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CORRECTION TO "ON THE PROBLEM OF BEST CONVERGENCE RATES OF DENSITY ESTIMATES"

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The assertion of Th. 1 in [1] should be replaced by

$$\limsup_{n\to\infty} a_n n^{k/(2k+m)} = \infty. \tag{A}$$

Since the proof of Th. 1 in [1] is somewhat in error, we give here a sketch of proof of (A). Choose $f \in C_{k\alpha}$ with $f(x) \geqslant a > 0$ for $\|x\| \leqslant \varepsilon > 0$, and define $h_\delta(x) = f(x) + e_{k\delta}(x)$, where $e_{k\delta}(x)$, as well as d and $C_{k\alpha}^{(n)}(d)$ to appear in the following, are the same as in [1]. Choose $\bar{\delta} > 0$ so that $h_\delta \in C_{k\alpha}$ for $\delta \in (0, \bar{\delta})$. For each δ in $(0, \bar{\delta})$, there exists an integer n such that $h_\delta \in C_{k\alpha}^{(n)}(d)$. Hence an integer N can be found such that for some sequence $\{\delta_i\} \subset (0, \bar{\delta})$ we have $g_i \triangleq h_{\delta_i} \in C_{k\alpha}^{(N)}(d)$, $i = 1, 2, \cdots$. Without lossing generality, we can assume that $\delta_i \downarrow 0$ (otherwise take another f), and hence we also can assume that $\delta_i \geqslant 2\delta_{i+1}$, $i = 1, 2, \cdots$. Define an integer n_i such that $\delta_i^{2k+m} \in \left[\frac{b}{n_i+1}, \frac{b}{n_i}\right)$, $i = 1, 2 \cdots$. Since $\delta_i \geqslant 2\delta_{i+1}$, we have $n_1 < n_2 < \cdots$. Replace $C(\delta_n)$, a_n, $h_{\bar{f},\delta_n}$, $\gamma_n(0)$ and $\bar{f}$ in (14)—(19) of [1] by 1, a_{n_i}, g_i, $\gamma_{n_i}(0)$ and f respectively. In this way we get

$$\limsup_{n\to\infty} 2a_n n^{k/(2k+m)} \geqslant \liminf_{i\to\infty} 2a_{n_i} n_i^{k/(2k+m)} \geqslant b^{k/(2k+m)}.$$

Since b can be chosen arbitrarily, we finally get (A).

Based on (A), the proof of Th. 2 in [1] is still valid, with an obvious modification.

We mention that it is wrong to define the function $e_{k\delta}(x)$ as in (2.18) of Farrell [2], for it does not satisfy (2.19) of [2]. An correct choice may be

$$e_{k\delta}(x) = g_{k\delta}(\|x\|^2 - 2^{k-1}\delta),$$

where $g_{k\delta}$ is defined in [2]. No change is needed in the proof under this new choice of $e_{k\delta}(x)$.

References

[1] Chen Xiru, *Chin. Ann. of Math.*, **5B** (2) (1984), 185—192.
[2] Farrell, R. H., *Ann. Math. Statist.*, **43** (1972), 170—180.