Chin. Ann. of Math.
7B (4) 1986

SOME PROPERTIES OF THE l_2-VALUED LONG JAMES BANACH SPACE $J(\eta, l_2)$

ZHAO JUNFENG (赵俊峰)*

Abstract

The main result of this paper is to show that the bidual $J(\eta, l_2)^{**}$ of the long James type l_2-valued Banach space $J(\eta, l_2)$ can be identified with transfinite matrices of scalars $[(b_{\alpha,i})i \in [0, \omega)]_{\alpha\in[0,\eta)}$ with some conditions and the norm of the element x^{**} in $J(\eta, l_2)^{**}$ equals $\sup\limits_{\gamma\in[0,\eta]} \|\sum\limits_{\alpha\in[0,\gamma)} \sum\limits_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i}\|_{J(\eta,l)^{**}}$.

Suppose η is an ordinal number. Let the set of ordinal numbers $[0, \eta]$ have the order topology. Let ω be the first infinite ordinal number. Suppose the unit vector e_i ($i\in[0, \omega)$) in the Banach space l_2 is the vector that has 1 in its $(i+1)$th slot and zeros elsewhere.

The vector valued long James type Banach space $J(\eta, l_2)$ defined in [6] consists of the vector valued functions F defined on $[0, \eta]$ and valued in l_2 which satisfy the following conditions:

(i) $F(0)=0$;

(ii) $\|F\|=\sup\left(\sum_{i=1}^{n}\|F(\alpha_i)-F(\alpha_{i-1})\|_{l_2}^2\right)^{1/2}$,

where the sup is taken over all finite sequences $\alpha_0<\alpha_1<\cdots<\alpha_n$ in $[0, \eta]$;

(iii) F is continuous in the norm of the space l_2.

In this paper we will investigate some properties of the dual $J(\eta, l_2)^*$ and prove that the elements of the bidual $J(\eta, l_2)^{**}$ can be identified with transfinite matrices with some conditions.

In [6] we prove that $[(\phi_{\alpha,i})\ i\in[0, \omega)]\alpha\in[0, \eta)$ is a transfinite basis of $J(\eta, l_2)$, where

$$\phi_{\alpha,i}(\gamma)=\begin{cases} e_i, & \text{if } \gamma\in(\alpha, \eta], \\ 0, & \text{if } \gamma\notin(\alpha, \eta]. \end{cases}$$

And for any $F\in J(\eta, l_2)$ we have

$$F=\sum_{\alpha\in[0,\eta)}\sum_{i\in(0,\omega)} C_{\alpha,i}\phi_{\alpha,i},$$

where $C_{\alpha,i}=F_{\alpha+1,i}-F_{\alpha,i}$.

Manuscript received April 16, 1984.
* Department of mathematics, Wuhan University, Wuhan, China.

Let us now consider the dual $J(\eta, l_2)^*$ of the Banach space $J(\eta, l_2)$. Suppose

$$g_{\alpha,i}(F)=F_{\alpha,i}$$

for any $F\in J(\eta, l_2)$, $\alpha\in[0, \eta]$, $i\in[0, \omega)$. Since $|F_{\alpha,i}|\leqslant\|F\|$, we have $\|g_{\alpha,i}\|\leqslant 1$. Therefore $g_{\alpha,i}$ is a linear bounded functional on $J(\eta, l_2)$, i. e.

$$((g_{\alpha,i})\ i\in[0, \omega))\ \alpha\in[0, \eta]\in J(,\eta\ l_2)^*.$$

Proposition 1. *$((\phi_{\alpha,i}, g_{\alpha,i}), i\in[0, \omega)), \alpha\in[0, \eta)$ is not a biorthogonal system.*

The proof is easy, since

$$g_{\alpha,i}\ (\phi_{\beta,j})=\begin{cases}1, \text{ if } i=j \text{ and } \alpha\in(\beta, \eta], \\ 0, \text{ if } i\neq j \text{ or } \alpha\notin(\beta, \eta].\end{cases} \tag{1}$$

Proposition 2.

$$[\phi_{\alpha,n+1}, \phi_{\alpha,n+2}, \cdots]\subsetneqq[g_{\alpha,n+1}, g_{\alpha,n+2},\cdots]\perp$$

for any $\alpha\in[0, \eta)$ and for any $\eta\in[0, \omega)$, where the notation $[x_1, x_2, \cdots]$ indicates the closure of linear span of the set $\{x_1, x_2, \cdots\}$. $[g_{\alpha,n+1}, g_{\alpha,n+2}, \cdots]\perp=\{\phi\in J(\eta, l_2)$: $\langle\phi, g\rangle=0$ for any $g\in[g_{\alpha,n+1}, g_{\alpha,n+2}, \cdots]\}$.

Proof Since $g_{\alpha,i}(\phi_{\beta,j})=0$ for any i, $j\in[0, \omega)$ when $\beta\geqslant\alpha$, in particular, we have $g_{\alpha,i}(\phi_{\alpha,j})=0$ for any i and j in $[0, \omega)$. So

$$[\phi_{\alpha,n+1}, \phi_{\alpha,n+2}, \cdots]\subseteq[g_{\alpha,n+1}, g_{\alpha,n+2}, \cdots]\perp.$$

Furthermore, $\phi_{\alpha,n}\notin[\phi_{\alpha,n+1}, \phi_{\alpha,n+2}, \cdots]$. It follows from $g_{\alpha,i}(\phi_{\alpha,n})=0$ that

$$\phi_{\alpha,n}\in[g_{\alpha,n+1}, g_{\alpha,n+2}, \cdots]\perp.$$

Therefore

$$[\phi_{\alpha,n+1}, \phi_{\alpha,n+2}, \cdots]\subsetneqq[g_{\alpha,n+1}, g_{\alpha,n+2}, \cdots]\perp.$$

Proposition 3. *For any $l\in J(\eta, l_2)^*$, $l=\sum_{\alpha\in[0,\eta)}\sum_{i\in[0,\omega)}(U_{\alpha,i}-U_{\alpha+1,i})\,g_{\alpha,i}$ holds in the topology $[J(\eta, l_2)^*, J(\eta, l_2)]$ in the Banach space $J(\eta, l_2)^*$ where*

$$U_{\alpha,i}=\begin{cases}l(\phi_{\alpha-1,i}), \text{ if } \alpha \text{ is a non-limit ordinal}, \\ \lim\limits_{\beta<\alpha} l(\phi_{\beta,i}), \text{ if } \alpha \text{ is a limit ordinal},\end{cases}$$

$\alpha\in[0, \eta)$, $i\in[0, \omega)$.

Proof Let $l\in J(\eta, l_2)^*$. We prove that $\lim\limits_{\beta<\alpha} l(\phi_{\beta,i})$ exists for any limit ordinal $\alpha\in[0, \eta)$ and any $i\in[0, \omega)$. Suppose it does not exist. Then there are real numbers $a<b$ and ordinals $\beta_0<\beta_1<\cdots<\alpha$ with $l(\phi_{\beta_{2j},i})<a$, $l(\phi_{\beta_{2j+1},i})>b$. For any integer n

$$\left\|\sum_{j=1}^{n}(\phi_{\beta_{2j-1},i}-\phi_{\beta_{2j},i})\right\|=(2n\|e_i-0\|_{l_2}^2)^{1/2}=(2n)^{1/2}.$$

$l(\phi_{\beta_{2j-1},i}-\phi_{\beta_{2j},i})>b-a$. Therefore

$$n(b-a)\leqslant l\Big(\sum_{j=1}^{n}\phi_{\beta_{2j-1},i}-\phi_{\beta_{2j},i}\Big)\leqslant\|l\|(2n)^{1/2}.$$

So $\|l\|=\infty$, a contradiction.

According to the definition of $U_{\alpha,i}$, we have

$$\lim_{\beta<r} U_{\alpha,i}=\begin{cases} l(\phi_{r-1,i}), & \text{if } r \text{ is non-limit ordinal,} \\ \lim_{\beta<r}\lim_{\alpha<\beta} l(\phi_{\alpha,i}), & \text{if } r \text{ is limit ordinal,}\end{cases}$$

$$=U_{r,i}.$$

In particular, $U_{\eta+1,j}=l(\phi_{\eta,j})=0$.

We now prove that

$$\sum_{\alpha\in[0,\eta)}\sum_{i\in[0,\omega)}(U_{\alpha,i}-U_{\alpha+1,i})g_{\alpha,i}$$

is w^*-convergent to l in the Banach space $J(\eta, l_2)^*$. It follows from (1) that

$$\langle\sum_{\alpha\in[0,\eta)}\sum_{i\in[0,\omega)}(U_{\alpha,i}-U_{\alpha+1,i})g_{\alpha,i},\ \phi_{\beta,j}\rangle$$

$$=\sum_{\alpha\in[0,\eta)}\sum_{i\in[0,\omega)}(U_{\alpha,i}-U_{\alpha+1,i})g_{\alpha,i}(\phi_{\beta,j})$$

$$=\sum_{\alpha\in(\beta,\eta)}(U_{\alpha,j}-U_{\alpha+1,j})=U^+{}_{\beta+1,j}-U_{\eta+1,j}$$

$$=U_{\beta+1,j}=l(\phi_{\beta,j})=\langle l,\ \phi_{\beta,j}\rangle$$

for any $\beta\in[0,\eta)$, any $j\in[0,\omega)$. Therefore

$$\sum_{\alpha\in[0,r)}\sum_{i\in[0,N)}(U_{\alpha,i}-U_{\alpha+1,i})g_{\alpha,i}\xrightarrow{w^*} l$$

as $N\to\omega$ and $r\to\eta$, and

$$\sum_{\alpha\in[0,\eta)}\sum_{i\in[0,\omega)}(U_{\alpha,i}-U_{\alpha+1,i})g_{\alpha,i}=l$$

holds in the topology $[J(\eta, l_2)^*, J(\eta, l_2)]$ in the Banach space $J(\eta, l_2)^*$.

Proposition 4. *Suppose $\beta\in[0,\eta]$ is a limit ordinal number in the topological space $[0,\eta]$, $\{r_n\}_{n=1}^{\infty}$ is a sequence of ordinal numbers in $[0,\eta]$ and $r_1<r_2<\cdots<r_n<\cdots\uparrow\beta$ whenn$\to\infty$. Let*

$$V_1=\{l\in J(\eta, l_2)^* \mid \|l\|_{r_n}=\|l|_{[((\phi_{\alpha,i})i\in[0,\omega)]\alpha\in[r_n,\beta)]}\|\to 0,\ as\ n\to\infty\},$$

$$V_2=\{l\in J(\eta, l_2)^* \mid \langle l, f_m\rangle\to 0,\ as\ m\to\infty\ for\ all\ sequences\ k_1<k_2<\cdots$$

and all

$$f_m\in((\phi_{\alpha,i})i\in[0,\omega))\ \alpha\in[r_{k_m}, r_{k_{m+1}})\}.$$

Then $V_1=V_2$.

Proof We show that $V_2\subseteq V_1$ first.

Suppose $V_2\nsubseteq V_1$. Then for any $l\in J(\eta, l_2)^*$ and $l\notin V_1$ there is an $\varepsilon>0$ such that

$$\|l\|_{r_1}\geq\|l\|_{r_2}\geq\cdots\geq\|l\|_{r_n}\geq\cdots>\varepsilon.$$

Since $\|l\|_{r_1}>\varepsilon$ we can choose

$$g_1\in[((\phi_{\alpha,i})\ i\in[0,\omega)]\alpha\in[r_1,\beta)]\ \text{and}\ \|g_1\|\leq\frac{1}{2}$$

such that $\langle l, g_1\rangle>\varepsilon$. There is $\phi_1\in\text{span}\{[(\phi_{\alpha,i})i\in[0,\omega)]\alpha\in[r_1,\beta)\}$ such that $\|g_1-\phi_1\|<\frac{1}{2}$ and $\langle l, \phi_1\rangle>\varepsilon$. Let the biggest index of the finite combination of ϕ_1 in $[r_1,\beta)$ be less than r_{k_2}. So

$$\phi_1\in[((\phi_{\alpha,i})i\in[0,\omega)]\ \alpha\in[r_1, r_{k_2})],$$

say

$$\phi_1=\sum_{\alpha\in[r_1,r_{k_2})}\sum_{i\in[0,\omega)}t^{(1)}_{\alpha,i}\phi_{\alpha,i}.$$

Since $\|l\|_{r_{k_2}}>\varepsilon$, choose

$$g_2\in[((\phi_{\alpha,i})i\in[0,\omega)]\alpha\in[r_{k_2},\beta)] \text{ and } \|g_2\|<\frac{1}{2}$$

such that $\langle l, g_2\rangle>\varepsilon$ and there is

$$\phi_2\in\operatorname{span}\{((\phi_{\alpha,i})\ i\in[0,\omega))\ \alpha\in[r_{k_2},\beta)\},$$

$\|g_2-\phi_2\|\leqslant\frac{1}{2}$, and $\langle l, \phi_2\rangle>\varepsilon$. Let the biggest index of the finite combination of ϕ_2 in $[r_{k_2},\beta)$ be less than r_{k_3}. So

$$\phi_2\in[((\phi_{\alpha,i})\ i\in[0,\omega)]\quad\alpha\in[r_{k_2}, r_{k_3})],$$

say

$$\phi_2=\sum_{\alpha\in[r_{k_2},r_{k_3})}\sum_{i\in[0,\omega)}t^{(2)}_{\alpha,i}\phi_{\alpha,i}.$$

Continuing by induction, choose

$$g_m\in[((\phi_{\alpha,i})\ i\in[0,\omega))\alpha\in[r_{k_m},\beta)] \text{ and } \|g_m\|<\frac{1}{2}$$

such that $\langle l, g_m\rangle>\varepsilon$, and there is

$$\phi_m\in\operatorname{span}\{((\phi_{\alpha,i})i\in[0,\omega))\ \alpha\in[r_{k_m},\beta)\},$$

$\|g_m-\phi_m\|<\frac{1}{2}$ and $\langle l, \phi_m\rangle>\varepsilon$. Let the biggest index of the finite combination of ϕ_m in $[r_{k_m},\beta)$ be less than $r_{k_{m+1}}$. So

$$\phi_m\in[((\phi_{\alpha,i})i\in[0,\omega))\alpha\in[r_{k_m},r_{k_{m+1}})],$$

say

$$\phi_m=\sum_{\alpha\in[r_{k_m},r_{k_{m+1}})}\sum_{i\in[0,\omega)}t^{(m)}_{\alpha,i}\phi_{\alpha,i},\ \cdots.$$

Now we get

$$\phi_m\in[((\phi_{\alpha,i})i\in[0,\omega))\alpha\in[r_{k_m}, r_{k_{m+1}})],$$

but $\langle l, \phi_m\rangle>\varepsilon$. So $l\notin V_2$.

Next show that $V_1\subseteq V_2$.

Suppose $l\in V_1$. Let

$$f_m\in[((\phi_{\alpha,i})\ i\in[0,\omega))\alpha\in[r_{k_m},r_{k_{m+1}})]$$

and

$$\|f_m\|\leqslant1,\ \|l\|_{r_{k_m}}\to0\quad(m\to\infty).$$

We have

$$\langle l, f_m\rangle\leqslant\|l\|_{r_{k_m}}\|f_m\|\to0, \text{ as } m\to\infty.$$

So $l\in V_2$.

Suppose $((C_{\alpha,i})\ i\in[0,\omega))\ \alpha\in[0,\eta)$ are the coefficient functionals associated with the transfinite basis $((\phi_{\alpha,i})\ i\in[0,\omega))\ \alpha\in[0,\eta)$ of $J(\eta, l_2)$ as we mentioned before stating Proposition 1.

Now we have

Theorem 5. *The bidual $J(\eta, l_2)^{**}$ can be identified with the Banach Space of all transfinite matrices of scalars $((b_{\alpha,i})\ i\in[0,\omega))\alpha\in[0,\eta)$ such that for any $\gamma\in[0,\eta]$ the series*

$$\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i}$$

is w^* convergent and

$$\sup_{\gamma\in[0,\eta]}\Big\|\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i}\Big\|_{J(\eta,l_2)^{**}}<+\infty.$$

This correspondence is given by

$$x^{**}\leftrightarrow((x^{**}(C_{\alpha,i})\ i\in[0,\omega)),\ \alpha\in[0,\eta),$$

for $x^{**}\in J(\eta, l_2)^{**}$. *The norm of x^{**} is equal to*

$$\sup_{\gamma\in[0,\eta]}\Big\|\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} x^{**}(C_{\alpha,i})\phi_{\alpha,i}\Big\|_{J(\eta,l_2)^{**}}.$$

Furthermore, for any $i\in[0,\omega)$, $\alpha\in[0,\eta)$ we have

$$|x^{**}(C_{\alpha,i})|\leqslant\|x^{**}\|,$$

where $\phi_{\alpha,i}$, $C_{\alpha,i}$ are defined as before.

Proof Suppose $[(b_{\alpha,i})i\in[0,\omega)]\alpha\in[0,\eta)$ is a transfinite matrix such that for any $\gamma\in[0,\eta]$ the series

$$\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i}$$

is w^* convergent and

$$\sup_{\gamma\in[0,\eta]}\Big\|\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i}\Big\|_{J(\eta,l_2)^{**}}<+\infty.$$

Let $x^{**}\in J(\eta, l_2)^{**}$ and let

$$\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i}\xrightarrow{w^*}x^{**}.$$

So for any $j\in[0,\omega)$ and any $\beta\in[0,\eta)$ we have

$$\Big\langle\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i},\ C_{\beta,j}\Big\rangle\xrightarrow[\gamma\to\eta]{}\langle x^{**}, C_{\beta,j}\rangle.$$

By the biorthogonality

$$\Big\langle\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} b_{\alpha,i}\phi_{\alpha,i},\ C_{\beta,j}\Big\rangle\xrightarrow[\gamma\to\eta]{}b_{\beta,j}.$$

So $\langle x^{**},C_{\beta,j}\rangle=b_{\beta,j}$ for any $j\in[0,\omega)$ and any $\beta\in[0,\eta)$.

Conversely, suppose $x^{**}\in J(\eta, l_2)^{**}$ and P_γ is the projection from $J(\eta, l_2)$ into $J(\eta, l_2)$ defined by

$$P_\gamma\phi=\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} C_{\alpha,i}\phi_{\alpha,i},$$

where $\phi=\sum_{\alpha\in[0,\eta)}\sum_{i\in[0,\omega)} C_{\alpha,i}\,\phi_{\alpha,i}$.

$P_\gamma\phi\xrightarrow[\gamma\to\eta]{\text{norm}}\phi$ in the space $J(\eta, l_2)$, since $((\phi_{\alpha,i})\ i\in[0,\omega))\ \alpha\in[0,\eta)$ is a transfinite basis of $J(\eta, l_2)$ (see [6]). For the adjoint operator P_γ^* we have

$$P_\gamma^* l=\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)} l(\phi_{\alpha,i})C_{\alpha,i}\xrightarrow[\gamma\to\eta]{\text{norm}}l$$

in the space $J(\eta, l_2)^*$, $l\in J(\eta, l_2)^*$. So

$$\langle P_\gamma^{**}x^{**}, l\rangle=\langle x^{**}, P_\gamma^* l\rangle \xrightarrow[\gamma\to\eta]{} \langle x^{**}, l\rangle.$$

Suppose now x^{**} is given in the space $J(\eta, l_2)^{**}$. Then for any $\varepsilon>0$ there is an $l\in J(\eta, l_2)^*$, $\|l\|=1$ such that

$$|\langle x^{**}, l\rangle|\geqslant\|x^{**}\|-\varepsilon.$$

Then

$$\lim_{\gamma\to\eta}\|P_\gamma^{**}x^{**}\|\,\|l\|\geqslant\lim_{\gamma\to\eta}\langle P_\gamma^{**}x^{**}, l\rangle\geqslant\|x^{**}\|-\varepsilon.$$

So

$$\lim_{\gamma\to\eta}\|P_\gamma^{**}x^{**}\|\geqslant\|x^{**}\|-\varepsilon.$$

Now we show that $\|P_\gamma\|=1$ for any $\gamma\in[0, \eta]$.

For any $\phi\in J(\eta, l_2)$ we have

$$\|P_\gamma\phi\|\leqslant\|P_\gamma\|\,\|\phi\|, \text{ so } \|P_\gamma\|\geqslant 1.$$

On the other hand, for any $\phi\in J(\eta, l_2)$, $\|P_\gamma\phi\|\leqslant\|\phi\|$. In fact, if γ is a partition point of a partition;

$$\alpha_0<\alpha_1<\cdots<\gamma<\cdots<\alpha_n$$

in $[0, \eta]$, then

$$\sum_{i=1}^n\|P_\gamma\phi(\alpha_i)-P_\gamma\phi(\alpha_{i-1})\|^2=\sum_{i=1}^n\|\phi(\alpha_i)-\phi(\alpha_{i-1})\|^2.$$

If γ is not a partition point of a partition, then

$$\sum_{i=1}^n\|P_\gamma\phi(\alpha_i)-P_\gamma\phi(\alpha_{i-1})\|^2\leqslant\sum_{i=1}^n\|\phi(\alpha_i)-\phi(\alpha_{i-1})\|^2$$

(by the definitions of P_γ and $\phi_{\alpha,i}$). So $\|P_\gamma\|=1$, $\gamma\in[0, \eta]$. Hence $\|P_\gamma^{**}\|=1$, $\gamma\in[0, \eta]$. Therefore

$$\|x^{**}\|-\varepsilon\leqslant\lim_{\gamma\to\eta}\|P_\gamma^{**}x^{**}\|\leqslant\|P_\gamma^{**}\|\,\|x^{**}\|=\|x^{**}\|.$$

So $\lim\limits_{\gamma\to\eta}\|P_\gamma^{**}x^{**}\|=\|x^{**}\|$.

Since

$$\Big\|\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)}x^{**}((C_{\alpha,i})\phi_{\alpha,i})\Big\|=\Big\|P_\gamma^{**}\Big(\sum_{\alpha\in[0,\gamma+1)}\sum_{i\in[0,\omega)}x^{**}(C_{\alpha,i})\phi_{\alpha,i}\Big)\Big\|$$
$$\leqslant\Big\|\sum_{\alpha\in[0,\gamma+1)}\sum_{i\in[0,\omega)}x^{**}(C_{\alpha,i})\phi_{\alpha,i}\Big\|,$$

we see that $\{\|P_\gamma^{**}x^{**}\|\}$ is increasing. Therefore

$$\lim_{\gamma\to\eta}\|P_\gamma^{**}x^{**}\|=\sup_{\gamma\in[0,\eta)}\Big\|\sum_{\alpha\in[0,\gamma)}\sum_{i\in[0,\omega)}x^{**}(C_{\alpha,i})\phi_{\alpha,i}\Big\|.$$

So $\|x^{***}\|=\sup\limits_{\gamma\in[0,\eta]}\Big\|\sum\limits_{\alpha\in[0,\gamma)}\sum\limits_{i\in[0,\omega)}x^{**}(C_{\alpha,i})\phi_{\alpha,i}\Big\|$.

Finally we show that

$$|x^{**}(C_{\alpha,i})|\leqslant\|x^{**}\|,\ i\in[0, \omega),\ \alpha\in[0, \eta).$$

Since

$$\|C_{\alpha,i}\|=\sup_{\|f\|\leqslant 1}|C_{\alpha,i}(f)|=\sup_{\|f\|\leqslant 1}\|f_{\alpha+1,i}-f_{\alpha,i}\|$$
$$\leqslant\sup_{\|f\|\leqslant 1}\|f\|\leqslant 1\ (\text{since } \|f_{\alpha+1,i}-f_{\alpha,i}\|\leqslant\|f\|_{J(\eta,l_2)},\ f\in J(\eta, l_2))$$

and $|C_{a,i}(\phi_{a,i})|=1$, $\|\phi_{a,i}\|=1$, we have

$$(x^{**}(C_{a,i})|\leqslant\|x^{**}\|\|C_{a,i}\|=\|x^{**}\|.$$

This completes the proof.

Remark 6. Theorem 5 remains true for the space $J(\eta, l_p)\,(1<p<\infty)$.

The author cordially thanks professor G. A. Edgar for his valuable advice while the author was visiting the Ohio State University.

References

[1] Dunford, N. and Schwartz, J. T., Linear operators, Part I. Interscience, 1958.

[2] Edgar, G. A., A long James space. In Measure Theary, Oberwolfach, 1979, edited by *v*. Kolzow, Lecture notes in Mathematics 794, Springer-Verlag, 1980.

[3] Edgar, G. A. and Zhao Junfeng(赵俊峰), The ordering structure on Banach spaces, *Pacific J. of Math.*, **116**, 2(1985), 255—263.

[4] Lindenstrauss, J. and Tzafriri, L., Classical Banach spaces I, Ergehniss der Mathematik und ihrr Grenzgebiete 92, Springer-Verlag, 1977.

[5] Singer, I., Bases in Banach spaces I, 1970.

[6] Zhao, Junfeng(赵俊峰), The vector valued long James type Banach spaces $J(\eta, l_p)$ $(1\leqslant p<\infty)$, *J. Sys. Sci. & Math. Scis.*, **5**: 1 (1985), 52—62.

[7] Zhao Junfeng (赵俊峰), The transfinite basis of the bidual space of the long James space, *Acta Mathemetica scientia*, **5**: 3(1985), 295—301.

[8] Zhao Junfeng(赵俊峰), The ordering structure on Banach Spaces II, *Journal of Wuhan University (Natural Science Edition)*, 3(1985), 11—18.