\pm}}(0,0,0,0,0)\mu_j^{\pm}(0,0,0,0,0) \\ \qquad (\bar p=1,\ \cdots,\ P), \\ \dot f(0)=0. \end{cases} \tag{4.7}$$

We assume that the corresponding conditions of solvability are satisfied, i. e., boundary conditions (4.2), (4.4) and interface conditions (1.15), (1.16) can be rewritten as follows:

$$\begin{cases} x=1: & u_{\bar r}^{+}=I_{\bar r}(t,\ u_{\bar s}^{+},\ v^{+}) & (\bar r=1,\ \cdots,\ R;\ \bar s=R+1,\ \cdots,\ n), \\ x=-1: & u_{\hat s}^{-}=\hat I_{\hat s}(t,\ u_{\hat r},\ v^{-}) & (\hat r=1,\ \cdots,\ S;\ \hat s=S+1,\ \cdots,\ n), \\ x=0: & u_{\hat q}^{+}=\hat J_{\hat q}(t,\ u_{\hat p}^{+},\ u_{\bar q}^{-},\ v^{+},\ v^{-}) & (\hat p=1,\ \cdots,\ Q;\ \hat q=Q+1,\ \cdots,\ n), \\ & u_{\bar p}^{-}=J_{\bar p}(t,\ u_{\hat p}^{+},\ u_{\bar q}^{-},\ v^{+},\ v^{-}) & (\bar p=1,\ \cdots,\ P;\ \bar q=P+1,\ \cdots,\ n). \end{cases} \tag{4.8}$$

We assume further that the coefficients of the system and the given functions in boundary conditions satisfy conditions of smoothness (i), (ii), (iii) (vi) in § 3 and

(iv) All the first order derivatives of $G_{\bar r}(t,\ u^{+},\ v^{+})$ $(\bar r=1,\ \cdots,\ R)$, $\hat G_{\hat s}(t, u^{-}, v^{-})$ $(\hat s=S+1,\ \cdots,\ n)$, $H_{\bar p}(t,\ u^{\pm},\ v^{\pm})$ $(\bar p=1,\ \cdots,\ P)$ and $\hat H_{\hat q}(t, u^{\pm}, v^{\pm})$ $(\hat q=Q+1, \cdots, n)$ belong to $\mathrm{Lip}^{\frac{\alpha}{2}}$ with respect to all the arguments.

(v) $F^{\pm}\in C^1$.

Then we have

Theorem 4.1. *Suppose the preceding assumptions are satisfied, then there exists a positive number* $\delta_*(\leqslant\delta_0)$ *such that on* $R(\delta_*)$ *the second initial-boundary value problem with interface* (1.4), (1.5), (4.1)—(4.5), (1.15)—(1.18) *admits a unique solution* $u^{\pm}\in C^{1+\frac{\alpha}{2}}(R^{\pm}(\delta_*))$, $v^{\pm}\in \bar C^{2+\alpha}(R^{\pm}(\delta_*))$.

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